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VOLUME-TEMPERATURE-PRESSURE RELATIONS FOR SEVERAL NON-VOLATILE LIQUIDS.

BY P. W. BRIDGMAN.

INVESTIGATIONS ON LIGHT AND HEAT MADE AND PUBLISHED WITH AID FROM THE RUMFORD FUND.

VOLUME-TEMPERATURE-PRESSURE RELATIONS FOR SEVERAL NON-VOLATILE LIQUIDS.

By P. W. Bridgman.

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INTRODUCTION.

The material of this paper constitutes an extension to new liquids of measurements of volume as a function of pressure and temperature by the sylphon method recently developed and described.[1] The apparatus, and methods of experiment and calculation were the same as in the first paper and need not be described again in detail.

Most of the liquids of this paper were chosen particularly with a view to finding organic liquids which should be as incompressible as possible. One of the interesting results which had appeared in the previous paper on summarizing all available data for the volume of organic liquids at high pressures was the exceptionally low compressibility of glycerine, which stands in a class by itself with a compressibility about one half as great as that of water, which in its turn is of the order of only half as compressible as the majority of all other liquids hitherto measured. The problem that suggested itself, therefore, was to find if possible other organic liquids as incompressible as glycerine. This was interesting on its own account as throwing some light on the reasons for low compressibility in organic liquids, and also had an incidental interest in that the sylphon method of measuring compressibilities could be extended to solid substances if liquids could be found of high incompressibility with which to transmit pressure to the solids in the sylphon. Pursuant to this program, four liquids were investigated in the same chemical series as glycerine, and in addition, glycerine itself was remeasured, the previous measurements having been made at only a single temperature. These measurements showed nothing less compressible than glycerine; the measurements made evident in what direction in the series incompressibility increases, and also that glycerine is the most incompressible liquid available in the series, the next members in the direction of increasing incompressibility being solid rather than liquid. Another clue was therefore followed in the further search for incompressibility. Low compressibility might reasonably be associated with low thermal expansion, and this in its turn is known to be associated with low vapor pressure. The Eastman Kodak Co., in their search for liquids suitable for diffusion pumps, has recently issued a list of organic liquids with vapor pressures less than that of mercury. Half a dozen of these liquids, of widely varying chemical composition, were selected for measurement. These all proved to be much more incompressible than the ordinary organic liquid, but nothing was found superior to glycerine.

In addition to the eleven liquids just described and chosen with a view to high incompressibility, three other liquids were investigated for various reasons. Eugenol was measured because of the interesting effects found by W. E. Danforth, Jr.,[2] in measurements of the effect of pressure on its dielectric constant, which demanded a knowledge of volume as a function of pressure to bring out their full significance. Iso-octane which has recently become available through the activities of one of the large commercial research laboratories, was measured in order to supplement the measurements of the preceding paper on normal octane. Isoprene was measured in order to supplement the studies previously made by Professor J. B. Conant, C. O. Tongberg, and W. R. Peterson[3] on the effect of pressure in causing it to polymerize to rubber.

DETAILS OF EXPERIMENT OR CALCULATION.

The measurements given in the following differ in only two important respects from those in the preceding paper. In the first place, a knowledge of the density of the liquid at 0° C at atmospheric pressure is necessary in order to reduce the sylphon readings to fractional changes of volume. The necessary data have been previously determined in only a few cases, so that special measurements had to be made. These usually consisted of a determination of the density at room temperature at atmospheric pressure by weighing a mass of copper immersed in the liquid. To reduce this density to 0° the thermal expansion is also necessary. This was not directly measured at atmospheric pressure, but was found by an extrapolation of the measurements under pressure of the volume at 0° and 50°. At 0° the volume was measured in the sylphon as a function of pressure down to atmospheric pressure, but at 50° the lowest pressure was usually about 200 kg/cm^2. The thermal expansions between 0° and 50° at 200, 400, 600, 800, and 1000 kg were obtained from the sylphon readings, and extrapolated back to atmospheric pressure. This could be done with very little uncertainty because the curvature was slight, and even sometimes the thermal expansion was sensibly a linear function of pressure over this range. The density at 0° was then calculated from the observed density at 25° at atmospheric pressure with the value just obtained for the thermal expansion, assuming that at atmospheric pressure the thermal expansion is linear in temperature between 0° and 50°. This again is not quite exact, but the error so introduced cannot be large.

In the following the necessary data are given so that in the future correction can be made for the various effects just discussed if it should prove necessary. The tables of volume given in the following ostensibly contain the relative volumes in terms of the volume at 0° C and atmospheric pressure as unity. The tables were actually constructed, however, in the first place by measuring the volume decrements of a certain weight of liquid, reducing these volume decrements by division to values corresponding to the weight of liquid which by calculation would occupy 1 cm^3 at 0°C at atmospheric pressure, and then finding the actual volumes by subtracting these volume decrements from 1.0000.

The way in which corrections should be applied in case of future improvements in the thermal expansion at atmospheric pressure may be shown by a detailed example. Consider methyl oleate. The decrements for this are stated to be for 0.8950 gm, which means that the density at 0° C at atmospheric pressure, D_0, was taken to be 0.8950. The measured density at atmospheric pressure at 26.0° is stated to be 0.8768. The extrapolated thermal expansion must therefore have been such as to increase the density from 0.8768 at 26° to 0.8950 at 0°. The mean thermal expansion between 0° and 26° was therefore $\left(\frac{1}{.8768} - \frac{1}{.8950}\right)\frac{1}{26} = \frac{1.14051 - 1.11732}{26} = \frac{.02319}{26}$ $= .000893$. Suppose now that future experiment shows that a better value for the mean thermal expansion in this range is 0.00095. Then $\frac{1}{D_0}[1 + 26 \times .00095] = \frac{1}{.8768}$. (The density at 26° must not be adjusted even if future measurements on methyl oleate give a better value, because the error of measurement of the density was here low enough to give valid values, and future disagreement must be ascribed to differences in the purity of the liquid.) Solving the last equation for D_0 gives $D_0 = .8768 \times 1.02470 = .8984$. The Table should therefore have been constructed for 0.8984 gm rather than 0.8950 gm. The volume decrements in the table must therefore be multiplied by $\frac{.8984}{.8950}$. This would give, for example, a volume at 0° and 1500 kg of $\left[1.0000 - \frac{.8984}{.8950} \times .0638\right] = 1.0000 - .0640 = .9360$. Again, to get the correct volume at 95° and 1500 kg the volume difference in the table between 95° and 0° at 1500 kg (i. e. $0.9886 - 0.9362 = 0.0524$) is to be multiplied by $\frac{.8984}{.8950}$ $(= 0.0526)$, and this added to the corrected volume at 0° and 1500 kg, giving for the final volume $0.9360 + 0.0526 = 0.9886$, accidentally the same as the original volume. The volume differences in the tables were constructed as far as possible on the isotherms at 0°; at pressures higher than the maximum pressure allowed at 0°, they were constructed from the maximum pressure of the 0° isotherm, which in the case of methyl oleate is 1500 kg, and the appropriate temperature as a fiducial point. Thus in the example above, the corrected volume at 95° and 10000 kg would be $.9886 - [.9886 - .8373]\frac{.8984}{.8950} = .8367$, instead of .8373.

In a second respect the experimental procedure was different from that in the preceding paper. The maximum pressures attainable are in many cases limited by the freezing of the liquid. In the previous paper there was often sufficient data already available to allow some guess to be made at what the freezing pressures might be, but so few of the physical properties of the liquids of this paper have been tabulated that a direct determination of the freezing pressures was undertaken for some of the liquids. This usually took the form of a freezing point determination at atmospheric pressure by the method of temperature arrest, the substance being first solidified at liquid air temperature, and then the rise of temperature, as given by a thermocouple imbedded in the substance, observed as a function of time, melting being indicated in the conventional way by a flat place in the curve. The freezing under pressure was usually determined at room temperature. This was done as simply as possible. A cylinder with a hole 5/16 of an inch, 0.79 cm, in diameter was filled with the liquid, the manganin pressure gauge being directly immersed in the liquid. The liquids were in all cases good enough insulators so that this could be done with no appreciable amount of short circuiting. Pressure was then increased by pushing in the piston, and the piston position was plotted against the pressure as given by the manganin gauge. Freezing was indicated by a flat place in the curve. After the substance had been frozen by increasing pressure, readings were usually repeated with decreasing pressure, in this way getting a better value for the freezing pressure without error from subcooling. The two points, at atmospheric pressure and at room temperature, were usually good enough to give a sufficient indication of the range of pressure at any temperature that might safely be applied without runnuing into the freezing curve.

Detailed Presentation of Data.

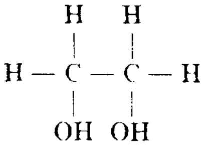
Ethylene Glycol.  $H-\overset{H}{\underset{OH}{C}}-\overset{H}{\underset{OH}{C}}-H$ This material was obtained from the Eastman Kodak Co., and was used without further purification. In order to avoid the possibility of freezing, runs were made first at 50° and 95° to 12000 kg, starting the run at 50° at atmospheric pressure. These runs were accomplished without incident, the points with increasing and decreasing pressure lying on the same smooth curve. After the run at 95° it was necessary to take the apparatus apart to renew some of the insulation, and on reassembling the apparatus and starting the run at 0° it was evident that some permanent distortion had been introduced into the sylphon, probably by releasing the pressure to atmospheric before the sylphon had sufficiently cooled. Furthermore, the run at 0° encountered stationary readings, indicating freezing, at about 6000 kg. This, however, was not the true freezing pressure, but was too high because of subcooling; in fact pressure was released as far as 3300 kg without signs of melting. So great a subcooling was not anticipated, and the apparatus was again dismantled for inspection, with the expectation of finding the sylphon ruined. No difficulty was obvious, however, and weighing showed that there had been no leak. The apparatus was then again reassembled, and the 0° readings checked. The run was then concluded with several readings at 50° and 95° and then back to 0°, in order to establish the temperature expansion, which could not be obtained between 50° and 95° because of the distortion on the first disassembly. This experiment provides a rather extreme example of the amount of abuse a sylphon will sometimes support without being put out of commission. The average deviation from a smooth curve of a single one of the 70 readings was 0.26% of the maximum effect, treating the readings after the distortion as belonging to a different series from those before it.

TABLE I.

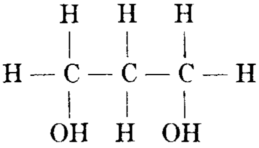
Relative Volumes of Ethylene Glycol.

Pressure kg/cm²	Temperature 0°	50°	95°
0	1.0000	1.0300	
500	.9841	1.0133	
1,000	.9703	.9979	1.0278
1,500	.9589	.9846	1.0091
2,000	.9486	.9724	.9938
3,000	.9304	.9519	.9708
4,000	.9154	.9343	.9515
5,000	.9032	.9197	.9352
6,000		.9073	.9215
7,000		.8959	.9089
8,000		.8851	.8982
9,000		.8753	.8883
10,000		.8664	.8789
11,000		.8585	.8706
12,000		.8503	.8627

The relative volumes are given in Table I. The volume decrements in the table are for 1.1263 gm. The density, as directly determined from the loss of weight of an immersed copper weight was 1.1137 at 20.25°. In reducing this to the density at 0°, 1.1263, the total thermal expansion of 1 gm of the liquid between 0° and 50° at atmospheric pressure was taken as 0.0267 cm^3, this being the value given directly by the sylphon measurements.

It is not at all improbable that the melting pressures at 50° and 95° may be below the maximum of 12000 kg reached in these measurements, the substance having remained liquid by subcooling. A similar effect is known to exist in glycerine, in which the subcooling is very marked, and it is not unreasonable to expect the same effect here.

Tri-methylene Glycol.



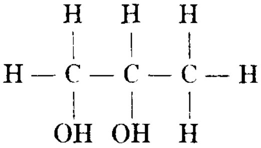
$$\begin{array}{ccccccc} & \mathrm{H} & & \mathrm{H} & & \mathrm{H} & \\ & | & & | & & | & \\ \mathrm{H}\,- & \mathrm{C} & - & \mathrm{C} & - & \mathrm{C} & -\,\mathrm{H} \\ & | & & | & & | & \\ & \mathrm{OH} & & \mathrm{H} & & \mathrm{OH} & \end{array}$$

This was obtained from the Eastman Kodak Co., and was most kindly further purified by fractional distillation by Professor Conant, who did not find, however, any marked change in its boiling point introduced by the redistillation.

Runs were made at 0° to a maximum of 5000 kg, and then at 50° and 95° to 12000 kg, with no sign of freezing at any temperature. There was an unexplained break between 0° and 50°, not due to the sylphon, which was taken down and checked between the two runs; the break was probably due to an inadvertent and unrecorded change in the electrical constants of the potentiometer. The return check readings were made at 50° and 0° as usual, and the thermal expansion between 0° and 50° taken from the check readings. The average departure from smooth curves of a single one of the 61 readings was 0.28% of the maximum effect.

The relative volumes are shown in Table II. The volume decrements listed in the table are for 1.063 gm. The density at 20.2° at atmospheric pressure determined by buoyancy measurements was 1.0528. This substance is listed in International Critical Tables, for which the density at 20° is given as 1.053, very good agreement.

Propylene Glycol.



$$\begin{array}{ccccccc} & \mathrm{H} & & \mathrm{H} & & \mathrm{H} & \\ & | & & | & & | & \\ \mathrm{H}\,- & \mathrm{C} & - & \mathrm{C} & - & \mathrm{C} & -\,\mathrm{H} \\ & | & & | & & | & \\ & \mathrm{OH} & & \mathrm{OH} & & \mathrm{H} & \end{array}$$

This material was obtained from the Eastman Kodak Co., and was used without further purification. Readings were made first at 0° to a maximum of 4200 kg, then at 50° to 12000, at 95° to 12000, a return check reading at 50° at 900 kg, and then the run at 0° was repeated to a maximum of 12000 and back, with perfect agreement with the first run in the common interval, and no indication of freezing at any temperature. The average departure from smooth curves of a single one of the 73 readings was 0.13% of the maximum effect.

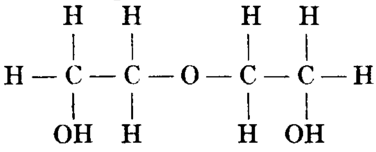
TABLE II.

Relative Volumes of Tri-methylene Glycol.

Pressure		Temperature	
kg/cm^2	0°	50°	95°
0	1.0000	1.0274	
500	.9830	1.0082	
1,000	.9671	.9929	1.0165
1,500	.9547	.9799	1.0016
2,000	.9432	.9678	.9879
3,000	.9237	.9472	.9640
4,000	.9072	.9300	.9450
5,000	.8922	.9150	.9292
6,000		.9020	.9154
7,000		.8898	.9030
8,000		.8792	.8919
9,000		.8707	.8817
10,000		.8630	.8729
11,000		.8553	.8655
12,000		.8474	.8582

The relative volumes are given in Table III; the volume decrements are for 1.0486 gm. The measured density at 20° was 1.0340. International Critical Tables gives 1.038 at 23°, not especially good agreement. The assumed value for the density at 0°, 1.0486, was obtained by taking the total expansion at atmospheric pressure between 0° and 50° to be 0.0325 cm^3, a value obtained by slight extrapolation of the sylphon readings.

Diethylene Glycol.



$$\begin{array}{ccccccccccccc} & & H & & H & & & & H & & H & & \\ & & \mid & & \mid & & & & \mid & & \mid & & \\ H & — & C & — & C & — & O & — & C & — & C & — & H \\ & & \mid & & \mid & & & & \mid & & \mid & & \\ & & OH & & H & & & & H & & OH & & \end{array}$$

This was obtained from the Eastman Kodak Co., and was used without further purification. The first run was at 0° to a maximum of 5800 kg. At this pressure a stationary reading was obtained, indicating freezing. Temperature was raised at once to 50° in order to avoid possible damage to the sylphon, but in spite of this precaution, some permanent set was produced. At 50°, pressure was lowered to 300 kg, increased back to 12000, and then lowered back to 400; all the points at 50° lay on the same smooth curve, indicating no further damage to the sylphon. The 95° run was next made, then the check reading at 50°, with perfect agreement, and finally a new curve was obtained at 0°, up to 4500 and back. The average deviation from smooth curves of a single one of the 62 readings was 0.25% of the maximum effect.

TABLE III.

RELATIVE VOLUMES OF PROPYLENE GLYCOL.

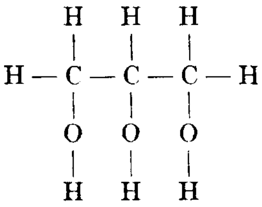
Pressure kg/cm^2	Temperature 0°	50°	95°
0	1.0000		
500	.9819	1.0129	
1,000	.9664	.9947	1.0201
1,500	.9540	.9794	1.0027
2,000	.9432	.9659	.9877
3,000	.9237	.9454	.9637
4,000	.9070	.9266	.9430
5,000	.8935	.9120	.9257
6,000	.8809	.8989	.9110
7,000	.8679	.8864	.8985
8,000	.8569	.8750	.8867
9,000	.8472	.8652	.8763
10,000	.8380	.8563	.8672
11,000	.8294	.8477	.8589
12,000	.8219	.8400	.8514

The relative volumes are given in Table IV. The density at 0° at atmospheric pressure was taken as 1.1298, which is therefore the number of grams to which the volume decrements of the table refer. The directly measured density at 25° was 1.1122. This does not agree particularly well with the value of International Critical Tables, which is 1.132 at 20°. In calculating the density at 0°, the total expansion of one gram between 0° and 25° was taken to be 0.0142 cm^3, a value calculated, as explained, by a slight extrapolation of the sylphon readings at 50°.

TABLE IV.

RELATIVE VOLUMES OF DIETHYLENE GLYCOL.

Pressure	Temperature		
kg/cm²	0°	50°	95°
0	1.0000	1.0320	
500	.9845	1.0135	
1,000	.9710	.9987	1.0223
1,500	.9604	.9861	1.0077
2,000	.9492	.9729	.9833
3,000	.9320	.9523	.9705
4,000	.9174	.9352	.9522
5,000	.9047	.9207	.9368
6,000		.9075	.9235
7,000		.8963	.9113
8,000		.8859	.9005
9,000		.8762	.8902
10,000		.8677	.8810
11,000		.8608	.8728
12,000		.8542	.8648

Glycerine. 
$\begin{array}{ccccccc} & & \mathrm{H} & & \mathrm{H} & & \mathrm{H} & \\ & & | & & | & & | & \\ \mathrm{H} & - & \mathrm{C} & - & \mathrm{C} & - & \mathrm{C} & - & \mathrm{H} \\ & & | & & | & & | & \\ & & \mathrm{O} & & \mathrm{O} & & \mathrm{O} & \\ & & | & & | & & | & \\ & & \mathrm{H} & & \mathrm{H} & & \mathrm{H} & \end{array}$
This was Kahlbaum's purest grade, "Zur Analyse," used without further purification. It was, however, doubtless, somewhat impure with water, as shown by the low value of the density, discussed in the next paragraph. This material had been previously measured, but only at a single temperature, and with the free piston piezometer, which is not capable of such great accuracy as the sylphon. Readings were first made at 0° to a maximum of 5000, then at 50° to 12000 and back, at 95° to 12000 and back, and then the run at 0° was repeated, this time to 12000 and back, with a perfect check on the common portions of the two 0° curves. Particular care was taken to change the pressure slowly, because the viscosity of gylcerine is known to increase greatly under pressure. The average deviation from smooth curves of a single one of the 65 readings was 0.31% of the maximum effect. Two thirds of the total irregularity was contributed by the 13 points of the 0° run above 6000 kg, and is without doubt the expected effect of viscosity.

The relative volumes are given in Table V. The density at atmospheric pressure at 0° was taken as 1.2385, and the volume decrements of the table are for this number of grams. The measured density at atmospheric pressure at 24.6° was 1.2226. International Critical Tables gives at 20° 1.260. The low value for this glycerine is to be explained by the presence of water. That this was the case was checked later by Mr. W. E. Danforth, Jr. who tried to measure the effect of pressure on the dielectric constant of this same material without success, because of its high conductivity, which means water. He was able to remove the water sufficiently for his purposes by chemical means; it might pay at some future time to redetermine the compressibility of entirely water-free glycerine. The effect of the water cannot be large, however, as may be seen by a comparison of the values of Table V with those given in the preceding paper for a 50% water-glycerine mixture.

TABLE V.

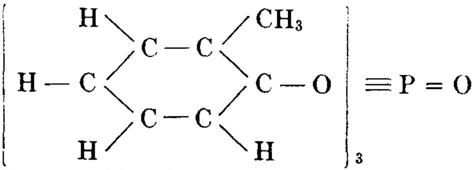
RELATIVE VOLUMES OF GLYCERINE.

Pressure kg/cm^2	Temperature		
	0°	50°	95°
0	1.0000	1.0266	
500	.9900	1.0136	
1,000	.9806	1.0025	1.0240
1,500	.9721	.9930	1.0125
2,000	.9641	.9843	1.0024
3,000	.9501	.9688	.9853
4,000	.9373	.9548	.9700
5,000	.9264	.9423	.9565
6,000	.9157	.9310	.9447
7,000	.9057	.9211	.9342
8,000	.8958	.9121	.9244
9,000	.8867	.9036	.9152
10,000	.8783	.8955	.9070
11,000	.8712	.8879	.8994
12,000	.8648	.8800	.8925

The density at 0° at atmospheric pressure was calculated from the measured density at 24.6° by taking 0.0215 cm^3 for the total expansion between 0° and 50° of one gram, a value obtained by extrapolation of the sylphon readings at 50°.

Comparison with the previous values of the compressibility of glycerine from the same source gives rather good agreement within the limits of error of the previous method. The former values for the relative volumes at 30° at 0, 2000, 4000, 6000, 8000, 10000, and 12000 kg respectively are: 1.000, 0.958, 0.932, 0.911, 0.893, 0.879, and 0.866. The values obtained by linear interpolation between 0° and 50° of the present measurements are: 1.000, 0.960, 0.933, 0.910, 0.891, 0.875, and 0.860. Compared with the sylphon, the former free piston seems to give volume changes which are too large at low pressures and too small at high pressures.

Tri-o-cresyl-Phosphate.



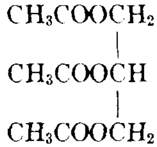
This was obtained from the Eastman Kodak Co. and was used without further purification. The first run at 0° indicated freezing somewhere between 2200 and 3000 kg. At the latter pressure temperature was raised immediately to 50°, and the run at 50° made to a maximum of 4900 kg, and then the run at 95° to a maximum of 6200. The return check reading at 0° now showed that a permanent distortion had taken place at the first freezing; the run at 0° was repeated to a maximum of 1800, giving a curve of the same shape as the first curve, but displaced by a constant amount. Points on the 50° and the 95° curve were then repeated, giving agreement with the first determinations. The average deviation from smooth curves of a single one of the 36 readings was 0.54% of the maximum effect. The greater irregularity than usual is to be explained in part by the much narrower range than usual.

TABLE VI.

RELATIVE VOLUMES OF TRI-O-CRESYL PHOSPHATE.

Pressure kg/cm²	Temperature 0°	50°	95°
0	1.0000	1.0302	
500	.9831	1.0109	
1,000	.9687	.9938	1.0163
1,500	.9573	.9790	.9994
2,000	.9478	.9669	.9858
3,000		.9468	.9630
4,000		.9308	.9448
5,000		.9180	.9300
6,000			.9166

The relative volumes are given in Table VI. The density at 0° at atmospheric pressure was taken to be 1.1987, which means that the volume decrements listed in the table are for this number of grams. The directly measured density was 1.1803 at 25°. The volume expansion of one gram at atmospheric pressure between 0° and 50° was taken to be 0.02528, obtained by extrapolation of the sylphon readings.



$$\begin{array}{l} \mathrm{CH_3COOCH_2} \\ \quad\quad\quad\; | \\ \mathrm{CH_3COOCH} \\ \quad\quad\quad\; | \\ \mathrm{CH_3COOCH_2} \end{array}$$

Tri-acetin. The material was obtained from the Eastman Kodak Co. and used without further purification. Runs were made at 0° to a maximum of 4900 kg, at 50° to 10000, and at 95° to 12000, with a return check reading at 0° with good agreement. The maxima at 0° and 50° were as high as it was thought safe to go in view of a preliminary freezing point examination in the special small apparatus; the indications of freezing were not very sharp, however, and it should not be assumed without further examination that the limits mentioned above are actually very close to the freezing points. The average departure from smooth curves of a single one of the 44 readings was 0.13%.

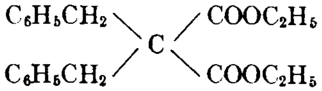
TABLE VII.

Relative Volumes of Tri-acetin.

Pressure kg/cm²	Temperature 0°	50°	95°
0	1.0000	1.0441	
500	.9793	1.0198	
1,000	.9627	.9981	1.0289
1,500	.9492	.9805	1.0071
2,000	.9374	.9653	.9893
3,000	.9177	.9412	.9628
4,000	.9012	.9225	.9432
5,000	.8861	.9068	.9258
6,000		.8932	.9098
7,000		.8807	.8956
8,000		.8694	.8831
9,000		.8591	.8721
10,000		.8498	.8624
11,000			.8533
12,000			.8445

The relative volumes are given in Table VII. The density at 0° at atmospheric pressure was taken as 1.2157, so that the volume decrements in the table are for this number of grams. The directly measured density at 26.9° was 1.1613; in reducing this to the density at 0° the expansion of one gram between 0° and 26.9° was taken to be 0.0194 cm^3, obtained by extrapolation of the sylphon readings. This substance is unusual in that the thermal expansion between 0° and 50° is linear in the pressure up to 1100 kg/cm^2 within the limits of experimental error.

Ethyl Dibenzyl Malonate.



$$\begin{matrix} C_6H_5CH_2 & \searrow & & \nearrow & COOC_2H_5 \\ & & C & & \\ C_6H_5CH_2 & \nearrow & & \searrow & COOC_2H_5 \end{matrix}$$

This was obtained from the Eastman Kodak Co. and used without further purification. A preliminary freezing exploration indicated that the freezing pressure was very low, and the limits of the measurements were set by this exploration. The first run was made at 50° to a maximum of 2100 kg, then at 95° to 5000, at 50° again to a maximum of 3000, at 0° to 1000, with a final check reading at 50° in perfect agreement. The average departure from smooth curves of a single one of the 31 readings was 0.48% of the maximum effect. The irregularity, which is somewhat greater than usual, is to be entirely explained by the small range, the average absolute irregularity in the readings being no greater than usual.

TABLE VIII.

Relative Volumes of Ethyl Dibenzyl Malonate.

Pressure kg/cm^2	Temperature 0°	50°	95°
0	1.0000	1.0361	
500	.9775	1.0111	
1,000	.9620	.9912	1.0172
1,500		.9748	.9979
2,000		.9616	.9823
3,000		.9412	.9581
4,000			.9381
5,000			.9179

The relative volumes are shown in Table VIII. The density at 0° at atmospheric pressure was taken to be 1.1143, so that the volume decrements in the table refer to this number of grams. The directly measured density at 25.9° was 1.0943; the density at 0° was calculated from this, using for the total expansion of one gram between 0° and

25.9° 0.0167 cm^3, a value obtained by extrapolation of the sylphon readings. The thermal expansion between 0° and 50° is again linear in the pressure up to 1000 kg.

Methyl Oleate. $CH_3(CH_2)_7CH = CH(CH_2)_7COOCH_3$. This was obtained from the Eastman Kodak Co. and used without further purification. A preliminary freezing point determination gave an unusually sharp freezing point between 2900 and 3300 kg at 24°; this may be accepted with some confidence as the proper freezing coordinate. In the compressibility measurements the range was fixed by this information, together with the freezing point at atmospheric pressure listed in tables of physical constants. Two sets of runs were made; the first was terminated by damage to the sylphon before the return check readings could be made. The second set-up with a new sylphon gave first a run at 95° to 10000 kg, one at 50° to 5900, one at 0° to 1800, and finally a return check reading at 95° with perfect agreement. The average deviation from smooth curves of a single one of the 30 readings, discarding a single one where there must have been a blunder in recording, was 0.20%.

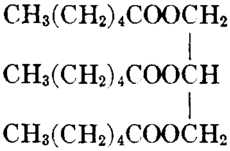
TABLE IX.

RELATIVE VOLUMES OF METHYL OLEATE.

Pressure kg/cm^2	Temperature 0°	50°	95°
0	1.0000	1.0401	
500	.9740	1.0103	
1,000	.9532	.9859	1.0110
1,500	.9362	.9669	.9886
2,000		.9509	.9711
3,000		.9251	.9421
4,000		.9044	.9159
5,000		.8874	.9004
6,000		.8716	.8845
7,000			.8703
8,000			.8583
9,000			.8475
10,000			.8373

The relative volumes are given in Table IX. The density at 0° at atmospheric pressure was taken as 0.8950, so that the volume decrements of the table are for this number of grams. The directly measured density at atmospheric pressure at 26.0° was 0.8768. In calculating the density at 0°, the total expansion of one gram between 0° and 26° was taken to be 0.0234 cm^3, as given by extrapolation of the sylphon readings.

Tri-caproin.



$$\begin{array}{r} \mathrm{CH_3(CH_2)_4COOCH_2} \\ | \\ \mathrm{CH_3(CH_2)_4COOCH} \\ | \\ \mathrm{CH_3(CH_2)_4COOCH_2} \end{array}$$

This was obtained from the Eastman Kodak Co. and used without further purification. The first run at 0° was to a maximum of 7500 kg, then at 50° to 11000, at 95° to 12000, and finally a check reading at 0° in perfect agreement. The average departure from a smooth curve of a single one of the 49 readings was 0.16%.

TABLE X.

RELATIVE VOLUMES OF TRI-CAPROIN.

Pressure kg/cm^2	Temperature		
	0°	50°	95°
0	1.0000	1.0412	
500	.9741	1.0095	
1,000	.9547	.9850	1.0120
1,500	.9388	.9656	.9905
2,000	.9251	.9490	.9723
3,000	.9025	.9229	.9431
4,000	.8846	.9031	.9208
5,000	.8698	.8862	.9024
6,000	.8565	.8714	.8867
7,000	.8442	.8584	.8726
8,000	.8333	.8471	.8601
9,000		.8373	.8493
10,000		.8285	.8395
11,000		.8207	.8309
12,000			.8230

The relative volumes are given in Table X. The density at 0° at atmospheric pressure was taken to be 0.9970 so that the volume decrements of the table are for this number of grams. The measured density at atmospheric pressure at 27.0° was 0.9754. In computing the density at 0° the thermal expansion at atmospheric pressure of one gram between 0° and 50° was taken to be 0.0413 cm^3, as given by extrapolation of the sylphon readings.

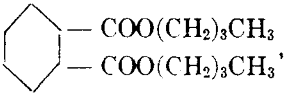
Normal-butyl Phthalate.  $C_{16}H_{22}O_4$ This was obtained from the Eastman Kodak Co.; it had been specially freed from all volatile impurities so as to be ready for immediate use in diffusion pumps, and it was used directly in the sylphon with no attempt at further purification. A first attempt with this failed because of the development of a leak in the sylphon. The first set-up established, however, the safe pressure limits. The second set-up gave first a run at 0° to 8200 kg, then at 50° to 12000, at 95° to 12000, and then return check readings at both 0° and 95° again with good agreement. The average departure from smooth curves of a single one of the 48 readings was 0.16%.

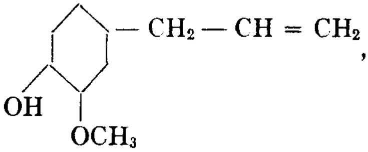
TABLE XI.

RELATIVE VOLUMES OF NORMAL-BUTYL PHTHALATE.

Pressure kg/cm²	Temperature 0°	50°	95°
0	1.0000	1.0369	
500	.9768	1.0095	
1,000	.9575	.9872	1.0172
1,500	.9419	.9690	.9928
2,000	.9297	.9540	.9759
3,000	.9096	.9301	.9489
4,000	.8931	.9125	.9277
5,000	.8779	.8964	.9099
6,000	.8649	.8811	.8945
7,000	.8534	.8682	.8809
8,000	.8447	.8574	.8690
9,000		.8479	.8586
10,000		.8392	.8494
11,000		.8313	.8409
12,000		.8241	.8340

The relative volumes are given in Table XI. The density at 0° at atmospheric pressure was taken to be 1.0643, so that the volume decrements of the table are for this number of grams. The measured density at atmospheric pressure at 25.0° was 1.0453. In reducing this to the density at 0° the thermal expansion at atmospheric pressure of one gram between 0° and 25.0° was taken to be 0.0173 cm³, as given by extrapolation of the sylphon readings.

Eugenol.



, $C_{10}O_2H_{12}$ This material was obtained from the Eastman Kodak Co. and used without further purification. It was colorless in appearance, contrasted with the yellow color of the material which had been previously used in measurements of the viscosity under pressure.[4] As already mentioned, the compressibility of this was measured to give greater point to the measurements by W. E. Danforth, Jr., of the effect of pressure on dielectric constant, who in turn was interested in it because of the previously discovered enormously large increase of viscosity produced by pressure. For the purpose in mind, it was sufficient to determine the compressibility at a single temperature, 0°, because this was the temperature at which the interesting dielectric effects had been found. The maximum pressure at this temperature was only 4600 kg, this being as high as it was thought safe to go because of the greatly increased viscosity. As it was, the sylphon was damaged, the points with decreasing pressure lying off the curve, and the zero was greatly displaced. On opening the apparatus there was a distinct smell of eugenol, showing that the sylphon had leaked. This one run gave sufficient data for the purpose in hand, however, and no attempt was made to repeat the measurements. This run gave 8 points with increasing pressure, which lay remarkably well on a smooth curve, the average departure of a single reading being only 0.03% of the maximum effect.

TABLE XII.

RELATIVE VOLUMES OF EUGENOL.

Pressure kg/cm²	Temperature 0°
0	1.0000
500	.9801
1,000	.9639
1,500	.9501
2,000	.9382
3,000	.9187
4,000	.9028
5,000	.8896

The relative volumes are given in Table XII. The density at atmospheric pressure at 25.9° was measured as 1.0515. No sylphon measurements were made from which the thermal expansion could be calculated, and there seem to be no values recorded in the literature. For the construction of the table the density at atmospheric pressure at 0° was assumed to be 1.065, using for the thermal expansion a mean from other organic compounds.

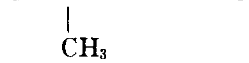
Iso-octane (2, 2, 4 tri-methyl pentane). This material I owe to the courtesy of Mr. A. E. Becker of the Standard Oil Development Co. The material had been refined in the research laboratory of this company in connection with experiments on motor fuels. They give for it the following physical constants. Boiling point: 99.3° within a range of 0.3°; specific gravity 0.6917 at 20° referred to water at 4°; freezing point — 107° C; aniline point 84°. It was used in the sylphon without further purification. A preliminary freezing point exploration indicated sharp freezing, evidence of high purity. Runs were made with a single filling of the apparatus, to 5600 kg at 0°, 8700 at 50°, 10200 at 95°, with a return check reading at 0° with very good agreement. The pressure limits of these runs were set by the preliminary freezing point examination. The average departure from smooth curves of a single one of the 40 readings was 0.075% of the maximum effect.

TABLE XIII.

Relative Volumes of Iso-octane.

Pressure		Temperature	
kg/cm²	0°	50°	95°
0	1.0000	1.0607	
500	.9529	.9949	
1,000	.9221	.9560	.9851
1,500	.9001	.9286	.9531
2,000	.8825	.9077	.9292
3,000	.8558	.8764	.8946
4,000	.8356	.8529	.8684
5,000	.8161	.8318	.8453
6,000	.8037	.8176	.8307
7,000		.8038	.8161
8,000		.7911	.8024
9,000		.7797	.7901
10,000			.7795

The relative volumes are given in Table XIII. The density at 0° at atmospheric pressure was taken to be 0.7086, and the volume decrements given in the table are for this number of grams. The directly measured density at atmospheric pressure was 0.6897 at 22.5°. In calculating the density at 0° the thermal expansion at atmospheric pressure between 0° and 22.5° was taken as 0.0385 cm^3 per gm, obtained by extrapolation of the sylphon readings.



$$CH_2 = \underset{\displaystyle CH_3}{\underset{|}{C}} - CH = CH_2$$

Isoprene. This material was synthesized at the Chemical Laboratory of Harvard University under the direction of Professor J. B. Conant. The effect of pressure in polymerizing this substance to a rubber-like material had already been extensively studied by Professor Conant[3], and it was of interest to determine the compressibility in connection with these studies. The material was used only a short time after it had been synthesized, and during this time as well as during the process of filling the sylphon it was maintained continuously at 0° in order to avoid danger of polymerization. The compressibility was attempted only at 0°, because at higher temperatures polymerization is known to proceed much more rapidly. No appreciable amount of polymerization was produced by the compressibility run, as shown by the agreement of the points obtained with increasing and decreasing pressure. The pressure range at 0° was safely taken to be the full 12000 kg, isoprene being a very volatile substance of low freezing point and low viscosity. The average departure from a smooth curve of a single one of the 18 readings was 0.16% of the maximum effect.

The relative volumes are given in Table XIV. The density at 0° at atmospheric pressure was taken to be 0.691. For this figure I am indebted to Dr. W. R. Peterson, who made a direct determination in the laboratory of Professor Conant.

DISCUSSION.

This discussion will be mainly occupied with the attempt to correlate differences in the compressibility with differences in the structural formulas. It is worth mentioning, however, that certain general features of the p-v-t- relations of liquids already emphasized in two preceding papers[5] continue to hold for these liquids also. There is in the first place the reversal in the sign of $\left(\frac{\partial^2 v}{\partial \tau^2}\right)_p$ at high pressures. At atmospheric and low pressures the sign of this is positive, but at pressures of the order of 3000 and over it had been found to become

TABLE XIV.

RELATIVE VOLUMES OF ISOPRENE.

Pressure kg/cm²	Temperature 0°
0	1.0000
500	.9524
1,000	.9208
1,500	.8973
2,000	.8781
3,000	.8478
4,000	.8252
5,000	.8070
6,000	.7911
7,000	.7776
8,000	.7658
9,000	.7551
10,000	.7450
11,000	.7357
12,000	.7272

negative, which means that at high pressures the thermal expansion of a liquid is less at high temperatures than at low temperatures at the same pressure. Inspection of the tables of volume of the new liquids of this paper will show that this continues to be true. A second general feature previously found was that $\left(\frac{\partial p}{\partial \tau}\right)_v$ is not a function of volume only, as had been assumed in various theoretical discussions by several authors. An application of the same method of analysis as that used in the preceding paper will show that for these new liquids also there are failures of the relation just as pronounced as found before.

Turning now to the relative behavior of the various compounds, the series terminating with glycerine is perhaps the most interesting. In Figure 1 are plotted the relative volume decrements at 50° of the five members of this series as a function of pressure, in terms of the respective volumes at 50° and atmospheric pressure as unity. 50° was chosen in order to allow comparison over a wider pressure range than would have been possible at 0°, where the range is often restricted by freezing. Included in the same diagram are the volumes of two of the alcohols and n-pentane for comparison. The most striking feature of the diagram is that the volume decrements of the four members of the series containing two OH groups in the molecule are very nearly alike.

The diagram also brings out in a very striking way that when an OH group is substituted into a hydrocarbon by far the most important factor in determining the compressibility is the number of OH's in

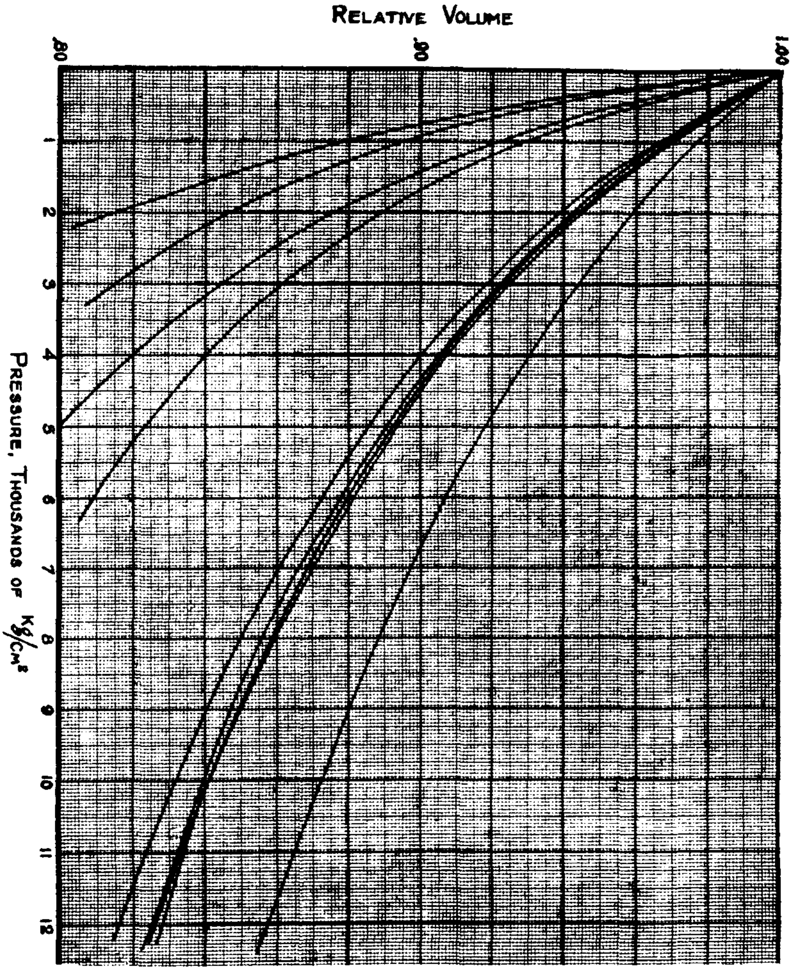


Figure 1. The relative volumes at 50° C of a number of liquids as a function of pressure. At 2000 kg/cm² the order of the curves, reading from the top down, is: glycerine, ethylene glycol, diethylene glycol, tri-methylene glycol, propylene glycol, amyl alcohol, methyl alcohol, n-hexane, and n-pentane.

the molecule, the compressibility decreasing as the number of OH's increases. This is shown in the diagram by the very marked tendency of the curves to fall into families, the unaltered hydrocarbons with no OH groups, typified by n-pentane, constituting the first family with the highest compressibility, the alcohols, which are hydrocarbon chains with one OH group, constituting the next family with lower compressibility, the four glycols of this paper with two OH groups, constituting the next family with still lower compressibility which is nearly the same for all members of the family, and finally glycerine, with three OH groups, and with the lowest compressibility of any organic liquid yet measured. It obviously suggests itself that in the search for organic compounds with low compressibility the natural course is to try those with a large number of OH groups in the molecule. Unfortunately, glycerine appears to be the last compound which is liquid under ordinary conditions, and even with it the liquidity is due only to its remarkable capacity for subcooling. It would be of interest, however, to study for its own sake the compressibility of solid compounds whose molecules contain more than three OH groups, for the same regularities which attach to the compressibility of the liquid phases will doubtless carry over to the solid phase. The study of solids is complicated, however, by the fact that for an exhaustive characterization of their behavior it will be necessary to measure the compressibility of single crystals in different directions.

Any detailed numerical study of the data presented in Figure 1 will encounter complications from temperature effects, the curves being in most cases drawn for the same temperature (50°) whereas doubtless a better method would have been to draw them for the same reduced temperature. This accounts for part of the effect of adding OH groups, for the addition of an OH group increases the critical temperature, so that 50° C becomes an increasingly smaller fraction of the critical temperature, and compressibility normally decreases on decreasing the reduced temperature. The same can be said for the behavior in a single family. On increasing the size of the molecule, keeping the number of OH groups constant, as, for example, in passing from methyl to amyl alcohol, the effect of increasing the length of the hydrocarbon chain is to raise the critical temperature. This is not by any means the entire effect, however, and the addition of the OH group does of itself markedly decrease the compressibility irrespective of the temperature effect, as shown, for example, by methyl alcohol and hexane, the critical temperature of the former being only 1% higher than that of the latter, while its compressibility is 16% less.

In seeking for an explanation of the effect of the OH group it is important to notice that the effect cannot be regarded as a specific contribution of the OH group only, without interaction with the rest of the molecule. If such were the case the compressibility of ethylene glycol [$C_2H_4(OH)_2$] should be practically the same as that of glycerine, the OH groups having .55 of the total mass of the molecule in the former compound, and .56 in the latter. Another example of the same sort is afforded by trimethylene and diethylene glycols. The OH groups constitute .46 of the weight of the molecule of the first and .32 of the second, yet the compressibilities are nearly the same. It is the number of OH groups in the molecule primarily rather than the fraction of the molecule occupied by OH groups which is determinative, although doubtless in very complicated compounds a single OH group may be swamped by the rest of the molecule. These facts suggest that in the molecule the OH groups interact with all the other parts of the molecule, consolidating them and decreasing the compressibility. Such effects are strongly suggested by wave mechanics.

A detailed comparison of the curves for the four glycols shows certain irregularities, of which the most prominent are the crossing of the ethylene and the diethylene glycol curves at 10000 kg, and the relatively large downward bulge in the trimethylene glycol curve between 6000 and 9000. Such irregularities are not at all unexpected in view of previous experience. A detailed explanation of them would be very complicated and doubtless beyond present day possibilities; I have suggested in a recent paper[6] that effects may be involved similar to the anomalies in the behavior of NH_4Cl and NH_4Br.

An interesting minor structural difference appears between propylene and trimethylene glycol, which differ only in the position of the OH groups; in the latter the two OH groups occupy the two ends of the chain, whereas in the former one OH occupies an end position and the other a mid position. The diagram shows that the propylene compound is distinctly more compressible.

Turning now to a comparison of the other substances, there are few structural similarities to suggest analogies of behavior. Of these, triacetin and tricaproin are the only ones with strictly similar structures. Both of these are in turn similar to glycerine, the difference being that in triacetin each OH group of glycerine is replaced by a CH_3COO group, and in tricaproin each OH group is replaced by a $CH_3(CH_2)_4COO$ group. One would expect similarities in the behavior of these three substances. This, however, is not the case. In the first place both triacetin and tricaproin are much more compressible than glycerine, and in fact their compressibility is in excess of the compressibility of the glycols by a quantity of the same order of magnitude as that by which the compressibility of the glycols is in excess of that of glycerine. This again emphasizes the abnormally low compressibility of the OH group. Furthermore, triacetin is noticeably less compressible than tricaproin, the difference being of the order of 15%. This is not what might at first be expected, the usual rule being for the compressibility to decrease as one proceeds in a family of chemical compounds from those with small to those with larger and more complicated molecules. The evident significance of this abnormal behavior is that the compressibility of the $(CH_2)_4$ group is greater than the average compressibility of the CH_3COO group, an effect for which we are already prepared from our discussion of the glycerine family, where it was suggested that the compressibility of the hydrocarbon chain is rather high. In general, the presence of an O atom in a molecule seems to decrease the compressibility.

N-butyl phthalate and eugenol are similar in that they are substitution products of the benzene ring. In the former two H's of benzene have been replaced by two $COO(CH_2)_3\,CH_3$ groups, $C_{10}H_{18}O_4$ altogether, and in eugenol three H's have been replaced by CH_3O, OH, and $-CH_2-CH=CH_2$, or $C_4H_8O_2$ altogether. Because of the somewhat larger percentage of O in the eugenol replacement group, and particularly because of the presence of the OH group, one would expect the compressibility of eugenol to be less than that of the phthalate. This indeed proves to be the case, the compressibility of the latter being about 11% greater than that of the former. That the OH group does not impart a still more marked incompressibility is evidently explained by the fact that it is swamped by the rest of the molecule, which is much more complex in the ring compounds than in the members of the glycerine family.

Two other benzene substitution compounds have been previously investigated,[7] C_6H_5Cl and C_6H_5Br. These two compounds can be measured only over a small range because of freezing by pressure, but in the short range open to comparison the compressibilities of both are very nearly equal to that of the phthalate.

The structure of ethyl dibenzyl malonate is complicated and involves two benzene rings in the same molecule. It is liquid over only a small range of pressure; in this range its compressibility is very nearly the same as that of eugenol.

Tri-ortho-cresyl-phosphate contains three benzene rings, but there is a new element, phosphorus, so that we do not know exactly what to expect. As a matter of fact it is one of the more incompressible compounds, and is very close indeed to ethylene glycol. It would be interesting to examine phosphates containing OH groups if such exist.

Methyl oleate is essentially a long hydro-carbon chain with a $COOCH_3$ termination. The proportion of O atoms is small, so that one expects a compressibility of the same order as that of the higher straight hydro-carbons. The compressibility of the hydro-carbons has been measured as far out as n-decane. If the volume decrements at 0° produced by 1000 kg are plotted for these hydro-carbons as a function of the number of C's in the chain, and if due account is taken of the fact that the odd members of the family follow a different progression from the even members, the difference between odd and even vanishing asymptotically for long chains, it will be found that a natural and unforced extrapolation of the curve can be made from $n = 10$ (decane) to the volume decrement of methyl oleate at $n = 19$, 19 being the total number of C atoms in methyl oleate.

It is a curious fact that the volumes of methyl oleate and tricaproin are almost identical functions of pressure and temperature. This must be to a certain extent coincidence, the structure of the two being unlike.

Isoprene, C_5H_8, is essentially an iso-chain compound, with the number of H atoms cut down by the presence of two double bonds. The effect of a double bond seems to be in general to cut down the compressibility. One therefore expects a compressibility not greatly different from that of i-pentane, but somewhat less. This turns out to be the case, the volume decrement of isoprene at 0° at 7000 kg, for example, being about 10% less than that of i-pentane, and almost exactly the same as that of n-hexane.

The reason for measuring such an unusual compound as isoprene was, as already mentioned, that the data was of interest in connection with the work of Professor Conant on polymerization by pressure. The polymerization product has under normal conditions a volume of the order of 20% less than isoprene, so that it is at first natural to think that the polymerization is a pure pressure effect connected with the change of volume. The rate of polymerization increases rapidly with increasing pressure, however, and at the same time the volume of isoprene approaches that of the polymerization product, because of the very much greater compressibility of isoprene, now established by these measurements. In fact, special experiments in which the polymerization was conducted at 20000 kg showed that at this pressure the rate is more rapid than at any lower pressure, but that the change of volume is not perceptible. It is therefore evident that some other effect than that of pure volume change must be responsible for the polymerization. The situation has been studied in full detail, and a solution proposed, in papers by Professor Conant.[3]

Finally, the measurements on i-octane must be mentioned. One would expect a compressibility very nearly the same as that of n-octane, but somewhat less, judging from the effect of the iso-structure as shown by the behavior of n- and i- pentane, for example. This turns out to be the case, the volume decrement at 0° and 5000 kg, for example, of i-octane, being 0.9% less than that of n-octane.

It is a pleasure to acknowledge generous assistance from the Rumford Fund of the American Academy of Arts and Sciences and from the Milton Fund of Harvard University. For the readings I am indebted to my assistant Mr. L. H. Abbot.

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PHYSICAL PROPERTIES OF SINGLE CRYSTAL MAGNESIUM.

By P. W. Bridgman.

Investigations on Light and Heat made and published with aid from the Rumford Fund.

PHYSICAL PROPERTIES OF SINGLE CRYSTAL MAGNESIUM.

By P. W. Bridgman.

Presented Oct. 14, 1931. Received Oct. 16, 1931.

Introduction.

The compressibility, specific electrical resistance, and pressure and temperature coefficients of resistance of single crystal magnesium in the crystallographically independent directions have already been reported in a previous paper.[1] To these are now added the five elastic constants, the thermo-electric properties, and the thermal expansion. Although the amount of this new material is not large, it has seemed better to give it in a small independent paper rather than to wait indefinitely until opportunity should present itself to publish it with related data for other materials.

The specimens whose thermo-electric properties and elastic constants are measured in this paper were the same as those whose electrical resistance was determined in the previous paper. The thermal expansion was measured on the two compressibility samples of the previous paper.

Detailed Data.

Thermo-Electric Properties. Measurements were made simultaneously on six crystal rods of different orientations in the same apparatus as previously used for thermo-electric measurements on a number of non-cubic crystals.[2] The ends of the rods were in contact with two different streams of rapidly circulating kerosene at different temperatures, and the thermo-electromotive force between the magnesium rods and copper leads was measured as a function of the variable temperature of the one stream, that of the other being maintained fixed at about 15.7° C. The upper temperatures were 37°, 56°, 75°, 96°, and a return check reading was made at 57°. Smooth curves of e.m.f. against temperature were then drawn through the observed points for each specimen, and the differences taken between the smooth curves and the straight lines joining the extreme points. The differences were then reproduced analytically by a three constant expression in the temperature of the form $at' + bt'^2 + ct'^3$, where the temperature of the cold junction is taken as 15.7°, and t' is the excess of the temperature of the hot junction above this. A three constant expression was adequate to reproduce the results within experimental error, but a two constant expression was very distinctly not adequate. For most of the previous metals two constant expressions had been adequate. The calculation of the constants a, b, and c was facilitated by the fact that the maximum departure from linearity of the smooth curves for all orientations occurred at the same temperature, 35°. The constants a, b, and c were then plotted separately against $\cos^2\theta$, where θ is the angle between the principal axis of the crystal and the length of the specimen. If the Kelvin-Voigt symmetry relations hold, these three constants should each be linear in $\cos^2\theta$. The experimental evidence indicates that the Kelvin-Voigt relation is only an approximate relation, and that in Bi and Sn there are deviations beyond experimental error, although in Zn, Cd, and Sb the relation does seem to be satisfied within experimental error.[2] It would therefore have been most desirable to test the Kelvin-Voigt relation in magnesium also, but unfortunately the experimental accuracy was not great enough for this, and all that was possible was to assume the relation and to draw through the experimental points the best straight lines.

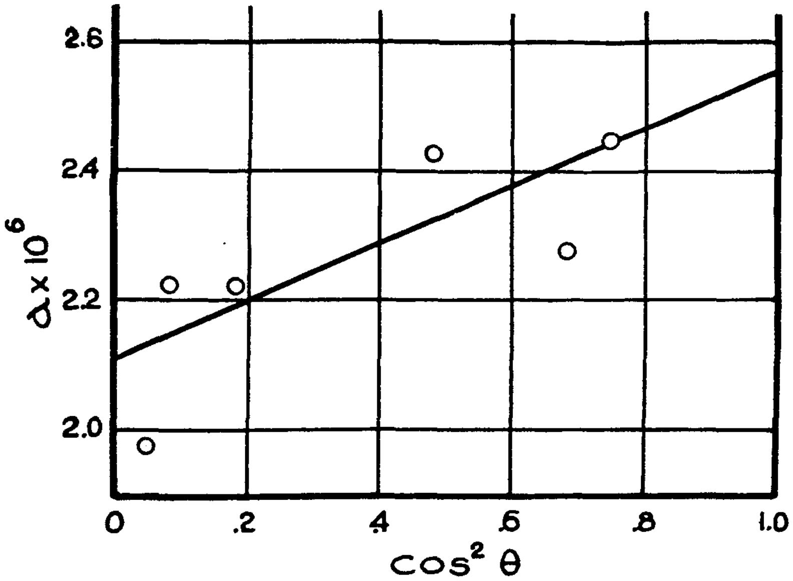

FIGURE 1. The constant "a" of the thermo-electric formula for rods of different orientations plotted as a function of the square of the cosine of the angle between the length of the rod and the crystal axis.

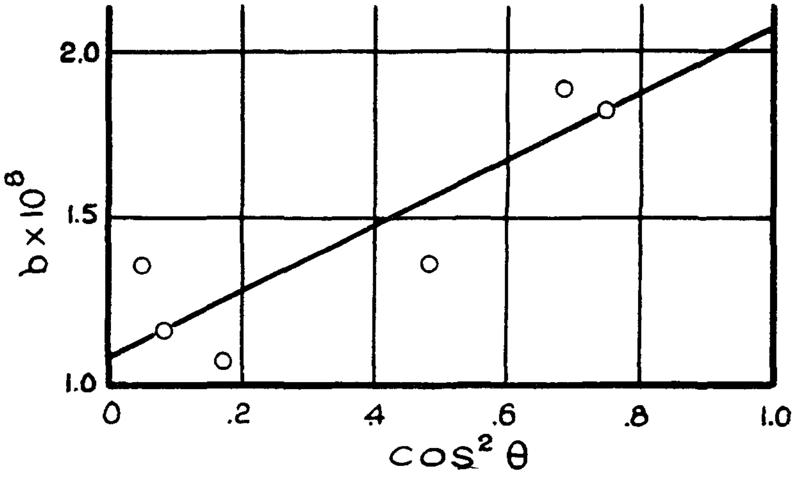

FIGURE 2. The constant "b" of the thermo-electric formula for rods of different orientations plotted as a function of the square of the cosine of the angle between the length of the rod and the crystal axis.

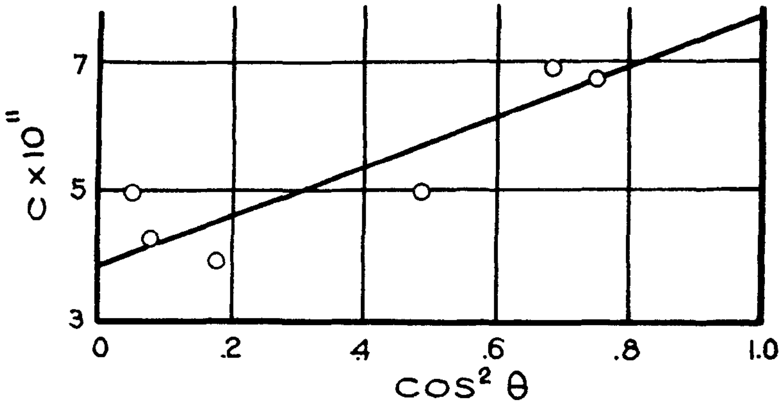

FIGURE 3. The constant "c" of the thermo-electric formula for rods of different orientations plotted as a function of the square of the cosine of the angle between the length of the rod and the crystal axis.

In figures 1, 2, and 3 the three constants a, b, and c are shown plotted against $\cos^2 \theta$, together with the straight lines taken to best represent the results. The intercepts of the lines at $\cos^2 \theta = 0$ and 1 give the values of a, b, and c for the parallel and perpendicular orientations. Finally, the origin of temperature may be changed to 0° C, giving the following formulas for the thermo-electromotive force in volts:

$$(\text{t.e.m.f.})_{\perp-\text{Cu}} = 1.65 \times 10^{-6}t + 12.5 \times 10^{-9}t^2 - 3.8 \times 10^{-11}t^3,$$

$$(\text{t.e.m.f.})_{\parallel-\text{Cu}} = 1.85 \times 10^{-6}t + 24.3 \times 10^{-9}t^2 - 7.7 \times 10^{-11}t^3,$$

where t is ordinary Centigrade temperature. From these expressions the Peltier and Thomson heats between the parallel and the perpendicular directions may be found by differentiating once and twice.

$$P_{\parallel-\perp} = \tau[0.20 \times 10^{-6} + 23.6 \times 10^{-9}t - 11.7 \times 10^{-11}t^2],$$

$$\sigma_{\parallel-\perp} = \tau[23.6 \times 10^{-9} - 23.4 \times 10^{-11}t],$$

τ is absolute temperature. The sign convention is the usual one. A positive $(\text{t.e.m.f.})_{\perp-\text{Cu}}$ means that positive current flows from the perpendicular orientation to copper at the hot junction.

The mean of these results differs very considerably from that previously found for the e.m.f. between extruded polycrystalline commercial magnesium and copper.[3] The former result was:

$$(\text{t.e.m.f.})_{\text{Mg}-\text{Cu}} = 2.872 \times 10^{-6}t + 4.79 \times 10^{-9}t^2.$$

The great difference is doubtless to be partly explained by the single crystal condition, but also no doubt in large part by the superior purity of the new material, the former material having been commercial magnesium with a mean temperature coefficient of resistance between 0° and 20° of 0.0039 against 0.00432 between 0° and 100° of the new material. The temperature ranges are not the same, so that the results are not strictly comparable, but there can be no doubt that the coefficient of the new material is significantly higher.

Elastic Constants. The elastic constants were determined by combining with the measurements already made of the linear compressibility parallel and perpendicular to the axis new measurements of the bending and twisting of seven rods, thus giving 16 measurements in all to determine five constants. Six of the seven new rods were the same as those on which the thermo-electric properties had been measured, except that after the thermo-electric measurements were completed the rods were machined to true cylinders, the rough castings being too irregular to permit accurate determinations of the elastic constants. The machining was done in a jeweller's lathe with great care, using a sharp tool and removing a chip of only 0.001 inch at a time. No perceptible distortion of the rods was produced by the machining operation, but one or two of the rods had been slightly bent in getting out of the molds, so that it was necessary to straighten them before the machining could be done. The seventh rod was the same in all respects, except orientation, as the other six; its basal plane was practically perpendicular to the length, which is the direction of greatest plasticity.

The pieces of apparatus for bending and twisting were the same as those already used and described in connection with the alkali halides,[4] so that no further description is necessary. The bending measurements were made with four different orientations of each rod with respect to the weight; no variation with orientation could be detected in any case. The rods in their finally machined form were about 2.5 mm. in diameter, and the clear length between supports in the bending measurements was 4.2 cm. The load was 42 gm; the magnification was such that the deflections were several centimeters, and readings could easily be made to 0.1 mm. The probable accuracy was 1 or 2 %, as shown by the agreement of results obtained on the same specimen at different times with different set-ups. Twisting measurements were made with two different weights and several repetitions. The accuracy was about the same as for the bending measurements. Especial attention was given to the question of remaining within the elastic limit; this was probably always accomplished in the bending measurements, but it is less certain that the condition was met in the twisting measurements, and in fact the seventh rod, with the basal plane perpendicular to the length, showed very perceptible creep under load and elastic recovery on removal of load, although in bending no such effect was apparent. The twisting measurements on this specimen were made with a maximum twisting moment of only 25 gm cm instead of the usual 50, and furthermore the load was allowed to act for only a few seconds before readings. Even with these precautions there were noticeable creep effects, and furthermore the strain was not proportional to the load, but increased more rapidly. In the calculations the results with the smallest load were taken as most probably accurate, but the internal evidence is that the effect of creep was not entirely eliminated.

The equations connecting the elastic constants with the distortions may be found in Voigt's book Lehrbuch der Kristallphysik. The linear compressibilities are:

Perpendicular to axis, $s_{11} + s_{12} + s_{13} = 0.98 \times 10^{-9}$,

Parallel to axis, $2s_{13} + s_{33} = 0.98 \times 10^{-9}$, The numerical values on the right hand sides of the equations are obtained from the linear compressibility measurements already reported.

For the bending of a rod of circular section, loaded at the center with weight Γ, and of total length L between end supports, Voigt gives on pages 746 and 747

$$s'_{33} = \frac{12\pi a^4}{\Gamma L^3} u,$$

where u is the deflection of the mid point and a the radius. Expressed in terms of the fundamental constants and the angle between the length of the rod and the axis,

$$s'_{33} = s_{11} \sin^4 \theta + (2s_{13} + s_{44}) \cos^2 \theta \sin^2 \theta + s_{33} \cos^4 \theta.$$

For the twist of a similar rod the formulas are:

$$n = \frac{N}{\pi a^4} (s'_{44} + s'_{55}),$$

where n is the twist per unit length, N the twisting moment, and

$$\tfrac{1}{2}(s'_{44} + s'_{55}) = s_{44} + (s_{11} - s_{12} - \tfrac{1}{2}s_{44}) \sin^2 \theta + 2(s_{11} + s_{33} - 2s_{13} - s_{44}) \cos^2 \theta \sin^2 \theta.$$

In making the computations, the results on linear compressibility were accepted as absolutely correct, this having been justified by previous experience and the fact that the cubic compressibility calculated from the linear compressibility agrees with that of other observers determined by essentially different methods. The effective constants s'_{33} and $s'_{44} + s'_{55}$, as given by the bending and twisting measurements, were now plotted against $\cos^2 \theta$, and curves drawn through these points in such a way as to pass as close to the points as possible and at the same time give values for the fundamental constants which should be consistent with the linear compressibility measurements. In figures 4 and 5 are shown the observed values of s'_{33} and $s'_{44} + s'_{55}$ plotted against $\cos^2 \theta$ and the curves drawn through them. The fundamental constants were calculated from the intercepts of the curves at the two extreme points and at the mid point, $\cos^2 \theta = 0.50$. Thus, from the bending curve, the value of s'_{33} when $\cos^2 \theta = 0$ is s_{11}, its value when $\cos^2 \theta = 0.50$ is $\frac{1}{4}[s_{11} + s_{33} + (2s_{13}+s_{44})]$, and its value when $\cos^2 \theta = 1$ is s_{33}. The values of the effective twisting modulus $(s'_{44} + s'_{55})$ for corresponding values of $\cos^2 \theta$ are

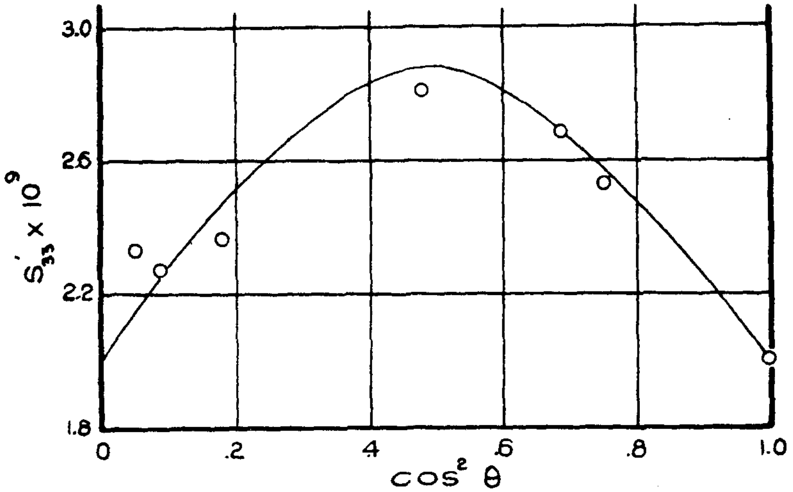

FIGURE 4. The effective elastic constant s'_{33} for rods of different orientations plotted as a function of the square of the cosine of the angle between the length of the rod and the crystal axis.

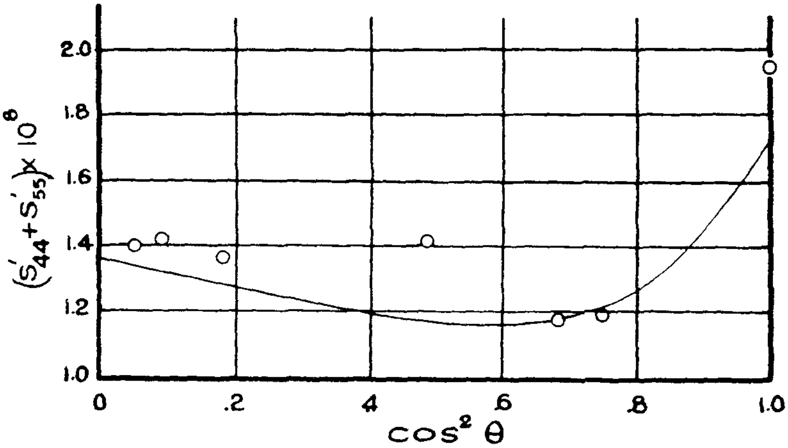

FIGURE 5. The effective elastic constant $s'_{44} + s'_{55}$ for rods of different orientations plotted as a function of the square of the cosine of the angle between the length of the rod and the crystal axis.

$$(2s_{11} - 2s_{12} + s_{44}),\quad \left(2s_{11} - s_{12} - 2s_{13} + s_{33} + \frac{s_{44}}{2}\right),$$

and $2s_{44}$.

The following are the values of the fundamental constants calculated in this way:

$$s_{11} = 2.00 \times 10^{-9} \quad \text{or} \quad 20.4 \times 10^{-13},$$
$$s_{33} = 2.00 \times 10^{-9} \quad \text{or} \quad 20.4 \times 10^{-13},$$
$$s_{13} = -\ 0.51 \times 10^{-9} \quad \text{or} \quad -\ 5.2 \times 10^{-13},$$
$$s_{12} = -\ 0.51 \times 10^{-9} \quad \text{or} \quad -\ 5.2 \times 10^{-13},$$
$$s_{44} = 8.62 \times 10^{-9} \quad \text{or} \quad 87.8 \times 10^{-13}.$$

The units in the first set of figures are gm cm and in the second Abs. C. G. S.

The self consistency of the experimental results, as shown by the closeness with which the curves pass through the experimental points, is in all cases satisfactory except for two points on the twisting curve. Both of these points are too high; the one corresponding to $\cos^2\theta = 1$ (basal plane perpendicular to the length) is doubtless in error because of the extreme softness of the specimen, as already explained, and the other point at $\cos^2\theta = 0.50$ lies off the curve by about the same amount and perhaps for the same reason.

Thermal Expansion. This was determined with the same apparatus as used in previous measurements, but it was freshly calibrated. The specimens, as already mentioned, were the same as those whose linear compressibility had already been measured. The temperature range was from about 35° to 15° C. The former procedure was somewhat modified in that instead of repeating measurements a number of times back and forth between the extremes of the temperature range, readings were made with smaller steps both with decreasing and increasing temperature, covering the entire range. Somewhat greater regularity was attained in this way.

There is an uncertainty in the final results due to the value assumed for the thermal expansion of the mild steel of which the apparatus was constructed, which was not directly measured. The value assumed was 12.0×10^{-6}; this value enters additively into the final result with a factor $\frac{1}{5}$, and affects measurements on both orientations equally, so that the resultant uncertainty is too small to be significant.

The results found for the linear expansion are:

Parallel to the axis,	27.1×10^{-6}.
Perpendicular to the axis,	24.3×10^{-6}.

This gives for the mean expansion of a polycrystalline aggregate 25.2×10^{-6}. International Critical Tables gives for ordinary polycrystalline magnesium 25.6×10^{-6}.

DISCUSSION.

For convenience the constants determined in the previous paper are reproduced here.

Linear Compressibility.

Parallel to the axis:

$$\text{At } 30^\circ, \quad -\Delta l/l_0 = 9.84 \times 10^{-7}p - 6.51 \times 10^{-12}p^2.$$

$$\text{At } 75^\circ, \quad -\Delta l/l_0 = 10.15 \times 10^{-7}p - 7.78 \times 10^{-12}p^2.$$

Perpendicular to the axis:

$$\text{At } 30^\circ, \quad -\Delta l/l_0 = 9.84 \times 10^{-7}p - 9.19 \times 10^{-12}p^2.$$

$$\text{At } 75^\circ, \quad -\Delta l/l_0 = 9.66 \times 10^{-7}p - 6.95 \times 10^{-12}p^2.$$

p is in kg/cm^2.

Electrical Resistance.

Specific resistance:

$$\left.\begin{array}{ll}\text{Parallel to the axis,} & 3.89 \times 10^{-6} \\ \text{Perpendicular to the axis,} & 4.60 \times 10^{-6}\end{array}\right\} \text{at } 22^\circ.5 \text{ C.}$$

Effect of pressure on resistance:

Axis 73° to length:

$$\text{At } 30^\circ, \quad -\frac{\Delta R}{R(0, 30^\circ)} = 5.61 \times 10^{-6}p - 8.4 \times 10^{-11}p^2.$$

$$\text{At } 75^\circ, \quad -\frac{\Delta R}{R(0, 75^\circ)} = 5.67 \times 10^{-6}p - 8.35 \times 10^{-11}p^2.$$

Axis 29° to length.

$$\text{At } 30^\circ, \quad -\frac{\Delta R}{R(0, 30^\circ)} = 5.48 \times 10^{-6}p - 7.8 \times 10^{-11}p^2.$$

$$\text{At } 75^\circ, \quad -\frac{\Delta R}{R(0, 75^\circ)} = 5.99 \times 10^{-6}p - 11.8 \times 10^{-11}p^2.$$

Temperature coefficient of resistance.

Axis 73° to length, mean coefficient 0°–100° = 0.00439.

Axis 29° to length, mean coefficient 0°–100° = 0.00420.

A paragraph is also quoted from the discussion in the previous paper:

"Magnesium is of interest because it is the first hexagonal metal measured in which the axial ratio corresponds to a packing of *spheres* in close packed hexagonal array. The piling in Zn and Cd, for example, is that of close packed ellipsoids. It is, of course, well known that the close packed hexagonal arrangement of spheres differs little from the close packed face centered cubic arrangement. In fact, if the two arrangements are built up by piling over each other layers perpendicular to the hexagonal axis, the only difference between the two arrangements is a slight relative displacement of the third layers, the first and second layers in the two arrangements being the same. Hence the forces on the atoms in the two arrangements can differ only by the contributions of the more distant atoms, which are unimportant because of the rapid falling off of atomic forces with increasing distance. It follows that a close packed hexagonal arrangement of spheres would not be expected to differ greatly in physical properties from a close packed cubical arrangement, and in particular, the compressibility of the hexagonal arrangement would be expected to be nearly the same parallel and perpendicular to the axis, because the compressibility of a cubic crystal is the same in all directions."

The expectation that magnesium would be like a cubic crystal in many respects is fulfilled in the first place by the linear compressibilities, which are the same in different directions within experimental error. Most of the elastic constants show the same sort of thing. Consider the "square" constants s_{11}, s_{33}, s_{12}, and s_{13}, called "square" because no shearing strains or stresses are involved, but these constants connect the elongations parallel and perpendicular to the axis with the normal stresses across the corresponding planes. For a cubic crystal these four square constants reduce to two, s_{11}, s_{11}, s_{12}, and s_{12} respectively. We notice at once that the four constants for hexagonal magnesium reduce in exactly the same way to two, that is, $s_{11} = s_{33}$, and $s_{13} = s_{12}$. s_{11} and s_{33} measure the effective Young's modulus for rods cut perpendicular and parallel to the crystal axis, so that we see that the extensibility under a one sided extending stress is the same for directions parallel and perpendicular to the axis, just like the linear compressibility. In the same way s_{12} and s_{13} measure the effective Poisson's ratio for rods parallel and perpendicular to the axis, and these again are equal. As far as this type of deformation is concerned, therefore, magnesium behaves much like a cubic substance.

The shearing constant, however, does not follow the type of the cubic crystal. In a cubic crystal the three principal shearing constants s_{44}, s_{55} and s_{66} are all equal. In a hexagonal crystal like magnesium, on the other hand, s_{44} and s_{55} are equal to each other, but s_{66} is different and identically equal to $2(s_{11} - s_{12})$. Hence if the shearing constants of magnesium were like those of a cubic crystal we would have for it $s_{44} = 2(s_{11} - s_{12})$. As a matter of fact, the numerical values are 87.8 and 51.2×10^{-13} respectively, so that the cubic relation fails by an amount far beyond possible experimental error.

The failure of the cubic relation for the shearing constants reacts on the effective Young's modulus of rods inclined to the axis, and it will be found that the Young's modulus of rods neither parallel nor perpendicular to the axis is not quite the same as for these two directions, so that with regard to the complete reaction to tension, magnesium does not behave like a cubic crystal, although it does with respect to hydrostatic pressure.

In view of the complete elastic symmetry with respect to hydrostatic pressure, it was a surprise to find the distinct failure of isotropy with respect to thermal expansion, which in the direction perpendicular to the axis is only 0.90 of the value parallel to the axis. This difference is consistent with the behavior of other metallic crystals, the thermal expansion usually being greatest across the basal plane, that is, the plane of easiest slip. For example, in Zn and Cd the ratio of the expansions perpendicular to parallel are 0.22 and 0.39 respectively. In the case of Zn and Cd, however, the ratio of the expansions is nearer to unity than the ratio of the compressibilities, whereas in Mg the reverse is the case. The behavior of the thermal expansion is somewhat in line with other temperature parameters, for it has already been found that the temperature coefficient of compressibility is markedly different in different directions, as is also the temperature coefficient of resistance. It is evident, therefore, that different effects are influenced in different degrees by the similarity of the structure of magnesium to the cubic structure.

With regard to thermo-electric properties, magnesium is like all the other non-cubic metals studied except tin, in that the Peltier heat at 0° between the parallel and perpendicular directions of flow is positive, but the effect is unusually small in magnesium, being only 0.55×10^{-4} against 4.90×10^{-4} for zinc, for example. In a cubic crystal the Peltier heat between any two directions is of course zero, so that with regard to Peltier heat magnesium again approaches a cubic crystal. The Thomson heat, however, is different, because $\sigma_{\|-\perp}$ for magnesium has the value 6.43×10^{-6}, and this is somewhat larger than for any of the other non-cubic metals except bismuth, where large effects are always to be expected. The sign of $\sigma_{\|-\perp}$ is the same for magnesium as for all the other non-cubic metals except bismuth. This confirms a conclusion previously reached on other grounds that there is not much similarity between the Peltier and Thomson heat mechanisms.

The manuscript of this paper was ready for publication when a preliminary notice, without experimental details, was published by Goens and Schmid[5] giving the specific resistance, temperature coefficient of resistance, and elastic constants of single crystal magnesium. The results of our two measurements are not as consistent as they ought to be. The greatest discrepancies are in the elastic constants. The values of Goens and Schmid are:

$$s_{11} = 22.3 \times 10^{-13}, \quad s_{33} = 19.8 \times 10^{-13}, \quad s_{44} = 59.5 \times 10^{-13},$$

$$s_{12} = -\ 7.7 \times 10^{-13}, \quad s_{13} = -\ 4.5 \times 10^{-13}.$$

The method of Goens and Schmid was a dynamic method in distinction from the static method used here. The greatest difference in our constants is in s_{44}. The values of Goens and Schmid of the constants would demand that the curve of figure 4 have the ordinate 22.7 at $\cos^2 \theta = 0.5$, and in figure 5 the ordinate at $\cos^2 \theta = 1$ would have to be 1.17; neither of these values seem possible for my material and my method.

The specific resistance of Goens and Schmid, parallel to the axis, reduced to 22.5°, is 3.84×10^{-6} against my 3.89×10^{-6}, and perpendicular to the axis they had 4.63 against my 4.60. The value to be calculated for polycrystalline material from these figures is almost identically the same for both of us. Their mean temperature coefficients of resistance parallel and perpendicular are 0.00427 and 0.00416 against my 0.00413 and 0.00441 respectively, a reversal of order to which they have already called attention, and which is not easy to account for.

Their thermal expansions parallel and perpendicular, making a linear correction for the difference of temperature range, are 25.5 ×

10^{-6} and 24.9×10^{-6} respectively, against my 27.1 and 24.3. The calculated values for volume expansions are 75.3 and 75.7 respectively.

The discrepancies are not easy to account for. The first thing to suspect is difference of purity. They do not give in their preliminary communication the method of casting the crystal or the sort of mold used, but state that the purity of the initial material was 99.95%. The values given above for the specific resistance would indicate no difference in the purity of our respective material, while the temperature coefficients would indicate a slight superiority of mine. The discrepancy in the elastic constants may be due to the difference of methods. It is to be noticed that their value for the shearing constant s_{44} is much more like that of a cubic metal than mine; it is important for theoretical reasons that the discrepancy be explained.

I am indebted to my assistant Mr. L. H. Abbot for the measurements of thermal expansion. I am also indebted for financial assistance to the Milton Fund of Harvard University and the Rumford Fund of the American Academy of Arts and Sciences.

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SOURCES OF AMERICAN DISCONTENT.

Communication to the American Academy of Arts and Sciences,
Wednesday, January 13, 1932.

BY H. ADDINGTON BRUCE.

SOURCES OF AMERICAN DISCONTENT.

By H. Addington Bruce.

On the eve of the turn from the nineteenth to the twentieth century an Englishman, James F. Muirhead, wrote a first-class book on the United States under the suggestive title, *The Land of Contrasts.* Were Mr. Muirhead writing his book to-day he would find still stronger justification for the title he selected. And were he writing it in the light of present scientific understanding of the forces governing human behavior, he would find the sharpest contrast of all in the paradox of the richest land in the world being at the same time the land of greatest discontent.

It is this despite the fact of the comparative freedom of the United States from the obvious, surface indications of social discontent—such indications as popular complaint and agitation for governmental change and for changes in the social structure. At the moment, to be sure, there is not a little complaining, as a consequence of the financial depression and widespread unemployment. But with the return of material prosperity, complaint and agitation will once more subside, thereby giving the politician renewed opportunity to "point with pride" to the fortunate state of the American people; who, nevertheless, will remain harried by a discontent far more mischievous than that which finds expression in mere complaint and agitation.

For it is a great mistake to assume, as many besides politicians still assume, that social discontent has its sources solely, or mainly, in economic, financial, or governmental conditions, and that it inevitably waxes or wanes with the character of these. This might perhaps be logically maintained if to complain and to agitate were the only and invariable manifestations of discontent, and were confined to the so-called poor and underprivileged. But it is noticeable that the poor and the underprivileged often accept their disadvantages in a curiously cheerful as well as philosophic spirit, while there may be much complaining among even the uncommonly well-to-do and privileged. More than this, as revealed by everyday observation and psychological research, well-to-do and poor alike, privileged and underprivileged, with or without a word of complaint, may be impelled by sheer discontent to all manner of aberrant behavior.

We find discontented of all classes turning to strong drink as an anodyne for the discontent that oppresses them; or, for the same purpose, becoming addicted to the use of narcotic drugs. We find them lapsing into immoral, even criminal ways of living. We find them, again under the stress of their discontent, hurrying to divorce courts, flocking to the consulting-rooms of nerve specialists, helping to swell the population of hospitals for the mentally diseased. Finally, when discontent is so extreme that life itself seems no longer tolerable, we find them committing suicide.

This is not to say that the social evils denoted by the terms vice, crime, divorce, nervous disease, mental disease, and suicide are wholly the product of discontent. But it is to say that discontent so commonly underlies them and is manifested by them that a country in which they are rife is essentially a discontented country. It remains in this category so long as it is beset by social evils, no matter what the national wealth or the status of its people with regard to institutional advantages. On this basis an appalling statistical array bears evidence to the prevalence of discontent in the United States; and, further, the story the statistics tell is of a rise in the occurrence-curve of social evils side by side with the rise in the curve of the nation's wealth. Seemingly, the richer we grow, the more discontented we become.

Take the evidence from the occurrence-curve of suicide. Whether the times have been "good" or "bad" this has risen pretty steadily. It took a sharp turn upward during the "depression" year 1921, when it rose from a total of 8,959 known suicides for the registration area of the United States in 1920 to a total of 11,136, or from 10.2 per 100,000 population to 12.6. This was popularly attributed to the strain of the economic situation, and even so experienced a student of suicide as Dr. Harry M. Warren, President of the Save-a-Life League, organized expressly to combat suicide, ventured the prediction:

"With improved business, which is sure to come, the suicide rate will be reduced and prosperity and happiness will return."[1]

For 1922 and 1923 there was a slight reduction, and while the rate rose again in 1924 it still was somewhat below that of 1921. But note what followed, coincident with the full development of the so-called Coolidge prosperity period. In 1926, 1927, and 1928, years of peak wages and peak dividends, years when politicians did not hesitate to suggest that we were in sight of the day when poverty might be abolished from the land, the suicide rate rose to 12.8, 13.3, and 13.6 per 100,000 population, with a total of 15,566 known suicides in 1928. These figures are taken from tabulations by the United States Census

Bureau. In 1929, according to figures assembled by Dr. Frederick L. Hoffman, there was a further rise to 17,000 known suicides, while in 1930, again according to Dr. Hoffman, the suicide total for the country was more than 18,000, or more than twice the total of ten years earlier.[2]

With regard to the occurrence of mental disease in the United States we again have a sorry record of a rising rate.

In 1880, or about fifty years ago, 31,973 persons were under treatment in State hospitals for the mentally diseased, or at a rate of 63.7 per 100,000 population. By 1904 the total had risen to 129,222, a rate of 158 per 100,000 general population. In 1928, the latest year for which I have Census Bureau statistics, there were no fewer than 264,226 mentally diseased persons in State hospitals, the rate then being 221.4 per 100,000 population, as against the 63.7 rate of 1880, when the total population of State hospitals for the mentally diseased was only 31,973.

That these figures indicate the actual rate of increase in mental disease in the United States during the past fifty years may properly be questioned, as it is questioned in a governmental bulletin issued in 1930. "The tremendous increase during the past generation in the numbers of mental patients receiving hospital treatment," the official comment runs, "has been largely the result of better methods of treatment, increased hospital facilities, and increased willingness on the part of relatives and friends of mental cases to utilize the hospitals to care for such patients."[3]

This is doubtless true, but it cannot safely be advanced, as it has been advanced by some, to support a claim that there has been only a seeming increase in mental disease in the United States. Nearly a quarter of a century ago I made for *The North American Review*[4] a study of the Census Bureau's statistics for mental disease at that time, and precisely the same objection was then raised to any suggestion that mental disease was increasing in this country at a rate beyond the rate of increase in the general population. Obviously such an objection would to-day be more impressive were it now being raised for the first time. In any case, that there has been an actual not merely a seeming increase is definitely shown by the findings of a survey undertaken in 1929 by the American Medical Association. As summarized in the bulletin of the Illinois State Department of Public Health, May 1, 1930, these findings show:

"During the two years ended with 1929 the number of hospital patients in the United States suffering from mental and nervous dis-orders increased 13 per cent. All other types of hospital patients increased less than 3 per cent. in number. The general population of the country increased from an excess of births over deaths by about 2 per cent. Thus it appears that the prevalence of mental and nervous incapacity is increasing at a rate over four times greater than that of all other kinds of illness which requires hospital care, and at a rate over six times greater than the rate of increase in population from natural sources."[5]

And, from comment in the American Medical Association's own *Journal*, for March 29, 1930:

"There are just nine more nervous and mental institutions than there were two years ago, but in that period their total capacity has increased from 373,364 to 414, 386, and the average number of patients and inmates has grown in the same period from 349,667 to 395,407. If the present rate continues, and there is no apparent reason for thinking it will not, by 1934 we will have more than one-half million persons in our nervous and mental institutions. This situation most seriously challenges the Government and the people of the United States."

According to the psychiatrist Karl A. Menninger, "Statistics say that one out of every twenty of us is, or has been, or will be, in a hospital for mental illness."[6] Writing somewhat earlier, Stewart Paton, of Princeton University, estimated that one of every five or six persons in a civilized community is afflicted in some degree with some form of nervous disorder.[7] The annual economic loss from nervous and mental disease in the United States to-day is incalculable (estimates have been given ranging from a minimum of $300,000,000 to a maximum of more than $2,200,000,000) and the burden on the American taxpayer tremendous. The maintenance cost of State hospitals alone is more than $75,000,000 a year;[3] and all the while old hospitals have to be enlarged and new ones constructed. In Massachusetts almost twenty cents of every tax dollar spent by the State goes for the care and housing of the mentally diseased and the mentally deficient;[8] and the Massachusetts spending for this purpose is not exceptionally heavy as compared with that of other States.

Turn now to divorce. If the increase in nervous and mental disease constitutes a situation that "most seriously challenges the Government and people of the United States," what are we to say of the situation created by the increase in divorce? Here there cannot be any quibbling as to whether the increase is seeming or actual.

Twenty years ago, when I had occasion to make a study of marriage failure in the United States, I found that nine of every 100 marriages terminated in divorce. Twenty years before that, according to divorce court statistics collected by the Federal Government, only six of every 100 marriages were ended by divorce. To-day divorce writes finis to seventeen of every 100 marriages, or a ratio nearly three times that of forty years ago.[9] Nor do divorce court statistics by any means tell the whole story of marriage failure. Many marriages culminate in voluntary separation, in desertion by husband or wife, even in the tragedy of murder by husband or wife. No complete statistical record is available of these other climaxes of marriage failure and of its sad sequel the broken home.

What of the children in the broken home? It is significant that recent studies of convicted criminals by Cooley, Glueck, and others, show that an average of 50 per cent. of prison inmates were as children bereft of one of their parents by death, desertion, or divorce. There are indeed many prison inmates in the United States to-day, so many that, as you know, prison construction is feverishly under way in the effort to lessen the evils—including prison riots—of overcrowding. Also, as in the case of hospitals for nervous and mental diseases, prison facilities are increasingly in demand. Twenty-five years ago the population of Federal and State prisons and reformatories was at the rate of about 70 per 100,000 general population. At the opening of 1929, the latest year for which I have governmental figures, the prison population in the United States was at the rate of nearly 100 per 100,000 general population. Nor does this take any account of inmates of city or county jails and workhouses.[10]

Prison statistics, of course, provide no sure measurement of the prevalence of crime. Particularly is this true of countries which, like the United States, send men and women to prison for behavior in no wise considered criminal in other lands. Yet, when we rigorously allow for this, and give heed only to offenses (against person or property) universally accepted as being criminal in character, the conclusion does seem unavoidable that crime, like other social evils, is increasing in our country. At all events, it now constitutes an intolerable strain on American social life, with a known murder occurring every forty-five minutes and the annual cost of crime in the United States running into the billions of dollars. No other country presents quite so dismal a picture in this respect.

In fact, with regard to social evils in general the situation in the United States as compared with that in other countries must be described as "markedly bad," to use the phrase applied by one American sociologist in comparing our crime situation with the crime situation elsewhere.[11] And the puzzling thing is that, while the occurrence-curve of social evils in general has been rising most rapidly here, it has risen in spite of the fact that in no other land has there been such organized effort to improve conditions. Here juvenile courts, child guidance clinics, habit clinics, psychopathic institutes, mental hygiene associations and other agencies for nipping social evils in the bud, have been at work to an extent not surpassed in any other country. But the greater the service they have rendered, the greater has grown the need for them to render service. Evidently there has been widespread failure to reckon with some influence specially provocative in the United States of growing discontent, with its varied manifestations in the social evils.

What is this influence? My own studies, carried through a long period of years, leave no doubt in my mind that it emanates from our excessive industrialization, which, while it has made us enormously wealthy as a nation, has been a disturbing factor in various ways.

For one thing, our excessive industrialization has led to a rapid and extreme shifting of the population from rural to urban life, with all the psychological ill effects of crowding. For another thing, it has caused, as a national trait, overemphasis on the importance of material goods, with a consequent dimming and lowering of national ideals and personal standards. And for still another thing, as national wealth has grown and the pursuit of wealth become more intense, there has been an unfortunate narrowing down of life activities in the interest of business-doing and the gaining of comforts and pleasures. As I see it, each of these consequences of excessive industrialization has been a special source of American discontent, particularly through interference with the upward urge innate in human beings.

Consider some of the effects on thought and feeling of the shifting of the population from rural to urban life—a shifting still under way as the extension of machine processes to agriculture lessens the vocational opportunities of agriculture for the small proprietor or tenant of the farm.

Few persons really appreciate how great the shifting has been and its rapidity since it was initiated by the application of steam and later of electricity to industrial purposes. In 1800 there were in all the United States only six cities with a population exceeding 8,000. Every decade since 1800 has witnessed a drift cityward, and a more rapid drift in each decade. In the ten year interval between the census of 1920 and that of 1930 the urban population increased more than six times as fast as the rural. So that whereas, in 1800, ninety-six of every 100 Americans lived on farms or in places of less than 8,000 population, in some States to-day nearly everybody is a town or city dweller.

In Rhode Island, for example, according to the 1930 census, the urban population is more than 92 per cent. of the total population of the State; in Massachusetts, it is 90.2 per cent.; in New York, 83.6 per cent.; in New Jersey, 82.6 per cent.; in Illinois, almost 74 per cent. Even in New Hampshire, which we still fondly consider a rural State, the State of the old homestead, nearly 60 per cent. of the total population is urban.[12]

Willy nilly, as a consequence, most of us of the present generation and of the rising generation live, move, and have our being largely in crowds; and with every advance in means of transportation and in the organization and mechanization of industry, we more and more live in crowds. We daily go in crowds to our places of work and daily return from them in crowds. Often we do our work in the midst of a crowd. More than this, we have not only grown accustomed to being in crowds, but have so acquired the crowd habit that we even incline to take our recreation in crowds, and come together in crowds on the slightest provocation. If the progress through our streets of a candidate for the Presidency draws an enormous crowd, so does the home-coming of a triumphant World Series baseball team. The funeral of a moving-picture celebrity means a crowd almost as surely as does the marriage of a millionaire's daughter.

Naturally the question suggests itself, "Are we as individuals and as members of society affected at all, for better or for worse, by the crowd habit, by the mere fact of our living and working and playing more and more in crowds?" One thing is certain: that crowd thinking differs conspicuously from the thinking of individuals not exposed to the contagion of crowds. It has been said by an eminent personage that mankind in the mass does not think at all, merely feels. That, however, is not a quite accurate statement of crowd mentation as the psychologist sees it. Crowds, according to those psychologists who have most closely studied them, do think—only they incline to think either as children think or as primitive peoples. They need not, they do not always think as children or primitives. Note that I say, they incline to think that way. A good deal depends on the degree of education of the individuals composing the crowd, and a good deal more on the purpose with which they have assembled. Still, given proper stimulus, crowd mentation is apt to develop even among the best educated.

For instance, a group of persons who have come together for the strictly intellectual purpose of hearing a scientific lecture, may be depended upon to remain a thoughtful, attentive audience, not a crowd, so long as the speaker confines himself to trying to meet their intellectual demand. But let him introduce something exceptionally disagreeable to them as members of a particular religious denomination or political party, and the crowd spirit is likely to develop in them, the spirit in which feeling unmistakably overrides thinking. When the assembling is for non-intellectual purposes, when those who assemble are not really well educated, then, naturally, the likelihood of the crowd spirit showing itself becomes all the greater.

And it is unfortunately a fact, substantiated by everyday observation, that with the growth of our cities occasions giving opportunity for feeling to override thinking draw the largest crowds. There never was a time when, for example, sporting events with the elements of conflict and potential gore, from prize fights to hockey matches, drew the crowds they are drawing even in this period of financial and economic depression. At such events, too, you are likely to see in full bloom the manifestations of crowd mentation and crowd behavior that drew from Everett Dean Martin the caustic observation, "A crowd is a device for indulging ourselves in a kind of temporary insanity by all going crazy together,"[13] and Boris Sidis's still more caustic description of the crowd as an instrument through which the credulous, uncontrolled, savage, primitive self slumbering in the depths of the civilized self gains momentary expression.

Here, fragmentarily, are Sidis's own words describing the primitive self, or, as he calls it, the subwaking self. Only, bear in mind that what he thus names is not identical with the subconscious self, the subliminal self of other psychologists. Sidis refers more strictly to what some Freudians picturesquely term, the cave man in us:

"The subwaking self is stupid; it lacks all critical sense. . . The subwaking self is extremely credulous; it lacks all sense of the true and rational. . . The subwaking self is most suggestible, most imitative; it is blown hither and thither by all sorts of incoming suggestions. . . The subwaking self is servile, cowardly, and devoid of scruple . . . ; it is essentially a brutal self."[14]

On first reading, one may be inclined to criticize this as extreme, exaggerated, an overstatement. But when one recalls the behavior of crowds in panics, in lynchings, in street riots, even in the excitement of sporting events, the inclination to criticize dies down. Also the question comes to mind, "Does the primitive, subwaking self find expression only in crowds? Given an increasing tendency in any people to live, to work, to take their amusements, even to spend their holidays in crowds, is there not a danger that that people will little by little acquire more and more the spirit and the ways of the crowd, will become infected, so to speak, with crowd mentality, even with regard to their thinking as individuals?"

This question, it is to be feared, must be answered in the affirmative. What, unhappily, is more to the point is that in our own case there is not lacking evidence that crowd thinking is to-day the thinking of the great majority of our people when out of crowds as well as when in crowds.

One has only to read the daily newspapers to appreciate the truth of this. With some outstanding exceptions our newspapers cater, and increasingly cater, to crowd mentality. The crowd mind longs for novelty, for excitement; craves to be amused, to be startled, to be shocked; takes delight in the misfortunes of others. In response, we have the average daily newspaper with its lurid headlines, its superfluity of crime news, its retailing of scandal, its cheap gossip, its many pages devoted to sports, its vulgar, crude, so-called comic cuts. Even the advertisements that overburden newspaper pages and the pages of magazines are in many cases frankly designed to appeal to crowd mentality.

Be sure that this would not be the case unless the advertisers found it profitable. For the matter of that, not so long ago I came across a book, "What About Advertising?", in which the authors, Kenneth M. Goode and Harford Powel, Jr., marshalled evidence from psychological and business research, going to show that most advertisers even to-day are overrating the mentation of most of their prospective customers. As Mr. Goode and Mr. Powel see it, the "average" American of to-day is on the intelligence level of the normal twelve- or fourteen-year-old child, and advertisers must govern themselves accordingly if they wish to make effective appeals through the printed word.

Personally, I think Mr. Goode and Mr. Powel misuse the word, intelligence, as some psychologists themselves do. Personally, I am convinced that the native intelligence of the mass of the American people is not one whit below that of their forefathers. The trouble is that the crowd habit—and other influences on which I will touch—has impeded and lowered the mental activity of great multitudes of people and has so diffused the crowd spirit that crowd thinking, with all its credulities, crudities, vulgarities, and brutalities has become widespread, to the growth directly and indirectly of discontent. For this our excessive industrialization, with its insistent shifting of the population from rural districts to herd in overgrown towns and cities, is largely to blame.

Our excessive industrialization, again, is to blame for a different but fully as harmful crowding—a crowding of life with things and with events. Especially during the past twenty-five years life has become so crowded with things that merely to enumerate the additional things brought into it during this period would require far more time than is at my disposal. Twenty-five years ago we had few of the extraordinary mechanical devices supposed to add to the comfort and ease of life but speeding it up and crowding it tremendously. And certainly we had next to none of the now never-ceasing propaganda designed to persuade us to evaluate things highly—designed indeed to persuade us that the more things we buy, at outright payment or on the instalment plan, the happier and better off we shall be.

Yet the plain truth is that things when made and pressed for sale in the enormous quantities of the present time, harmfully affect both those induced to buy them to excess and those who compete desperately, at times brutally, to sell them to excess. And the more science and invention add to the mountainous mass of things to be sold, the greater the threat to the human mind and spirit, the greater the breeding of discontent. So far has the competition to crowd life with things already progressed that many, I might fairly say the majority of business men, not only chronically wound their self-respect but give little real thought to anything outside of their particular occupation. Once, at a small luncheon gathering, I ventured to call this fact to the attention of those present, themselves mostly business men. For a moment there was a shocked silence, then one said:

"I'm afraid you're right. Lately I was traveling from New York to Chicago, and went to the smoking-compartment to enjoy a cigar. One other man was there. I greeted him, was answered pleasantly, and started, or attempted to start, a conversation. Every subject I brought up found him unresponsive. As I was about to give up, he turned to me, and, with an oddly pathetic look, quietly suggested, 'Try me with leather.'"

Here is one reason the art of conversation languishes to-day. Another is the influence of the crowd mentation already discussed.

People to all intents and purposes on a fourteen-year-old mental level, people who have the crowd habit, either agree thoughtlessly or disagree thoughtlessly, but do not readily converse on a variety of subjects in an interested or an interesting way. And the art of conversation suffers, as much else suffers, when people's attention is concentrated much of the time on the petty and the trivial and mere things. It needs no exhaustive research to determine that the attention of most persons to-day is concentrated most of the time on the petty and the trivial and mere things.

Crowding our lives with things, we necessarily tend to crowd them with events, with doings, if it be merely the looking at things, the selecting, rejecting, and buying of things, and the caring for the innumerable things with which we clutter our homes. Also we tend to crowd our lives with doings if only because of the decline in the art of thinking and the art of conversation.

Consider how most of us occupy our leisure. At work, we necessarily and rightly do. Free from work we equally, in many cases far more eagerly, give ourselves to doing. If we are poor company for ourselves and for one another, if we cannot converse in any really stimulating, satisfying way, we can at least play bridge, or dance, or go to the movies, the musical comedy, the hockey match, the prize fight, or other aid to passive amusement. Yet, by a cruel irony, the more desperately we seek satisfaction, seek joy in these, the greater the likelihood of our achieving boredom and discontent. To the point is a story told in *The Christian Century* concerning a drive taken by the teller of the story with a young business man:

"In the intimacy of the closed car, the driver began to talk of his home, his varied interests, his social experiences. He told, with enthusiasm, of the discovery which he and his wife had made of the satisfaction to be derived from spending evenings at home reading. 'We've come to the point,' he said, 'where we've just about exhausted the ways of having a good time. There's a crowd of us have played around together for years—gone to the theatre together, played cards together, danced together, all that sort of thing. The other night we were over at the home of one of the group. We started playing bridge, but after a couple of hands of that everybody had had enough. Then somebody turned on the radio, and we danced through a couple of numbers. And then we just sat and looked at each other, until finally Bill, who was the host, said, 'What the — shall we do now'?"[15]

Even those who have not succumbed to the self-centredness and materialism bred of the dominance of life by modern industry and business, those who do think and intelligently converse and retain a rational idea of the meaning of existence, still in many cases make the mistake of overcrowding life with doing, though it be highly useful doing. Such persons have the excuse that presentday conditions—especially as outlined in our brief survey of the prevalence of social evils in the United States—incite, almost compel the socially-minded to uncommon activity. Within certain limits it undoubtedly is good for them as well as for society that they thus exert themselves. It is not good, either for them or for society, that they go to excess in well-doing. To them, as to the aimless pleasure-seekers and the ultra-eager business-doers so numerous in the United States of to-day, real personal harm results from the overcrowding of life with doings.

Though most of us seem to have forgotten this, life never was meant to be wholly a matter of doing, of accomplishing, however beneficial or useful the doing from a social point of view; just as it never was meant to be wholly or chiefly a matter of amusing oneself or of acquiring as much wealth or other material possessions as possible. Undeniably, to do, to play, and to acquire are activities springing from natural human impulses—from instincts with which man has been endowed for useful ends. But—here is the significant point—man has been endowed with other instincts too. If life is to be found truly satisfying these other instincts—such as, and particularly, the creative as distinct from the procreative instinct, the instinct to grow in knowledge, the instinct for the beautiful, the altruistic instinct, the religious instinct—must be given adequate opportunity for expression.

Here, it seems to me—in the narrowing down of life activities plus the crowding of life, both resulting in chief measure from our excessive industrialization—are to be found the main sources of the discontent which gains such varied expression in our burden of social evils. Even the work in which most of us engage for the earning of a livelihood—the work of specialized tending or feeding of machines, the work of specialized distribution of machine-made products, the work of a routine clerkism—usually means a direct thwarting and repressing of at least one major instinct, the creative. This would not be so bad if we made it a point to give this instinct some opportunity for expression in our leisure hours. But most of us fail to do this—as most of us fail to give anything like adequate expression to the instinct to grow in knowledge, the instinct for the beautiful, the altruistic instinct, and the religious instinct.

So life becomes for great numbers—subconsciously if not consciously—a medley of discontent and of the demoralization born of surrender to the crowd spirit. Literally, we become sick of soul, and our soul sickness shows itself in the ugly symptomatology of the restlessness, nervousness, mental collapse, alcoholism, drug addiction, divorce, and suicide that are grim and growing features of the national life. How true it is that, in the main, these are rooted in our narrow, unbalanced, and materialistic mode of living, is evinced by the brilliant therapeutic results again and again obtained through psychological and spiritual methods of treatment of cases that have not taken the final flight from discontent to suicide.

Those cured are not cured merely because their thoughts have been turned from self, but because they have been aided to a better philosophy, and aided to more balanced living through the satisfaction of some instinctive tendency that had been left unsatisfied. Nor does one have to go to the case-books of psychological and spiritual therapeutists for indications of the discontent-preventing value of a life thus balanced. The joy in living displayed by the folk of Old World lands not as yet excessively industrialized has long been cause for remark by visitors from the New World. Their "standard of living," as the politicians phrase it, is distinctly below ours, they lack most of the "conveniences" of our highly industrialized land, they have long working-days largely at hand labor. Somehow they seem to get out of life much that the mass of our people fail to get.

Study them for a while, go with them into their cathedrals and churches, join them in their folk festivals, gain some knowledge of their philosophy of life, and the conviction will grow that their freer response to instinctive urgings—including especially the religious instinct and the instinct for the beautiful—is what most of all enables them to find greater joy in living. Recall, if you please, William McDougall's statement:

"The instincts are the prime movers of all human activity; by the conative or impulsive force of some instinct, every train of thought, however cold and passionless it may seem, is borne along toward its end, and every bodily activity is initiated and sustained. The instinctive impulses determine the ends of all activities and supply the driving power by which all mental activities are sustained; and all the complex intellectual apparatus of the most highly developed mind is but the instrument by which these impulses seek their satisfactions, while pleasure and pain do but serve to guide them in their choice of the means. . . These impulses are the mental forces that maintain and shape all the life of individuals and societies, and in them we are confronted with the central mystery of life and mind and will."[16]

Holding to this, it is only logical that McDougall should insist, "The first chapter of any useful psychology, whether normal or morbid, must be a study of the instinctive nature of the species *homo sapiens*."[17] Not all psychologists, not all sociologists, are of McDougall's belief. So far is his view from being universally accepted that some psychologists endeavor to dispense with instinct entirely, or almost entirely, in their explanations of why human beings behave as they do. Nevertheless, and particularly since the exploring of the obscure realm of the subconscious, an exploring begun by Pierre Janet and continued by Sigmund Freud, Boris Sidis, Morton Prince, McDougall himself and numerous other investigators, the supreme importance of reckoning with the instinctive nature of man in dealing with the ills and problems of society as of the individual, would seem to be soundly established. Until we have such reckoning, not by scientists alone, but still more by statesmen, educators, and industrial leaders themselves, we need not hope for the formulation of policies that will be effective in lessening American discontent.

The remedies commonly tried or proposed—such as prohibitive and punitive legislation, and action to bring about a more equitable distribution of the national wealth—leave the real need untouched.

A more equitable distribution of wealth would, obviously, weaken the influence of poverty as a source of discontent; and is fast becoming a necessity if our industrialists wish to continue mass production of manufactured goods on the scale of recent years. After all, the consumptive abilities of a millionaire are pretty limited so far as the personal use of manufactured goods is concerned; and, with Federal Trade Commission figures indicating that 13 per cent of the people own 90 per cent of the wealth,[18] it is hard to see how mass distribution of mass-produced goods can much longer be maintained satisfactorily in the absence of a foreign trade that it may be somewhat difficult to regain. But, as it has been my special endeavor to show, the problem of discontent goes far beyond financial and economic considerations. It is precisely because financial and economic considerations have been allowed to determine national thought and national policy that discontent has become the urgent problem it is in the United States of to-day.

That it has become a problem quite beyond solution I am not prepared to concede, though well aware of the difficulties of solution in a land where national programs and policies rest ultimately on the decision of a majority infused with the crowd spirit and on a fourteen-year-old level of mental activity. Still, the ultra-suggestibility of the crowd spirit may itself be used to contribute to acceptance of a wiser philosophy of life than that summed up in making money, spending money, and amusing oneself. Admittedly it is a Herculean undertaking to attempt to change a people's philosophy. But it can be done, for in the memory of many still living it has actually been done in the case of at least two peoples, the people of Germany and the people of Japan; in the former case to the hurt, in the latter thus far to the gain, of the people involved. In both cases intensive education of a far-spreading sort was the agency employed, and would have to be the agency employed to bring about the changed national philosophy needful for a lessening of discontent in the United States.

Into this further question of the possibilities and means of successful solution of the problem I cannot now go. In the time available I have, in fact, been able to present only in the baldest outline the major sources, as I see them, of the discontent that, with every passing year, adds to the population of our prisons, our hospitals for the nervously and mentally wrecked, our refuges for alcoholics and drug addicts; that overwhelms with business our criminal courts and our divorce courts; that takes a mounting toll in suicide; and also a mounting toll, though hitherto I have not mentioned this, in lives shortened by bodily diseases promoted, if not actually caused, by discontented states of mind.

Often one hears it said that the average life-expectation to-day is longer than it was a hundred, fifty, even twenty-five years ago. This is true only in so far as the expectation has been favorably affected by medical measures lessening the mortality among children and youths. It distinctly is not true with regard to the ages beyond fifty. As revealed by a statistical study undertaken by Professor C. H. Forsyth, of Dartmouth College, the life-expectation trend of the fifties and later is now unmistakably downward. "The expectation from age forty-five or fifty on," is the way Professor Forsyth puts it, writing in a 1929 issue of *Science*, "is the lowest of which we have any record—far lower than it was even forty years ago—and it still is going down, not up."

And, from Professor Forsyth also, by way of explanatory comment: "To me, the whole picture, from our earliest records in 1890 to the present time, points consistently and inevitably to a future of a declining average length of life until the American adult wakes up to the fact that the odds are at present heavily against his living as long as his father or grandfather. Some will say—and no doubt truly—that it is all a natural consequence of the great drift to the cities. Others will go farther and say life has become too fast and strenuous, and that we do not know as yet how to adjust ourselves to such a life.

"To the medical authorities the whole problem will loom as one of relieving the strain upon the heart. But little will be accomplished until the American adult himself is duly informed and made to realize that he is in the midst of a decidedly losing fight, and that the situation will continue unless he applies himself energetically to be superior to his environment. . . There is surely no worse influence than that wielded by well-meaning authorities who go around airing their ill-founded beliefs that all is going fine and that before long everybody is going to be living seventy-five or a hundred years."[19]

This, as stated, was written in 1929. Speaking as recently as December, 1931, in an address to the American Statistical Association in convention at Washington, Mr. Rollo H. Britten, statistician of the United States Public Health Service, declared to similar effect:

"Figures on sickness and death in this country since its first settlement show the reduction in mortality has come entirely at the younger age levels. Within the last thirty years this reduction has been phenomenal, but has only pertained to ages below fifty years. Because the United States has been a nation of young people, the trend of mortality in those ages has largely determined that for all ages. Even compared to ancient times, our present expectation of life at the higher ages is not favorable. . . Ancient Rome showed an expectation at birth of about twenty-one years, against ours of about fifty-five years; but as age advanced the difference rapidly decreased, until from age sixty on the Roman expectation was definitely above that of to-day."

If only from the point of view of gaining greater length of life for the individual, changes in the prevalent modes of thinking and of living are unmistakably in order. And, above all, as the evidence from the social evils goes to emphasize, the American people need to regain their lost sense of proportion as regards life's values and life's goods. It is not a question merely for theoretical consideration, this question of discontent in the United States. It is a question that bears directly on the larger question of national survival. By some means the people must be brought to a saner philosophy, a more rational, a better balanced mode of living, or the time will surely come when we shall justify the Spenglerian prediction and go the way of ancient Babylon and Rome.

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POSTULATES FOR SEPARATION OF POINT-PAIRS (REVERSIBLE ORDER ON A CLOSED LINE).

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POSTULATES FOR SEPARATION OF POINT-PAIRS (REVERSIBLE ORDER ON A CLOSED LINE).[1]

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Introduction.

This paper extends to the tetradic relation ABCD the same detailed treatment that was given to the triadic relation ABC in two papers on Betweenness published in 1917 and 1924.[2]

The great variety of logical processes involved in the several hundred theorems contained in these papers, combined with the extremely simple nature of the fundamental concepts involved, make the whole subject of order a veritable gold-mine of illustrative material for the logician. It is hoped that the present paper, which is the joint product of a worker in mathematics and a worker in philosophy, may serve to stimulate further the growing interest in the problems of logic which stand on the borderland between these two fields. Certainly no such wealth of source-material of this particular kind has ever before been available, either in mathematics or in philosophy.

The methods here followed are the usual methods of postulational theory. Some improvements will be found, however, in the form of the two tables presented at the end of the paper; and the new sign *vs* ("versus"), meaning "conflicts with," will be found to be a convenient abbreviation. Thus ABCD . VS . ABDC is an alternative notation for ABCD . ABDC : = : 0. The sign ⊃ means "If . . . then," the sign ᴗ means "or" (in the sense of "at least one"), and the dot means "and."

From the purely geometrical point of view, this paper completes the postulational study of the four most important types of order:

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[1] Presented to the American Mathematical Society by E. V. Huntington on April 11, 1925, and subsequently revised by the joint authors. An abstract of the paper, with special reference to Set X, was published in the Proceedings of the National Academy of Sciences, vol. 11 (1925), pp. 687–689.

[2] E. V. Huntington and J. R. Kline, Sets of independent postulates for betweenness, Transactions Amer. Math. Soc., vol. 18 (1917), pp. 301–325; and E. V. Huntington, A new set of postulates for betweenness with proof of complete independence, ibid., vol. 26 (1924), pp. 257–282. See also W. E. Van de Walle, On the complete independence of the postulates for betweenness, ibid., vol. 26 (1924), pp. 249–256.

(1) *serial order*, of points on a directed open line; (2) *cyclic order*, of points on a directed closed line;[3] (3) *betweenness*, of points on an undirected open line; (4) *separation*, of pairs of points on an undirected closed line. The theory of separation of point-pairs, or reversible order on a closed line, was first systematically studied by Pasch in 1882. The first set of postulates for this theory was given by Vailati in 1895.

BASIC LIST OF SEVENTEEN POSTULATES.

In the following general list, we collect together all the seventeen postulates whose logical inter-relations we propose to study in the present paper. These postulates relate to a system (K, R) in which K is a class of at least four elements A, B, C, . . .; and R is a tetradic relation, denoted by R (ABCD), or simply ABCD.

The most familiar example of a system (K, R) in which all these postulates are true is that in which K means the class of points on a closed line, and R (ABCD) means that the pair of points AC separates the pair of points BD. In other words, these postulates characterize the theory of reversible order on a closed line.

The first postulate (Postulate D) serves to exclude obviously trivial cases and will be assumed without further mention throughout the paper. The lettered postulates, F, F′, G, H, and R, R′, concern four elements. The numbered postulates, 10–19, concern five elements. The numbering of these postulates is so chosen as to avoid confusion with the numbering already used in the papers on betweenness.

POSTULATE D. If ABCD, then A, B, C, D are distinct. (Distinctness.)

In all the following postulates, it is understood that distinct letters denote distinct elements of the class K.

POSTULATE F. If A, B, C, D are any distinct elements, then at least one of the twenty-four permutations ABCD, ABDC, . . . DCBA, will form a true tetrad. ("Foursomeness.")

POSTULATE F′. At least one tetrad is true, say XYZW.

POSTULATE G. ABCD . ⊃ . BCDA. (Cyclicity.)

POSTULATE H. ABCD . ABDC : = : 0. ("Homogeneity.")

POSTULATE R. Every true tetrad is reversible: ABCD . ⊃ . DCBA. (Reversibility.)

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[3] See E. V. Huntington, Sets of completely independent postulates for cyclic order, Proceedings National Academy of Sciences, vol. 10 (1924), pp. 74–78.

Postulate R′. At least one true tetrad is reversible.
Postulate 10. ABCD . X . ⊃ . AXCD ∨ ABCX. ("Connexity.")
Postulate 11. ABXC . ABCY . ⊃ . ABXY.
Postulate 12. ABXC . ABCY . ⊃ . BXCY.
Postulate 13. ABXC . ABCY . ⊃ . AXCY.
Postulate 14. ABCX . ABCY . ⊃ . ABXY ∨ ABYX.
Postulate 15. ABCX . ABCY . ⊃ . ACXY ∨ ACYX.
Postulate 16. ABCX . ABCY . ⊃ . BCXY ∨ BCYX.
Postulate 17. ABCX . ABCY . ⊃ . (ABXY ∨ ACYX) . (ABYX ∨ ACXY).
Postulate 18. ABCX . ABCY . ⊃ . (ABXY ∨ BCYX) . (ABYX ∨ BCXY).
Postulate 19. ABCX . ABCY . ⊃ . (ACXY ∨ BCYX) . (ACYX ∨ BCXY).

The list as a whole contains, of course, many redundancies. In particular, Postulate F′ is merely a weaker form of F, and Postulate R′ is merely a weaker form of R.

In the main part of the paper, all the ways in which any one of these seventeen postulates can be deduced from any others are established, in 267 theorems on deducibility.

In the second part of the paper, 88 examples of systems (K, R) are given, which are used, in the usual way, to provide proofs of consistency and independence. These examples yield 154 theorems on non-deducibility.

As the main result of these two sets of theorems, it is found that from the complete list of seventeen postulates it is possible to select *ten and only ten sets of independent* postulates, as follows:

I. F, G, H, R, 10.
II. F, G, H, R, 12.
III. F, G, H, R, 13.
IV. F, G, H, R, 11, 14.
V. F, G, H, R, 11, 15.
VI. F, G, H, R, 11, 16.
VII. F, G, H, R, 11, 17.
VIII. F, G, H, R, 11, 18.
IX. F, G, H, R, 11, 19.
X. F′, R′, G, H, 10.

The simplest of these sets is the last: F′, R′, G, H, 10. In this set, Postulate F′ serves merely to exclude the trivial case of a system (K, R) in which no tetrad is true. Postulates G, H, 10 apply to any system of order on a closed line, whether reversible or non-reversible. [It can be shown (see Theorem R. 1, Corollary) that in any system which satisfies G, H, 10 either every true tetrad is reversible or no true tetrad is reversible.] Postulate R′ serves to exclude the non-reversible case.[4]

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[4] If Postulate R′ is replaced by the following: Postulate S′: At least one true tetrad is non-reversible; then the postulates F′, S′, G, H, 10 will form a set of independent postulates for cyclic order. See a forthcoming paper by E. V. Huntington on "The inter-relations of four types of order."

In the proofs which follow, every step of the demonstration is given explicitly, except in the case of two theorems (14.31 and 15.45) in which the proof is left to the reader. It will be observed that there are no proofs for Postulates D, G and H. The only proofs for F′ and R′ are the obvious proofs from F and R respectively.

EIGHT PROOFS OF POSTULATE F.

THEOREM F. 1. *Proof of* F *from* F′, G, H, 10.

To prove: If A, B, C, D are distinct, then at least one of the twenty-four permutations, ABCD, ABDC, . . . , DCBA, will form a true tetrad.
By Postulate F′, there exists at least one true tetrad, XYZW.
By 10, XYZW . A . ⊃ . XAZW ∪ XYZA. By G, AZWX ∪ AXYZ.
If AZWX, then by 10, AZWX . Y . ⊃ . AYWX ∪ AZWY.
If AXYZ, then by 10, AXYZ . W . ⊃ . AWYZ ∪ AXYW.
Hence at least one of the following four cases is true:

(1) AZWX . AYWX; by G, ZWXA . WXAY.
(2) AZWX . AZWY; by G, WXAZ . ZWYA.
(3) AXYZ . AWYZ; by G, XYZA . YZAW.
(4) AXYZ . AXYW; by G, YZAX . XYWA.

Now by G, XYZW . ⊃ . YZWX.
By 10, YZWX . A . ⊃ .YAWX ∪ YZWA. By G, AWXY ∪ AYZW.
If AWXY, then by 10, AWXY . Z . ⊃ . AZXY ∪ AWXZ.
If AYZW, then by 10, AYZW . X . ⊃ . AXZW ∪ AYZX.
Hence at least one of the following four cases is true:

(1′) AYZW . AYZX; by G, ZWAY . YZWA . YZXA.
(2′) AWXY . AZXY; by G, WXYA . XYAW . XYAZ.
(3′) AWXY . AWXZ; by G, WXYA . XYAW . WXZA.
(4′) AYZW . AXZW; by G, ZWAY . YZWA . ZWAX.

But by H, Case 1 conflicts with 2′ and 3′ (since WXAY . vs . WXYA) and also with 4′ (since ZWXA . vs . ZWAX). Hence 1 . ⊃ . 1′.
By H, Case 2 conflicts with 1′ and 4′ (since ZWYA . vs . ZWAY), and also with 3′ (since WXAZ . vs . WXZA). Hence 2 . ⊃ . 2′.
By H, Case 3 conflicts with 1′ and 4′ (since YZAW . vs . YZWA), and also with 2′ (since XYZA . vs . XYAZ). Hence 3 . ⊃ . 3′.
By H, Case 4 conflicts with 2′ and 3′ (since XYWA . vs . XYAW), and also with 1′ (since YZAX . vs . YZXA). Hence 4 . ⊃ . 4′.
Therefore at least one of the following four cases is true:

(a) AYZW . AZWX . AYWX . AYZX.
(b) AZWY . AZWX . AWXY . AZXY.
(c) AWYZ . AWXZ . AWXY . AXYZ.
(d) AYZW . AXZW . AXYW . AXYZ.

Hence in any case, AYZW ∨ AZWY ∨ AWYZ.
Also, AZWX ∨ AWXZ ∨ AXZW.
Also, AYWX ∨ AWXY ∨ AXYW.
Also, AYZX ∨ AZXY ∨ AXYZ.

Introducing the notation P (A, B, C, D) to mean: "at least one of the twenty-four tetrads formed from the letters A, B, C, D is true," these results may be written:[5]

XYZW . A . ⊃ . P (A, Y, Z, W) . P (X, A, Z, W) . P (Z, Y, A, W) . P (X, Y, Z, A).

But the conclusion of this proposition clearly does not depend on the order of the letters X, Y, Z, W. Hence we have the general formula:

P (X , Y, Z, W) . A . ⊃ . P (A, Y, Z, W) . P (X, A, Z, W) . P (X, Y, A, W) . P (X, Y, Z, A).

By successive applications of this formula, we have:

XYZW . A. ⊃ . P (A, Y, Z, W).
P (A, Y, Z, W) . B . ⊃ . P (A, B, Z, W).
P (A, B, Z, W) . C . ⊃ . P (A, B, C, W).
P (A, B, C, W) . D . ⊃ . P (A, B, C, D).

Therefore, XYZW . A . B . C . D . ⊃ . P (A, B, C, D), which was to be proved.

THEOREM F. 2. *Proof of* F *from* F′, G, 10, 12.

To prove: If A, B, C, D are distinct, then at least one of the twenty-four tetrads which can be formed from the elements A, B, C, D will be true; that is, prove P (A, B, C, D).
By F′, at least one tetrad is true, say XYZW. By G, YZWX.

Case 1. Let A, B, C, D be distinct from X, Y, Z, W.
Suppose P (A, Y, Z, W) = 0; that is, suppose every tetrad formed from the four elements A, Y, Z, W is false.
By 10, YZWX . A . ⊃ . YAWX ∨ YZWA. But YZWA is false, by supposition. Hence YAWX; whence, by G, XYAW.

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[5] This notation is due to Professor C. H. Langford (See Trans. Amer. Math. Soc. **26:** p. 279, 1924).

By 10, $XYZW . A . \supset . XAZW \smile XYZA$. But if $XYZA$, then by 12, $XYZA . XYAW . \supset . YZAW$, which is false. Hence $XAZW$; whence, by G, $WXAZ$.

By G, $XYAW . \supset . WXYA$. By 12, $WXYA . WXAZ . \supset . XYAZ$.

By 12, $XYAZ . XYZW . \supset . YAZW$, which is false.

Hence the supposition is false; that is, $P(A, Y, Z, W)$ is true.

Now by G, $XYZW . \supset . YZWX . ZWXY . WXYZ$, and, exactly as above, $YZWX . A . \supset . P(A, Z, W, X)$ and $ZWXY . A . \supset . P(A, W, X, Y)$ and $WXYZ . A . \supset . P(A, X, Y, Z)$.

Hence $XYZW . A . \supset . P(A, Y, Z, W) . P(X, A, Z, W) . P(X, Y, A, W) . P(X, Y, Z, A)$.

But the conclusion of this proposition clearly does not depend on the order of the letters X, Y, Z, W, so that in the premiss $XYZW$ might have been any other tetrad formed from the letters X, Y, Z, W. Hence we have the general formula:

$$P(X, Y, Z, W) . A :\supset: P(A, Y, Z, W) . P(X, A, Z, W) . P(X, Y, A, W) . P(X, Y, Z, A).$$

By applying the second, third, and fourth part of this formula, we have

$P(A, Y, Z, W) . B . \supset . P(A, B, Z, W)$,
$P(A, B, Z, W) . C . \supset . P(A, B, C, W)$,
$P(A, B, C, W) . D . \supset . P(A, B, C, D)$.

Therefore $P(A, B, C, D)$, which was to be proved.

Case 2. If any of the elements A, B, C, D are equal to any of the elements X, Y, Z, W, certain steps of the above proof are unnecessary.

Hence in any case, $P(A, B, C, D)$.

THEOREM F. 3. *Proof of* F *from* F′, G, 10, 13.

To prove: If A, B, C, D are distinct, then at least one of the twenty-four tetrads which can be formed from the elements A, B, C, D will be true; that is, prove $P(A, B, C, D)$.

By F′, at least one tetrad is true, say $XYZW$. By G, $YZWX$.

Case 1. Let A, B, C, D be distinct from $XYZW$.

Suppose $P(A, Y, Z, W) = 0$; that is, suppose every tetrad formed from the four elements A, Y, Z, W is false.

By 10, $YZWX . A . \supset . YAWX \smile YZWA$. But $YZWA$ is false. Hence $YAWX$; whence by G, $XYAW$ and $WXYA$.

Now by G, $YZWX . \supset . ZWXY$. By 10, $ZWXY . A . \supset . ZAXY \smile ZWXA$.

But if ZWXA, then by G, WXAZ, and by 13, $WXYA . WXAZ . \supset . WYAZ$, which is false.

Hence ZAXY, whence by G, XYZA. By 13, $XYZA . XYAW . \supset . XZAW$, whence, by G, WXZA.

Now by G, $XYZW . \supset . WXYZ$. By 13, $WXYZ . WXZA . \supset . WYZA$, which is false.

Hence the supposition is false; that is, P (A, Y, Z, W) is true.

From this point, the proof (including Case 2) duplicates the proof of Theorem F. 2.

Therefore P (A, B, C, D).

THEOREM F. 4. *Proof of* F *from* F′, G, 10, 14.

To prove: If A, B, C, D are distinct, then P (A, B, C, D).

By F′, XYZW. By G, $YZWX . ZWXY . WXYZ$.

Case 1. Let A, B, C, D be distinct from X, Y, Z, W.

Suppose P (A, Y, Z, W) = 0.

By 10, $YZWX . A . \supset . YAWX \vee YZWA$. But YZWA is false. Hence YAWX; whence by G, WXYA and AWXY.

By 14, $WXYA . WXYZ . \supset . WXAZ \vee WXZA$.

If WXAZ, then by G, ZWXA, whence by 14, $ZWXA . ZWXY . \supset . ZWAY \vee ZWYA$, both of which are false.

If WXZA, then by G, AWXZ, whence by 14, $AWXZ . AWXY . \supset . AWZY \vee AWYZ$, both of which are false.

Hence the supposition is false; that is P (A, Y, Z, W) is true.

From this point, the proof duplicates the proof of Theorem F. 2.

Therefore P (A, B, C, D).

THEOREM F. 5. *Proof of* F *from* F′, G, 10, 15.

To prove: If A, B, C, D are distinct, then P (A, B, C, D).

By F′, XYZW. By G, ZWXY.

Case 1. Let A, B, C, D be distinct from X, Y, Z, W.

Suppose P (A, Y, Z, W) = 0.

By 10, $ZWXY . A . \supset . ZAXY \vee ZWXA$.

If ZAXY, then by G, AXYZ, whence by 10, $AWYZ \vee AXYW$. But AWYZ is false, and if AXYW, then by 15, $AXYW . AXYZ . \supset . AYWZ \vee AYZW$, both of which are false.

If ZWXA, then by G, WXAZ, whence by 10, $WYAZ \vee WXAY$. But WYAZ is false, and if WXAY, then by 15, $WXAY . WXAZ . \supset . WAYZ \vee WAZY$, both of which are false.

Hence the supposition is false; that is, P (A, Y, Z, W) is true.
From this point, the proof duplicates the proof of Theorem F. 2.
Therefore P (A, B, C, D).

THEOREM F. 6. *Proof of* F *from* F′, G, 10, 16.

To prove: If A, B, C, D are distinct, then P (A, B, C, D).
By F′, XYZW. By G, YZWX and WXYZ.

Case 1. Let A, B, C, D be distinct from X, Y, Z, W.
Suppose P (A, Y, Z, W) = O.
By 10, YZWX . A . $\supset$. YAWX $\smile$ YZWA. But YZWA is false. Hence YAWX; whence, by G, WXYA and XYAW.
By 16, WXYA . WXYZ . $\supset$. XYAZ $\smile$ XYZA. But if XYZA, then by 16 XYZA . XYZW . $\supset$. YZAW $\smile$ YZWA, both of which are false. Hence XYAZ.
By 16, XYAW . XYAZ . $\supset$. YAWZ $\smile$ YAZW, both of which are false.
Hence the supposition is false; that is, P (A, Y, Z, W) is true.
From this point the proof duplicates the proof of Theorem F. 2.
Therefore P (A, B, C, D).

THEOREM F. 7. *Proof of* F *from* F′, G, 10, 11, 17.

Proof: By Theorem 14. 15, Postulates G, 11, 17 give 14; and by Theorem F. 4, Postulates F′, G, 10, 14 give F.

THEOREM F. 8. *Proof of* F *from* F′, G, 10, 11, 19.

Proof: By Theorem 16. 15, Postulates G, 11, 19 give 16; and by Theorem F. 6, Postulates F′, G, 10, 16 give F.

ONE PROOF OF POSTULATE R.

THEOREM R. 1. *Proof of* R *from* G, H, 10, R′.

To prove: ABCD . $\supset$. DCBA; that is, that every true tetrad is reversible.
If Postulate F′ is false, then there is no true tetrad in the system, and Postulate R is satisfied.
If Postulate F′ is true, then by Postulate R′, we know that there is at least one true tetrad which is reversible, or that we have, say, XYZW and WZYX.
Combining XYZW with the element A, as in the proof of Theorem F, we know that at least one of the four cases, a, b, c, d (mentioned in that proof) is true. Using G, and re-arranging the terms, we may rewrite these four cases as follows:

(a') YZXA . YWXA . ZWXA . YZWA; by G, AYWX . XAYW.
(b') ZXYA . WXYA . ZWXA . ZWYA; by G, AZXY . YAZX.
(c') XYZA . WXYA . WXZA . WYZA; by G, AWYZ . ZAWY.
(d') XYZA . XYWA . XZWA . YZWA; by G, AXZW . WAXZ.

By combining WZYX with the element A in a similar way (simply replacing X by W, and Y by Z, and Z by Y, and W by X, in cases c, b, a, d, in that order) we see that at least one of the following four cases is true:

(a'') AXZY . AXWY . AXWZ . AWZY; by G, YAXZ.
(b'') AYXZ . AYXW . AXWZ . AYWZ; by G, ZAYW.
(c'') AZYX . AYXW . AZXW . AZYW; by G, WAZX.
(d'') AZYX . AWYX . AWZX . AWZY; by G, XAWY.

But by H, a' conflicts with b'' and c'' (since AYWX . VS . AYXW), and also with d'' (since XAYW . VS . XAWY). Hence $a' . \supset . a''$.
By H, b' conflicts with c'' and d'' (since AZXY . VS . AZYX), and also with a'' (since YAZX . VS . YAXZ). Hence $b' . \supset . b''$.
By H, c' conflicts with a'' and d'' (since AWYZ . VS . AWZY), and also with b'' (since ZAWY . VS . ZAYW). Hence $c' . \supset . c''$.
By H, d' conflicts with a'' and b'' (since AXZW . VS . AXWZ), and also with c'' (since WAXZ . VS . WAZX). Hence $d' . \supset . d''$.
Therefore at least one of the following four cases is true:

(I) AXZY . YZXA . AXWY . YWXA . AXWZ . ZWXA . AWZY . YZWA.
(II) AYXZ . ZXYA . AYXW . WXYA . AXWZ . ZWXA . AYWZ . ZWYA.
(III) AZYX . XYZA . AYXW . WXYA . AZXW . WXZA . AZYW . WYZA.
(IV) AZYX . XYZA . AWYX . XYWA . AWZX . XZWA . AWZY . YZWA.

Hence AXZY . YZXA $: \vee :$ AYXZ . ZXYA $: \vee :$ AZYX . XYZA.
Also AXWY . YWXA $: \vee :$ AYXW . WXYA $: \vee :$ AWYX . XYWA.
Also AXWZ . ZWXA $: \vee :$ AZXW . WXZA $: \vee :$ AWZX . XZWA.
Also AWZY . YZWA $: \vee :$ AYWZ . ZWYA $: \vee :$ AZYW . WYZA.
Introducing the notation Q (A, B, C, D) to mean "at least one tetrad formed from the letters A, B, C, D, is true *and reversible*," these results may be written:

XYZW . WZYX . A $: \supset :$ Q (A, Y, Z, W) . Q (X, A, Z, W) . Q (X, Y, A, W) . Q (X, Y, Z, A).

The conclusion of this proposition is clearly not affected by changing the order of the letters X, Y, Z, W. Hence we have the general formula:

$$Q(X, Y, Z, W) . A :\supset: Q(A, Y, Z, W) . Q(X, A, Z, W) . Q(X, Y, A, W) . Q(X, Y, Z, A).$$

By successive applications of this general formula we have:

$XYZW . WZYX . A :\supset: Q(A, Y, Z, W)$;
then $Q(A, Y, Z, W) . \supset . Q(A, B, Z, W)$;
then $Q(A, B, Z, W) . \supset . Q(A, B, C, W)$;
then $Q(A, B, C, W) . \supset . Q(A, B, C, D)$.
That is, $XYZW . WZYX . A :\supset: Q(A, B, C, D)$.

This result tells not only that at least one tetrad formed from the letters A, B, C, D, is true (which we knew already from the hypothesis of the theorem), but also that at least one such true tetrad is reversible.

Now the twenty-four tetrads which can be formed from the letters A, B, C, D may be arranged in six groups, as follows:

(1) ABCD, BCDA, CDAB, DABC;
(2) ABDC, BDCA, DCAB, CABD;
(3) ACBD, CBDA, BDAC, DACB;
(4) CBAD, BADC, ADCB, DCBA;
(5) DBAC, BACD, ACDB, CDBA;
(6) BCAD, CADB, ADBC, DBCA.

By G, all tetrads in group (1) are true (since ABCD is true); and by G and H, all in groups (2) and (3) are false. Suppose group (4) is false; then the only tetrads which can be true will be found in group (1), (5), or (6). Since at least one of these is reversible, at least one tetrad in groups (4), (2), or (3) must be true, contrary to the supposition. Hence group (4) must be true.

Therefore DCBA.

Corollary. In any system which satisfies G, H, and 10, either every true tetrad is reversible or else no true tetrad is reversible.

For, in such a system, if R′ is true then R is true (by Theorem R. 1); that is, every true tetrad is reversible; and if R′ is false, then no true tetrad is reversible.

Eight Proofs of Postulate 10.

THEOREM 10. 1. *Proof of* 10 *from* R, F, G, 12.

To prove: $ABCD . X . \supset . AXCD \vee ABCX$.

By F, at least one of the twenty-four permutations of the elements A, X, C, D will form a true tetrad.

These twenty-four permutations may be arranged in three lines, as follows:

AXCD, XCDA, CDAX, DAXC; CXAD, XADC, ADCX, DCXA.
AXDC, XDCA, DCAX, CAXD; DXAC, XACD, ACDX, CDXA.
ACXD, CXDA, XDAC, DACX; XCAD, CADX, ADXC, DXCA.

By R, G, if any tetrad in the first line is true, then AXCD.
By R, G, if any tetrad in the second line is true, then AXDC.
By R, G, if any tetrad in the third line is true, then ACXD.
Hence, by F, R, G, we have AXCD ∨ AXDC ∨ ACXD.
Suppose AXCD = 0. Then AXDC ∨ ACXD. By G, DCAX ∨ DACX.

Case 1. Suppose DCAX. By R, ABCD . ⊃ . DCBA.
By 12, DCBA . DCAX . ⊃ . CBAX. By R, G, ABCX.

Case 2. Suppose DACX. By G, ABCD . ⊃ . DABC.
By 12, DABC . DACX . ⊃ . ABCX.
Therefore AXCD ∨ ABCX.

THEOREM 10. 2. *Proof of* 10 *from* R, F, G, 13.

To prove: ABCD . X . ⊃ . AXCD ∨ ABCX.
By F, R, G (as in the preceding theorem), AXCD ∨ AXDC ∨ ACXD.
By G, ABCD . ⊃ . CDAB. By R, G, ABCD . ⊃ . ADCB.
Suppose AXCD = 0. Then AXDC ∨ ACXD. By R, G, CDXA ∨ ADXC.

Case 1. If CDXA, by13, CDXA . CDAB . ⊃ . CXAB. By G, ABCX.

Case 2. If ADXC, by 13, ADXC . ADCB . ⊃ . AXCB. By R, G, ABCX.
Therefore AXCD ∨ ABCX.

THEOREM 10. 3. *Proof of* 10 *from* R, F, G, 11, 14.

Proof: Postulate 15 follows from R, G, 11, 14, by Theorem 15. 25, (below); then 10 follows from R, F, G, 11, 15, by Theorem 10. 4 (below).

THEOREM 10. 4. *Proof of* 10 *from* R, F, G, 11, 15.

To prove: ABCD . X . ⊃ . AXCD ∨ ABCX.
By R, G, ABCD . ⊃ . ADCB. By R, G, ABCD . ⊃ . BADC.
By R, ABCD . ⊃ . DCBA. By G, ABCD . ⊃ . DABC.
By G, ABCD . ⊃ . CDAB. By G, ABCD . ⊃ . BCDA.
By F, R, G (as in Theorem 10. 1), AXCD ∨ AXDC ∨ ACXD.
Suppose AXCD = 0. Then AXDC ∨ ACXD. By G, DCAX ∨ DACX.

Case 1. Suppose DCAX. By 11, DCBA . DCAX . ⊃ . DCBX.
By 15, DCBX . DCBA . ⊃ . DBXA ∨ DBAX. By G, ADBX ∨ BAXD.

If ADBX, by 11, ADCB . ADBX . ⊃ . ADCX; by R, G, AXCD.
If BAXD, by 11, BAXD . BADC . ⊃ . BAXC; by R, G, ABCX.
Hence in Case 1, AXCD ∨ ABCX.

Case 2. Suppose DACX. By 11, DABC . DACX . ⊃ . DABX.
By 15, DABX . DABC . ⊃ . DBXC ∨ DBCX. By G, CDBX ∨ BCXD.
If CDBX, by 11, CDAB . CDBX . ⊃ . CDAX; by G, AXCD.
If BCXD, by 11, BCXD . BCDA . ⊃ . BCXA; by G, ABCX.
Hence in Case 2, AXCD ∨ ABCX.
Therefore AXCD ∨ ABCX.

THEOREM 10. 5. *Proof of* 10 *from* R, F, G, 11, 16.

This theorem follows from Theorem 14. 20 (below) by aid of Theorem 10. 3.

THEOREM 10. 6. *Proof of* 10 *from* R, F, G, 11, 17.

This theorem follows from Theorem 15. 27 (below), by aid of Theorem 10. 4.

THEOREM 10. 7. *Proof of* 10 *from* R, F, G, 11, 18.

To prove: ABCD . X . ⊃ . AXCD ∨ ABCX.
By R, ABCD . ⊃ . DCBA. By G, ABCD . ⊃ . DABC.
By F, R, G (as in Theorem 10. 1), AXCD ∨ AXDC ∨ ACXD.
Suppose AXCD = 0. Then AXDC ∨ ACXD. By G, DCAX ∨ DACX.

Case 1. If DCAX, by 11, DCBA . DCAX . ⊃ . DCBX.
By 18, DCBX . DCBA . ⊃ . DCXA ∨ CBAX. By R, G, AXCD ∨ ABCX.

Case 2. If DACX, by 11, DABC . DACX . ⊃ . DABX.
By 18, DABX . DABC . ⊃ . DAXC ∨ ABCX. By G, AXCD ∨ ABCX.
Therefore AXCD ∨ ABCX.

THEOREM 10. 8. *Proof of* 10 *from* R, F, G, 11, 19.

This follows from Theorem 15. 28 (below), by aid of Theorem 10. 4.

SIX PROOFS OF POSTULATE 11.

THEOREM 11. 1. *Proof of* 11 *from* G, H, 10.

To prove: ABXC . ABCY . ⊃ . ABXY.
By 10, ABCY . X . ⊃ . AXCY ∨ ABCX. But ABCX . VS . ABXC, by H.
Hence AXCY, whence, by G, XCYA.

By 10, $ABXC . Y . \supset . AYXC \smile ABXY$.
But if $AYXC$, then by G, $XCAY$, and $XCAY . vs . XCYA$, by H.
Therefore $ABXY$.

THEOREM 11. 2. *Proof of* 11 *from* G, 12.

To prove: $ABXC . ABCY . \supset . ABXY$.
By 12, $ABXC . ABCY . \supset . BXCY$, whence by G, $CYBX$.
By G, $ABCY . \supset . CYAB$.
By 12, $CYAB . CYBX . \supset . YABX$. Hence by G, $ABXY$.

THEOREM 11. 3. *Proof of* 11 *from* G, 13.

To prove: $ABXC . ABCY . \supset . ABXY$.
By 13, $ABXC . ABCY . \supset . AXCY$, whence by G, $XCYA$.
By G, $ABXC . \supset . XCAB$.
By 13, $XCYA . XCAB . \supset . XYAB$. Hence by G, $ABXY$.

THEOREM 11. 4. *Proof of* 11 *from* R, 12, 13.

To prove: $ABXC . ABCY . \supset . ABXY$.
By 12, $ABXC . ABCY . \supset . BXCY$, whence by R, $YCXB$.
By R, $ABCY . \supset . YCBA$.
By 13, $YCXB . YCBA . \supset . YXBA$, whence by R, $ABXY$.

THEOREM 11. 5. *Proof of* 11 *from* R, H, 10.

To prove: $ABXC . ABCY . \supset . ABXY$.
Suppose $ABXY = 0$. Then by R, $YXBA = 0$.
By 10, $ABXC . Y . \supset . AYXC \smile ABXY$. But $ABXY = 0$. Hence $AYXC$.
By R, $ABXC . \supset . CXBA$ and by R, $ABCY . \supset . YCBA$.
By 10, $ABCY . X . \supset . AXCY \smile ABCX$. But $ABCX . vs . ABXC$ by H.
Hence $AXCY$. By R, $YCXA$.
By 10, $CXBA . Y . \supset . CYBA \smile CXBY$, whence by R, $ABYC \smile YBXC$.
But $ABYC . vs . ABCY$ by H. Hence $YBXC$.
By 10, $YCBA . X . \supset . YXBA \smile YCBX$. By R, $ABXY \smile XBCY$.
But $ABXY = 0$. Hence $XBCY$.
By 10, $XBCY . A . \supset . XACY \smile XBCA$. By R, $YCAX \smile ACBX$.
But $YCAX . vs . YCXA$ by H. Hence $ACBX$.
By 10, $ACBX . Y . \supset . AYBX \smile ACBY$. By R, $XBYA \smile YBCA$.
Suppose $XBYA$. Then by 10, $XBYA . C . \supset . XCYA \smile XBYC$, whence by R, $AYCX \smile XBYC$.
But $AYCX . vs . AYXC$ by H, and $XBYC . vs . XBCY$ by H.

Therefore YBCA.
By 10, YBCA . X . ⊃ . YXCA ∨ YBCX. But YBCX . vs . YBXC by H.
Hence YXCA. By R, ACXY.
By 10, ACXY . B . ⊃ . ABXY ∨ ACXB. But ACXB . vs . ACBX by H.
Therefore ABXY.

THEOREM 11. 6. *Proof of* 11 *from* R, 10, 12, 17.

To prove: ABXC . ABCY . ⊃ . ABXY.
Suppose ABXY = O. By R, YXBA = O.
By 10, ABXC . Y . ⊃ . AYXC ∨ ABXY. Hence AYXC. By R, CXYA.
By R, ABXC . ⊃ . CXBA. By R, ABCY . ⊃ . YCBA.
By 10, YCBA . X . ⊃ . YXBA ∨ YCBX. Hence YCBX. By R, XBCY.
By 12, ABXC . ABCY . ⊃ . BXCY. By R, YCXB.

Case 1. Suppose YBAX = O. By R, XABY = O.
By 17, YCBX . YCBA . ⊃ . YCXA ∨ YBAX. Hence YCXA. By R, AXCY.
By 17, YCXA . YCXB . ⊃ . YCAB ∨ YXBA. Hence YCAB. By R, BACY.
By 12, YCXA . YCAB . ⊃ . CXAB.
By 12, YCAB . YCBX . ⊃ . CABX.
Now CAXY = O (for if not, by 12, CABX . CAXY . ⊃ . ABXY). By R, YXAC = O.
Also, CXBY = O (for if not, by 12, CXAB . CXBY . ⊃ . XABY). By R, YBXC = O.
Then AYBX = O (for if not, by 12, AYBX . AYXC . ⊃ . YBXC). By R, XBYA = O.
By 10, CXBA . Y . ⊃ . CYBA ∨ CXBY. Hence CYBA.
Now CYAX = O (for if not, by 12, CYBA . CYAX . ⊃ . YBAX). By R, XAYC = O.
Also YBAC = O (for if not, by 10, YBAC . X . ⊃ . YXAC ∨ YBAX, while YXAC = O and YBAX = O).
Also XBYC = O (for if not, by 10, XBYC . A . ⊃ . XAYC ∨ XBYA, while XAYC = O and XBYA = O).
By 10, CABX . Y . ⊃ . CYBX ∨ CABY. By R, XBYC ∨ YBAC.
But XBYC = O and YBAC = O. Hence the supposition in case 1 is impossible.
Therefore YBAX. By R, XABY.

Case 2. Suppose YCAX = O. From Case 1, YBAX.
By 17, YCBA . YCBX . ⊃ . YCAX ∨ YBXA. Hence YBXA.
By 10, YBAX . C . ⊃ . YCAX ∨ YBAC. Hence YBAC.
By 12, YBXA . YBAC . ⊃ . BXAC.

By 12, BXAC . BXCY . ⊃ . XACY. By R, YCAX. But YCAX = o.

Hence the supposition in case 2 is impossible.

Therefore YCAX. By R, XACY.

By 12, YCAX . YCXB . ⊃ . CAXB. By R, BXAC.

By 12, YCBA . YCAX . ⊃ . CBAX.

Case 3. Suppose CABX. From Case 1, YBAX. From Case 2, YCAX.

Now CAXY = o (for if not, by 12, CABX . CAXY . ⊃ . ABXY).

By 10, CAXB . Y . ⊃ . CYXB ∨ CAXY. Hence CYXB. By R, BXYC.

Now CYBA = o (for if not, by 12, CYXB . CYBA . ⊃ . YXBA). By R, ABYC = o.

Then XAYC = o (for if not, by 12, XABY . XAYC . ⊃ . ABYC). By R, CYAX = o.

Then BXAY = o (for if not, by 12, BXAY . BXYC . ⊃ . XAYC). By R, YAXB = o.

By 10, CBAX . Y . ⊃ . CYAX ∨ CBAY. Hence CBAY.

Now CBXA = o (for if not, by 12, CBXA . CBAY . ⊃ . BXAY).

Then YCXA = o (for if not, by 12, YCBX . YCXA . ⊃ . CBXA).

By 10, YCXB . A . ⊃ . YAXB ∨ YCXA. But YAXB = o and YCXA = o.

Therefore CABX = o. By R, XBAC = o.

Now YCAB = o (for if not, by 12, YCAB . YCBX . ⊃ . CABX). By R, BACY = o.

By 10, YCXB . A . ⊃ . YAXB ∨ YCXA.

But if YCXA, then by 17, YCXA . YCXB . ⊃ . YCAB ∨ YXBA, where YCAB = o and YXBA = o. Hence YAXB. By R, BXAY.

Case 4. Suppose CYXB. By R, BXYC.

Now CYBA = o (for if not, by 12, CYXB . CYBA . ⊃ . YXBA). By R, ABYC = o.

By 12, BXAY . BXYC . ⊃ . XAYC. From Case 1, XABY.

By 12, XABY . XAYC . ⊃ . ABYC. But ABYC = o.

Hence the supposition in Case 4 is impossible.

Therefore CYXB = o. By R, BXYC = o.

By 17, BXAY . BXAC . ⊃ . BXYC ∨ BACY. Hence BACY. But BACY = o.

Hence the supposition that ABXY = o is impossible.

Therefore ABXY.

THIRTEEN PROOFS OF THE POSTULATE 12.

THEOREM 12. 1. *Proof of* 12 *from* G, H, 10.

To prove: ABXC . ABCY . ⊃ . BXCY.

By G, ABXC . ⊃ . BXCA.

By 10, BXCA . Y . ⊃ . BYCA ∨ BXCY. But if BYCA, then by G, ABYC, which conflicts with ABCY, by H.

Therefore BXCY.

Theorem 12. 2. *Proof of* 12 *from* G, 13.

To prove: ABXC . ABCY . ⊃ . BXCY.

By 13, ABXC . ABCY . ⊃ . AXCY, whence by G, XCYA.

By G, ABXC . ⊃ . XCAB. By G, XCYA . ⊃ . YAXC.

By 13, XCYA . XCAB . ⊃ . XYAB, whence by G, YABX.

By 13, YABX . YAXC . ⊃ . YBXC, whence by G, BXCY.

Theorem 12. 3. *Proof of* 12 *from* G, H, 11, 15.

To prove: ABXC . ABCY . ⊃ . BXCY.

By 11, ABXC . ABCY . ⊃ . ABXY, whence by G, YABX.

By G, ABCY . ⊃ . YABC. By G, ABCY . ⊃ . BCYA.

By 15, YABX . YABC . ⊃ . YBXC ∨ YBCX, whence by G, BXCY ∨ BCXY.

But if BCXY, then by 11, BCXY . BCYA . ⊃ . BCXA, whence by G, ABCX, which conflicts with ABXC, by H.

Therefore BXCY.

Theorem 12. 4. *Proof of* 12 *from* G, H, 11, 17.

To prove: ABXC . ABCY . ⊃ . BXCY.

By 11, ABXC . ABCY . ⊃ . ABXY, whence by G, YABX.

By 17, ABXY . ABXC . ⊃ . ABYC ∨ AXCY. But ABYC . VS . ABCY, by H.

Therefore AXCY, whence by G, YAXC. By G. ABCY . ⊃ . YABC.

By 17, YABC . YABX . ⊃ . YACX ∨ YBXC. But YACX . VS . YAXC, by H.

Therefore YBXC, whence by G, BXCY.

Theorem 12. 5. *Proof of* 12 *from* H, 11, 18.

To prove: ABXC . ABCY . ⊃ . BXCY.

By 11, ABXC . ABCY . ⊃ . ABXY.

By 18, ABXY . ABXC . ⊃ . ABYC ∨ BXCY.

But ABYC . VS . ABCY, by H. Therefore BXCY.

Theorem 12. 6. *Proof of* 12 *from* G, H, 11, 19.

To prove: ABXC . ABCY . ⊃ . BXCY.

By 11, ABXC . ABCY . ⊃ . ABXY, whence by G, YABX.

By G, ABCY . ⊃ . YABC.

By 19, YABX . YABC . ⊃ . YBXC ∨ ABCX. But ABCX . VS . ABXC, by H.

Therefore YBXC, whence by G, BXCY.

THEOREM 12. 7. *Proof of* 12 *from* H, 10, 11, 19.

To prove: ABXC . ABCY . ⊃ . BXCY.
By 10, ABCY . X . ⊃ . AXCY ∨ ABCX.
But ABCX . VS . ABXC, by H. Therefore AXCY.
By 11, ABXC . ABCY . ⊃ . ABXY.
By 19, ABXY . ABXC . ⊃ . AXYC ∨ BXCY.
But AXYC . VS . AXCY. Therefore BXCY.

THEOREM 12. 8. *Proof of* 12 *from* H, 11, 13, 19.
To prove: ABXC . ABCY . ⊃ . BXCY.
By 11, ABXC . ABCY . ⊃ . ABXY.
By 13, ABXC . ABCY . ⊃ . AXCY.
By 19, ABXY . ABXC . ⊃ . AXYC ∨ BXCY.
But AXYC . VS . AXCY, by H. Therefore BXCY.

THEOREM 12. 9. *Proof of* 12 *from* H, 11, 17, 19.

To prove: ABXC . ABCY . ⊃ . BXCY.
By 11, ABXC . ABCY . ⊃ . ABXY.
By 17, ABXY . ABXC . ⊃ . ABYC ∨ AXCY.
But ABYC . VS . ABCY, by H. Therefore AXCY.
By 19, ABXY . ABXC . ⊃ . AXYC ∨ BXCY.
But AXYC . VS . AXCY, by H. Therefore BXCY.

THEOREM 12. 10. *Proof of* 12 *from* R, G, H, 11, 14.

To prove: ABXC . ABCY . ⊃ . BXCY.
By 11, ABXC . ABCY . ⊃ . ABXY, whence by R, G, XBAY.
By R, G, ABXC . ⊃ . XBAC.
By 14, XBAC . XBAY . ⊃ . XBCY ∨ XBYC, whence by R, G, BCYX ∨ BXCY.
Suppose BCYX. By G, ABCY . BCYA. By G, ABXC . ⊃ . XCAB.
By 14, BCYA . BCYX . ⊃ . BCAX ∨ BCXA, whence by R, G, XBCA ∨ XCBA.
But XBCA . VS . XBAC, by H; and XCBA . VS . XCAB, by H.
Hence BCYX is false. Therefore BXCY.

THEOREM 12. 11. *Proof of* 12 *from* R, G, H, 11, 16.

To prove: ABXC . ABCY . ⊃ . BXCY.
By 11, ABXC . ABCY . ⊃ . ABXY. By G, ABXC . ⊃ . BXCA.
By 16, ABXY . ABXC . ⊃ . BXYC ∨ BXCY.
Suppose BXYC. Then by G, YCBX. By R, ABCY . ⊃ . YCBA.
By 16, YCBA . YCBX . ⊃ . CBAX ∨ CBXA, whence by R, G, ABCX ∨ BXAC.

But ABCX . VS . ABXC, by H; and BXAC . VS . BXCA, by H.
Hence BXYC is false. Therefore BXCY.

THEOREM 12. 12. *Proof of* 12 *from* R, H, 10, 18.

Postulate 11 follows from R, H, 10, by Theorem 11. 5.
Postulate 12 follows from H, 11, 18, by Theorem 12. 5.
Therefore 12 follows from R, H, 10, 18.

THEOREM 12. 13. *Proof of* 12 *from* R, H, 10, 19.

Postulate 11 follows from R, H, 10 by Theorem 11. 5.
Postulate 12 follows from H, 10, 11, 19, by Theorem 12. 7.
Therefore 12 follows from R, H, 10, 19.

THIRTEEN PROOFS OF POSTULATE 13.

THEOREM 13. 1. *Proof of* 13 *from* H, 10.

To prove: ABXC . ABCY . ⊃ . AXCY.
By 10, ABCY . X . ⊃ . AXCY ∨ ABCX. But ABCX . VS . ABXC, by H.
Therefore AXCY.

THEOREM 13. 2. *Proof of* 13 *from* G, 12.

To prove: ABXC . ABCY . ⊃ . AXCY.
By 12, ABXC . ABCY . ⊃ . BXCY, whence by G, CYBX.
By G, ABCY . ⊃ . CYAB.
By 12, CYAB . CYBX . ⊃ . YABX, whence by G, BXYA.
By 12, BXCY . BXYA . ⊃ . XCYA, whence by G, AXCY.

THEOREM 13. 3. *Proof of* 13 *from* G, H, 11, 15.

To prove: ABXC . ABCY . ⊃ . AXCY.
By 11, ABXC . ABCY . ⊃ . ABXY.
By G, ABXC . ⊃ . CABX.
By 15, ABXY . ABXC . ⊃ . AXYC ∨ AXCY, whence by G, CAXY ∨ AXCY.
But if CAXY, then by 11, CABX . CAXY . ⊃ . CABY, whence by G, ABYC, which conflicts with ABCY, by H.
Therefore AXCY.

THEOREM 13. 4. *Proof of* 13 *from* H, 11, 17.

To prove: ABXC . ABCY . ⊃ . AXCY.
By 11, ABXC . ABCY . ⊃ . ABXY.

By 17, ABXY . ABXC . ⊃ . ABYC ∨ AXCY.
But ABYC . vs . ABCY, by H.
Therefore AXCY.

Theorem 13. 5. *Proof of* 13 *from* G, H, 11, 18.

To prove: ABXC . ABCY . ⊃ . AXCY.
By 11, ABXC . ABCY . ⊃ . ABXY, whence by G, YABX.
By G, ABCY . ⊃ . YABC.
By 18, YABX . YABC . ⊃ . YAXC ∨ ABCX. But ABCX . vs . ABXC, by H.
Therefore YAXC, whence by G, AXCY.

Theorem 13. 6. *Proof of* 13 *from* G, H, 11, 19.

To prove: ABXC . ABCY . ⊃ . AXCY.
By 11, ABXC . ABCY . ⊃ . ABXY, whence by G, YABX.
By G, ABCY . ⊃ . YABC.
By 19, YABX . YABC . ⊃ . YBXC ∨ ABCX. But ABCX . vs . ABXC, by H.
Therefore YBXC, whence by G, BXCY.
By 19, ABXC . ABXY . ⊃ . AXCY ∨ BXYC. But BXYC . vs . BXCY, by H.
Therefore AXCY.

Theorem 13. 7. *Proof of* 13 *from* H, 11, 12, 19.

To prove: ABXC . ABCY . ⊃ . AXCY.
By 12, ABXC . ABCY . ⊃ . BXCY.
By 11, ABXC . ABCY . ⊃ . ABXY.
By 19, ABXC . ABXY . ⊃ . AXCY ∨ BXYC.
But BXYC . vs . BXCY, by H.
Therefore AXCY.

Theorem 13. 8. *Proof of* 13 *from* H, 11, 18, 19.

To prove: ABXC . ABCY . ⊃ . AXCY.
By 11, ABXC . ABCY . ⊃ . ABXY.
By 18, ABXY . ABXC . ⊃ . ABYC ∨ BXCY.
But ABYC . vs . ABCY, by H. Therefore BXCY.
By 19, ABXC . ABXY . ⊃ . AXCY ∨ BXYC.
But BXYC . vs . BXCY, by H.
Therefore AXCY.

Theorem 13. 9. *Proof of* 13 *from* R, G, H, 11, 14.

Postulate 12 follows from R, G, H, 11, 14, by Theorem 12. 10.
Postulate 13 follows from G, 12, by Theorem 13. 2.
Therefore 13 follows from R, H, G, 11, 14.

Theorem 13. 10. *Proof of* 13 *from* R, G, H, 11, 16.

Postulate 12 follows from R, G, H, 11, 16, by Theorem 12. 11.
Postulate 13 follows from G, 12, by Theorem 13. 2.
Therefore 13 follows from R, G, H, 11, 16.

Theorem 13. 11. *Proof of* 13 *from* R, H, 11, 18.

To prove: ABXC . ABCY . ⊃ . AXCY.
By 11, ABXC . ABCY . ⊃ . ABXY.
By 18, ABXY . ABXC . ⊃ . ABYC ∨ BXCY. But ABYC . vs . ABCY, by H.
Therefore BXCY, whence by R, YCXB. By R, ABCY . ⊃ . YCBA.
By 11, YCXB . YCBA . ⊃ . YCXA, whence by R, AXCY.

Theorem 13. 12. *Proof of* 13 *from* R, 11, 12.

To prove: ABXC . ABCY . ⊃ . AXCY.
By 12, ABXC . ABCY . ⊃ . BXCY, whence by R, YCXB.
By R, ABCY . ⊃ . YCBA.
By 11, YCXB . YCBA . ⊃ . YCXA, whence by R, AXCY.

Theorem 13. 13. *Proof of* 13 *from* R, 10, 12, 17.

Postulate 11 follows from R, 10, 12, 17, by Theorem 11. 6.
Postulate 13 follows from R, 11, 12, by Theorem 13. 12.
Therefore 13 follows from R, 10, 12, 17.

Thirty-Two Proofs of Postulate 14.

Theorem 14. 1. *Proof of* 14 *from* G, H, 10.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ ABYX.
Suppose ABXY = 0 and ABYX = 0. By G, BXYA = 0 and BYXA = 0.
By G, ABCX . ⊃ . BCXA. By G, ABCY . ⊃ . BCYA.
By 10, BCXA . Y . ⊃ . BYXA ∨ BCXY. Therefore BCXY.
By 10, BCYA . X . ⊃ . BXYA ∨ BCYX. Therefore BCYX.
But BCXY . vs . BCYX, by H. Therefore ABXY ∨ ABYX.

Theorem 14. 2. *Proof of* 14 *from* G, 12, 10.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ ABYX.
Suppose ABXY is false, whence by G, BXYA is false.
By G, ABCY . ⊃ . BCYA. By G, ABCX . ⊃ . BCXA.
By 10, BCYA . X . ⊃ . BXYA ∨ BCYX. But BXYA = 0. Therefore BCYX.
By 12, BCYX . BCXA . ⊃ . CYXA. By G, ABCY . ⊃ . CYAB.

By 12, CYXA . CYAB . ⊃ . YXAB, whence by G, ABYX.
Therefore ABXY ∨ ABYX.

THEOREM 14. 3. *Proof of* 14 *from* G, 12, 15.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ ABYX.
By G, ABCX . ⊃ . CXAB. By G, ABCY . ⊃ . CYAB.
By 15, ABCX . ABCY . ⊃ . ACXY ∨ ACYX, whence by G, CXYA ∨ CYXA.
If CXYA, then by 12, CXYA . CXAB . ⊃ . XYAB, whence by G, ABXY.
If CYXA, then by 12, CYXA . CYAB . ⊃ . YXAB, whence by G, ABYX.
Therefore ABXY ∨ ABYX.

THEOREM 14. 4. *Proof of* 14 *from* G, 12, 16.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ ABYX.
By G, ABCY . ⊃ . BCYA. By G, ABCX . ⊃ . BCXA.
By 16, ABCX . ABCY . ⊃ . BCXY ∨ BCYX.

Case 1. If BCXY, then by 12, BCXY . BCYA . ⊃ . CXYA.
By G, ABCX . ⊃ . CXAB.
By 12, CXYA . CXAB . ⊃ . XYAB, whence by G, ABXY.

Case 2. If BCYX, then by 12, BCYX . BCXA . ⊃ . CYXA.
By G, ABCY . ⊃ . CYAB.
By 12, CYXA . CYAB . ⊃ . YXAB, whence G, ABYX.
Therefore ABXY ∨ ABYX.

THEOREM 14. 5. *Proof of* 14 *from* G, 12, 17.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ ABYX.
Suppose ABYX is false.
By 17, ABCY . ABCX . ⊃ . ABYX ∨ ACXY. Therefore ACXY.
By G, ACXY . ⊃ . CXYA. By G, ABCX . ⊃ . CXAB.
By 12, CXYA . CXAB . ⊃ . XYAB, whence by G, ABXY.
Therefore ABXY ∨ ABYX.

THEOREM 14. 6. *Proof of* 14 *from* G, 12, 18.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ ABYX.
Suppose ABXY is false.
By 18, ABCX . ABCY . ⊃ . ABXY ∨ BCYX. Therefore BCYX.
By 12, BCYX . BCXA . ⊃ . CYXA. By G, ABCY . ⊃ . CYAB.
By 12, CYXA . CYAB . ⊃ . YXAB, whence by G, ABYX.
Therefore ABXY ∨ ABYX.

THEOREM 14. 7. *Proof of* 14 *from* G, 12, 19.

To prove: $ABCX . ABCY . \supset . ABXY \vee ABYX$.
By G, $ABCX . \supset . BCXA$. By G, $ABCY . \supset . CYAB$.
By 19, $ABCX . ABCY . \supset . ACXY \vee BCYX$.

Case 1. If $ACXY$, then by G, $CXYA$. By G, $ABCX . \supset . CXAB$.
By 12, $CXYA . CXAB . \supset . XYAB$, whence by G, $ABXY$.

Case 2. If $BCYX$, then by 12, $BCYX . BCXA . \supset . CYXA$.
By 12, $CYXA . CYAB . \supset . YXAB$, whence by G, $ABYX$.
Therefore $ABXY \vee ABYX$.

THEOREM 14. 8. *Proof of* 14 *from* G, 13, 10.

THEOREM 14. 9. *Proof of* 14 *from* G, 13, 15.

THEOREM 14. 10. *Proof of* 14 *from* G, 13, 16.

THEOREM 14. 11. *Proof of* 14 *from* G, 13, 17.

THEOREM 14. 12. *Proof of* 14 *from* G, 13, 18.

THEOREM 14. 13. *Proof of* 14 *from* G, 13, 19.

The six Theorems, 14. 8 to 14. 13, follow from Theorems 14. 2 to 14. 7, by the aid of Theorem 12. 2 (which says that Postulate 12 is a consequence of Postulates G and 13).

THEOREM 14. 14. *Proof of* 14 *from* G, 11, 15.

To prove: $ABCX . ABCY . \supset . ABXY \vee ABYX$.
By G, $ABCY . \supset . YABC$. By G, $ABCX . \supset . XABC$.
By 15, $ABCX . ABCY . \supset . ACXY \vee ACYX$. By G, $YACX \vee XACY$.
If $YACX$, then by 11, $YABC . YACX . \supset . YABX$; by G, $ABXY$.
If $XACY$, then by 11, $XABC . XACY . \supset . XABY$; by G, $ABYX$.
Therefore, $ABXY \vee ABYX$.

THEOREM 14. 15. *Proof of* 14 *from* G, 11, 17.

To prove: $ABCX . ABCY . \supset . ABXY \vee ABYX$.
Suppose $ABXY$ is false.
By 17, $ABCX . ABCY . \supset . ABXY \vee ACYX$. Therefore $ACYX$.
By G, $ACYX . \supset . XACY$. By G, $ABCX . \supset . XABC$.

By 11, XABC . XACY . ⊃ . XABY; by G, ABYX.
Therefore ABXY ∨ ABYX.

THEOREM 14. 16. *Proof of* 14 *from* G, H, 16.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ ABYX.
Suppose ABXY = 0 and ABYX = 0. Then by G, XYAB = 0 and XABY = 0.
By G, ABCX . ⊃ . BCXA. By G, ABCY . ⊃ . CYAB.
By G, ABCY . ⊃ . BCYA. By G, ABCX . ⊃ . CXAB.
By 16, ABCX . ABCY . ⊃ . BCXY ∨ BCYX.

Case 1. Suppose BCXY. By G, BCXY . ⊃ . XYBC.
By 16, BCXY . BCXA . ⊃ . CXYA ∨ CXAY. By G, BCXY . ⊃ . CXYB.
But if CXYA, then by 16, CXYA . CXYB . ⊃ . XYAB ∨ XYBA, whence XYBA (since XYAB = 0). Then by 16, XYBA . XYBC . ⊃ . YBAC ∨ YBCA, whence by G, CYBA ∨ BCAY, which conflicts with CYAB and BCYA by H.
Therefore CXAY. By 16, CXAY . CXAB . ⊃ . XAYB ∨ XABY, whence XAYB (since XABY = 0). By G, CXAY . ⊃ . XAYC.
By 16, XAYB . XAYC . ⊃ . AYBC ∨ AYCB, whence by G, BCAY ∨ AYCB.
But BCAY . vs . BCYA, by H. Therefore AYCB. By G, CXAY . ⊃ . AYCX.
By 16, AYCB . AYCX . ⊃ . YCBX ∨ YCXB. By G, XYCB ∨ CXBY.
But XYCB . vs . XYBC, by H; and CXBY . vs . CXYB, by H.
Therefore Case 1 is impossible.

Case 2. Suppose BCYX. By similar reasoning (merely interchanging X and Y), Case 2 is impossible.
Therefore ABXY ∨ ABYX.

THEOREM 14. 17. *Proof of* 14 *from* H, 17.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ ABYX.
Suppose both ABXY and ABYX are false.
By 17, ABCX . ABCY . ⊃ . ABXY ∨ ACYX. Hence ACYX.
By 17, ABCY . ABCX . ⊃ . ABYX ∨ ACXY. Hence ACXY.
But ACYX . vs . ACXY by H. Therefore ABXY ∨ ABYX.

THEOREM 14. 18. *Proof of* 14 *from* H, 18.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ ABYX.
Suppose both ABXY and ABYX are false.
By 18, ABCX . ABCY . ⊃ . ABXY ∨ BCYX. Hence BCYX.
By 18, ABCY . ABCX . ⊃ . ABYX ∨ BCXY. Hence BCXY.
But BCYX . vs . BCXY by H. Therefore ABXY ∨ ABYX.

THEOREM 14. 19. *Proof of* 14 *from* G, H, 19.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ ABYX.
By 19, ABCX . ABCY . ⊃ . ACXY ∨ BCYX.
By 19, ABCY . ABCX . ⊃ . ACYX ∨ BCXY.

Case 1. If ACXY is true, then ACYX is false, by H. Therefore BCXY, whence by G, CXYB. By G, ACXY . ⊃ . CXYA.
By G, ABCY . ⊃ . CYAB.
By 19, CXYB . CXYA . ⊃ . CYBA ∨ XYAB.
But CYBA . VS . CYAB, by H. Therefore XYAB, whence by G, ABXY.

Case 2. If BCYX is true, then BCXY is false, by H. Therefore ACYX, whence by G, CYXA. By G, BCYX . ⊃ . CYXB.
By G, ABCX . ⊃ . CXAB.
By 19, CYXB . CYXA . ⊃ . CXBA ∨ YXAB.
But CXBA . VS . CXAB by H. Therefore YXAB, whence by G, ABYX.
Therefore ABXY ∨ ABYX.

THEOREM 14. 20. *Proof of* 14 *from* R, G, 16.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ ABYX.
By R, ABCX . ⊃ . XCBA, whence by G, CBAX.
By R, ABCY . ⊃ . YCBA, whence by G, CBAY.
By 16, CBAX . CBAY . ⊃ . BAXY ∨ BAYX.
If BAXY, then by R, G, ABYX; and if BAYX, then by R, G, ABXY.
Therefore ABXY ∨ ABYX.

THEOREM 14. 21. *Proof of* 14 *from* R, G, 11, 19.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ ABYX.
By R, G, ABCX . ⊃ . CBAX. By R, G, ABCY . ⊃ . CBAY.
By G, ABCX . ⊃ . XABC.
By 19, CBAX . CBAY . ⊃ . CAXY ∨ BAYX, whence by R, G, XACY ∨ ABXY.
But if XACY, then by 11, XABC . XACY . ⊃ . XABY; by G, ABYX.
Therefore ABXY ∨ ABYX.

THEOREM 14. 22. *Proof of* 14 *from* R, G, 10, 15, 17.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ ABYX.
Suppose ABXY = O and ABYX = O. By G, BXYA = O and BYXA = O.
By 17, ABCX . ABCY . ⊃ . ABXY ∨ ACYX. Hence ACYX. By G, XACY.
By 17, ABCY . ABCX . ⊃ . ABYX ∨ ACXY. Hence ACXY. By G, YACX.
By G, ABCY . ⊃ . BCYA.
By 10, BCYA . X . ⊃ . BXYA ∨ BCYX. Hence BCYX. By G, XBCY.

By 15, BCYX . BCYA . ⊃ . BYXA ∨ BYAX. Hence BYAX, whence by G, AXBY.

By G, ABCX . ⊃ . BCXA.

By 10. BCXA . Y . ⊃ . BYXA ∨ BCXY. Therefore BCXY, whence by G, YBCX.

By 15, BCXY . BCXA . ⊃ . BXYA ∨ BXAY. Therefore BXAY, whence by G, AYBX.

By 10, AXBY . C . ⊃ . ACBY ∨ AXBC.

Case 1. Suppose ACBY. Then by G, YACB, and by R, YBCA.

Since ABXY = 0 and ABYX = 0, by R, G, YABX = 0 and YBAX = 0.

By 17, YACB . YACX . ⊃ . YABX ∨ YCXB. Hence YCXB.

By 17, YBCA . YBCX . ⊃ . YBAX ∨ YCXA. Hence YCXA.

Then by 15, YCXB . YCXA . ⊃ . YXBA ∨ YXAB, whence by R, G, ABXY ∨ ABYX.

Case 2. Suppose AXBC. Then by R, G, XACB, and by G, XBCA.

Since ABYX = 0 and ABXY = 0, by R, G, XABY = 0 and XBAY = 0.

By 17, XACB . XACY . ⊃ . XABY ∨ XCYB. Therefore XCYB.

By 17, XBCA . XBCY . ⊃ . XBAY ∨ XCYA. Therefore XCYA.

Then by 15, XCYA . XCYB . ⊃ . XYAB ∨ XYBA, whence by G, R, ABXY ∨ ABYX.

Therefore ABXY ∨ ABYX.

THEOREM 14. 23. *Proof of* 14 *from* R, G, 10, 15, 19,

Postulate 17 follows from R, G, 19, as we shall see in Theorem 17. 27 (below). Postulate 14 follows from R, G, 10, 15, 17, by Theorem 14. 22.

Therefore 14 follows from R, G, 10, 15, 19.

THEOREM 14. 24. *Proof of* 14 *from* R, G, 15, 17, 18.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ ABYX.

Suppose ABXY = 0 and ABYX = 0.

By 17, ABCX . ABCY . ⊃ . ABXY ∨ ACYX. Hence ACYX. By G, CYXA.

By 18, ABCY . ABCX . ⊃ . ABYX ∨ BCXY. Hence BCXY.

By 18, ABCX . ABCY . ⊃ . ABXY ∨ BCYX. Hence BCYX. By G, CYXB.

Since ABXY = 0 and ABYX = 0, by R, G, YXAB = 0 and AYXB = 0.

By 18, CYXB . CYXA . ⊃ . CYBA ∨ YXAB. Hence CYBA. By G, ACYB.

By 17, ACYB . ACYX . ⊃ . ACBX ∨ AYXB. Hence ACBX.

By G, ABCX . ⊃ . BCXA. Since ABXY = 0, by G, BXYA = 0.

By 17, BCXA . BCXY . ⊃ . BCAY ∨ BXYA. Hence BCAY. By R, G, ACBY.

By 15, ACBX . ACBY . ⊃ . ABXY ∨ ABYX.
Therefore ABXY ∨ ABYX.

Theorem 14. 25. *Proof of* 14 *from* R, G, 15, 18, 19.

Postulate 17 follows from R, G, 19, as we shall see in Theorem 17. 27 (below). Postulate 14 follows from R, G, 15, 17, 18, by Theorem 14. 24. Therefore 14 follows from R, G, 15, 18, 19.

Theorem 14. 26. *Proof of* 14 *from* R, F, G, 12.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ ABYX.
By F, at least one of the twenty-four permutations of the elements A, B, X, Y will form a true tetrad. These twenty-four permutations may be arranged in three lines, as follows:

ABXY, BXYA, XYAB, YABX; XBAY, BAYX, AYXB, YXBA.
ABYX, BYXA, YXAB, XABY; YBAX, BAXY, AXYB, XYBA.
AXBY, XBYA, BYAX, YAXB; BXAY, XAYB, AYBX, YBXA.

By R, G, if any tetrad in the first line is true, then ABXY is true.
By R, G, if any tetrad in the second line is true, then ABYX is true.
By R, G, if any tetrad in the third line is true, then AXBY is true.
Hence, by F, R, G, we have ABXY ∨ ABYX ∨ AXBY.
Suppose both ABXY and ABYX are false. Then AXBY.
By R, G, ABCX . ⊃ . AXCB. By R, G, ABCY . ⊃ . CBAY.
By 12, AXCB . AXBY . ⊃ . XCBY, whence by G, CBYX.
By 12, CBAY . CBYX . ⊃ . BAYX; By R, G, ABYX; which is false.
Therefore ABXY ∨ ABYX.

Theorem 14. 27. *Proof of* 14 *from* R, F, G, 13.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ ABYX.
By F, R, G (as in the preceding theorem), ABXY ∨ ABYX ∨ AXBY.
Suppose both ABXY and ABYX are false.
Then AXBY, whence by G, YABX. By G, ABCY . ⊃ . YABC.
By 13, YAXB . YABC . ⊃ . YXBC, whence by G, BCYX.
By G, ABCX . ⊃ . BCXA.
By 13, BCYX . BCXA . ⊃ . BYXA, by R, G, ABYX; which is false.
Therefore ABXY ∨ ABYX.

Theorem 14. 28. *Proof of* 14 *from* R, 10, 12, 17.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ ABYX.
In view of Theorem 18. 30 (below), we may use Postulate 18 as well as R, 10, 12, 17 in this proof.

Suppose ABXY = 0 and ABYX = 0. By R, YXBA = 0 and XYBA = 0.
By 17, ABCX . ABCY . ⊃ . ABXY ∨ ACYX. Hence ACYX. By R, XYCA.
By 18, ABCX . ABCY . ⊃ . ABXY ∨ BCYX. Hence BCYX. By R, XYCB.
By 17, XYCB . XYCA . ⊃ . XYBA ∨ XCAB. Hence XCAB.
By 18, XYCB . XYCA . ⊃ . XYBA ∨ YCAB. Hence YCAB.
By R, ABCX . ⊃ . XCBA. By R, ABCY . ⊃ . YCBA.
By 10, XCBA . Y . ⊃ . XYBA ∨ XCBY. Hence XCBY.
By 10, YCBA . X . ⊃ . YXBA ∨ YCBX. Hence YCBX.
By 12, XCAB . XCBY . ⊃ . CABY.
By 12, YCAB . YCBX . ⊃ . CABX.
By 18, CABX . CABY . ⊃ . CAXY ∨ ABYX. Hence CAXY.
By 12, CABX . CAXY . ⊃ . ABXY. But ABXY = 0.
Therefore ABXY ∨ ABYX.

THEOREM 14. 29. *Proof of* 14 *from* R, 10, 11, 13, 15, 18.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ ABYX.
In view of Theorem 17. 37 (below), we may use Postulate 17 in this proof, as well as R, 10, 11, 13, 15, 18.
Suppose ABXY = 0 and ABYX = 0. By R, YXBA = 0 and XYBA = 0.
By 17, ABCX . ABCY . ⊃ . ABXY ∨ ACYX. Hence ACYX. By R, XYCA.
By 17, ABCY . ABCX . ⊃ . ABYX ∨ ACXY. Hence ACXY. By R, YXCA.
By 18, ABCX . ABCY . ⊃ . ABXY ∨ BCYX. Hence BCYX. By R, XYCB.
By 18, ABCY . ABCX . ⊃ . ABYX ∨ BCXY. Hence BCXY. By R, YXCB.
By R, ABCX . ⊃ . XCBA. By R, ABCY . ⊃ . YCBA.
By 10, XCBA . Y . ⊃ . XYBA ∨ XCBY. Hence XCBY. By R, YBCX.
By 10, YCBA . X . ⊃ . YXBA ∨ YCBX. Hence YCBX. By R, XBCY.
Now ACBX = 0 (for if not, by 13, ACBX . ACXY . ⊃ . ABXY). By R, XBCA = 0.
Also, ACBY = 0 (for if not, by 13, ACBY . ACYX . ⊃ . ABYX). By R, YBCA = 0.
By 10, XBCY . A . ⊃ . XACY ∨ XBCA. Hence XACY. By R, YCAX.
By 10, YBCX . A . ⊃ . YACX ∨ YBCA. Hence YACX. By R, XCAY.
Now XCYB = 0 (for if not, by 13, XCYB . XCBA . ⊃ . XYBA).
Also, YCXB = 0 (for if not, by 13, YCXB . YCBA . ⊃ . YXBA).
By 18, XYCB . XYCA . ⊃ . XYBA ∨ YCAB. Hence YCAB.
By 18, YXCB . YXCA . ⊃ . YXBA ∨ XCAB. Hence XCAB.
By 18, YCAX . YCAB . ⊃ . YCXB ∨ CABX. Hence CABX.
By 18, XCAY . XCAB . ⊃ . XCYB ∨ CABY. Hence CABY.
By 15, CABX . CABY . ⊃ . CBXY ∨ CBYX. By R, YXBC ∨ XYBC.
If YXBC, then by 11, YXBC . YXCA . ⊃ . YXBA, which is false.

If XYBC, then by 11, XYBC . XYCA . ⊃ . XYBA, which is false.
Therefore, ABXY ∨ ABYX.

Theorem 14. 30. *Proof of* 14 *from* R, 10, 13, 17, 18.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ ABYX.
Suppose ABXY = 0 and ABYX = 0. By R, YXBA = 0 and XYBA = 0.
By 17, ABCX . ABCY . ⊃ . ABXY ∨ ACYX. Hence ACYX. By R, XYCA.
By 17, ABCY . ABCX . ⊃ . ABYX ∨ ACXY. Hence ACXY.
By 18, ABCX . ABCY . ⊃ . ABXY ∨ BCYX. Hence BCYX. By R, XYCB.
By 17, XYCB . XYCA . ⊃ . XYBA ∨ XCAB. Hence XCAB.
By 18, XYCB . XYCA . ⊃ . XYBA ∨ YCAB. Hence YCAB.
By R, ABCX . ⊃ . XCBA. By R, ABCY . ⊃ . YCBA.
By 10, XCBA . Y . ⊃ . XYBA ∨ XCBY. Hence XCBY. By R, YBCX.
By 10, YCBA . X . ⊃ . YXBA ∨ YCBX. Hence YCBX. By R, XBCY.
Now ACBY = 0 (for if not, by 13, ACBY . ACYX . ⊃ . ABYX). By R, YBCA = 0.
Also, ACBX = 0 (for if not, by 13, ACBX . ACXY . ⊃ . ABXY). By R, XBCA = 0.
By 10, YBCX . A . ⊃ . YACX ∨ YBCA. Hence YACX. By R, XCAY.
By 10, XBCY . A . ⊃ . XACY ∨ XBCA. Hence XACY. By R, YCAX.
Now XCYB = 0 (for if not, by 13, XCYB . XCBA . ⊃ . XYBA).
And YCXB = 0 (for if not, by 13, YCXB . YCBA . ⊃ . YXBA).
By 18, XCAY . XCAB . ⊃ . XCYB ∨ CABY. Hence CABY.
By 18, YCAX . YCAB . ⊃ . YCXB ∨ CABX. Hence CABX.
By 18, CABY . CABX . ⊃ . CAYX ∨ ABXY. Hence CAYX.
By 13, CABY . CAYX . ⊃ . CBYX. By R, XYBC.
By 13, XYBC . XYCA . ⊃ . XBCA, which is false.
Therefore ABXY ∨ ABYX.

Theorem 14. 31. *Proof of* 14 *from* R, 10, 12, 15, 18.

The proof of this theorem is left to the reader.

Theorem 14. 32. *Proof of* 14 *from* R, 10, 13, 17, 19.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ ABYX.
Suppose ABXY = 0 and ABYX = 0.
By R, XCBA, YCBA, YXBA = 0, XYBA = 0.
By 10, YCBA . X . ⊃ . YXBA ∨ YCBX. But YXBA = 0. Hence YCBX. By R, XBCY.
By 10, XCBA . Y . ⊃ . XYBA ∨ XCBY. But XYBA = 0. Hence XCBY. By R, YBCX.

By 13, if XCYB, then XCYB . XCBA . ⊃ . XYBA, which = 0. Hence XCYB = 0.

By 13, if YCXB, then YCXB . YCBA . ⊃ . YXBA, which = 0. Hence YCXB = 0.

By 17, ABCX . ABCY . ⊃ . ABXY ∨ ACYX. But ABXY = 0. Hence ACYX. By R, XYCA.

By 17, ABCY . ABCX . ⊃ . ABYX ∨ ACXY. But ABYX = 0. Hence ACXY. By R, YXCA.

By 13 and R, if YBCA, then ACBY . ACYX . ⊃ . ABYX, which = 0. Hence YBCA = 0.

By 13 and R, if XBCA, then ACBX . ACXY . ⊃ . ABXY, which = 0. Hence XBCA = 0.

By 10, XBCY . A . ⊃ . XACY ∨ XBCA. But XBCA = 0. Hence XACY. By R, YCAX.

By 10, XYCA . B . ⊃ . XBCA ∨ XYCB. But XBCA = 0. Hence XYCB.

By 10, YBCX . A . ⊃ . YACX ∨ YBCA. But YBCA = 0. Hence YACX. By R, XCAY.

By 10, YXCA . B . ⊃ . YBCA ∨ YXCB. But YBCA = 0. Hence YXCB. By R, BCXY.

By 17, YXCB . YXCA . ⊃ . YXBA ∨ YCAB. But YXBA = 0. Hence YCAB.

By 17, XYCB . XYCA . ⊃ . XYBA ∨ XCAB. But XYBA = 0. Hence XCAB.

By 13, if XAYB, then XACY . XAYB . ⊃ . XCYB, which = 0. Hence XAYB = 0.

By 13, if YAXB, then YACX . YAXB . ⊃ . YCXB, which = 0. Hence YAXB = 0.

By 13 and R, if CBXY, then YXBC . YXCA . ⊃ . YBCA, which = 0. Hence CBXY = 0.

By 19, XCAY . XCAB . ⊃ . XAYB ∨ CABY. But XAYB = 0. Hence CABY.

By 19, YCAX . YCAB . ⊃ . YAXB ∨ CABX. But YAXB = 0. Hence CABX.

By 19, CABX . CABY . ⊃ . CBXY ∨ ABYX. But CBXY = 0. And ABYX = 0. Hence the supposition is false. Therefore ABXY ∨ ABYX.

Forty-Five Proofs of Postulate 15.

Theorem 15. 1. *Proof of* 15 *from* G, H, 10.

To prove: ABCX . ABCY . ⊃ . ACXY ∨ ACYX.

By G, ABCX . ⊃ . XABC. By G, ABCY . ⊃ . BCYA.

By 10, XABC . Y . ⊃ . XYBC ∨ XABY, whence by G, YBCX ∨ ABYX.

Case 1. If YBCX, then by 10, YBCX . A . ⊃ . YACX ∨ YBCA, whence by G ACXY ∨ BCAY. But BCYA . vs . BCAY, by H. Therefore ACXY.

Case 2. If ABYX, then by 10, ABYX . C . ⊃ . ACYX ∨ ABYC.
But ABYC . VS . ABCY, by H. Therefore ACYX.
Therefore ACXY ∨ ACYX.

THEOREM 15. 2. *Proof of* 15 *from* G, 10, 12.

To prove: ABCX . ABCY . ⊃ . ACXY ∨ ACYX.
By G, ABCY . ⊃ . BCYA. By G, ABCX . ⊃ . BCXA.
By 10, BCYA . X . ⊃ . BXYA ∨ BCYX.

Case 1. If BXYA, then by G, ABXY. By 12, ABCX . ABXY . ⊃ . BCXY.
By 12, BCXY . BCYA . ⊃ . CXYA, whence by G, ACXY.

Case 2. If BCYX, then by G, BCYX . BCXA . ⊃ . CYXA, whence by G, ACYX.
Therefore ACXY ∨ ACYX.

THEOREM 15. 3. *Proof of* 15 *from* G, 12, 14.

To prove: ABCX . ABCY . ⊃ . ACXY ∨ ACYX.
By G, ABCY . ⊃ . BCYA. By G, ABCX . ⊃ . BCXA.
By 14, ABCX . ABCY . ⊃ . ABXY ∨ ABYX.

Case 1. If ABXY, then by 12, ABCX . ABXY . ⊃ . BCXY.
By 12, BCXY . BCYA . ⊃ . CXYA, whence by G, ACXY.

Case 2. If ABYX, then by 12, ABCY . ABYX . ⊃ . BCYX.
By 12, BCYX . BCXA . ⊃ . CYXA, whence by G, ACYX.
Therefore ACXY ∨ ACYX.

THEOREM 15. 4. *Proof of* 15 *from* G, 12, 16.

To prove: ABCX . ABCY . ⊃ . ACXY ∨ ACYX.
By G, ABCY . ⊃ . BCYA. By G, ABCX . ⊃ . BCXA.
By 16, ABCX . ABCY . ⊃ . BCXY ∨ BCYX.
If BCXY, then by 12, BCXY . BCYA . ⊃ . CXYA, whence by G, ACXY.
If BCYX, then by 12, BCYX . BCXA . ⊃ . CYXA, whence by G, ACYX.
Therefore ACXY ∨ ACYX.

THEOREM 15. 5. *Proof of* 15 *from* G, 12, 17.

To prove: ABCX . ABCY . ⊃ . ACXY ∨ ACYX.
By 17, ABCY . ABCX . ⊃ . ABYX ∨ ACXY.
If ABYX, then by 12, ABCY . ABYX . ⊃ . BCYX. By G, ABCX . ⊃ . BCXA.
By 12, BCYX . BCXA . ⊃ . CYXA, whence by G, ACYX.
Therefore ACXY ∨ ACYX.

THEOREM 15. 6. *Proof of* 15 *from* G, 12, 18.

To prove: ABCX . ABCY . ⊃ . ACXY ∨ ACYX.
By G, ABCY . ⊃ . BCYA. By G, ABCX ⊃ . BCXA.
By 18, ABCX . ABCY . ⊃ . ABXY ∨ BCYX.

Case 1. If ABXY, then by 12, ABCX . ABXY . ⊃ . BCXY.
By 12, BCXY . BCYA . ⊃ . CXYA, whence by G, ACXY.

Case 2. If BCYX, then by 12, BCYX . BCXA . ⊃ . CYXA, whence by G, ACYX.
Therefore ACXY ∨ ACYX.

THEOREM 15. 7. *Proof of* 15 *from* G, 12, 19.

To prove: ABCX . ABCY . ⊃ . ACXY ∨ ACYX.
Suppose ACXY is false. By G, ABCX . ⊃ . BCXA.
By 19, ABCX . ABCY . ⊃ . ACXY ∨ BCYX. Therefore BCYX.
By 12, BCYX . BCXA . ⊃ . CYXA, whence by G, ACYX.
Therefore ACXY ∨ ACYX.

THEOREM 15. 8. *Proof of* 15 *from* G, 10, 13.

This follows from Theorem 15. 2, by aid of Theorem 12. 2.

THEOREM 15. 9. *Proof of* 15 *from* 13, 14.

To prove: ABCX . ABCY . ⊃ . ACXY ∨ ACYX.
By 14, ABCX . ABCY . ⊃ . ABXY ∨ ABYX.
If ABXY, then by 13, ABCX . ABXY . ⊃ . ACXY.
If ABYX, then by 13, ABCY . ABYX . ⊃ . ACYX.
Therefore ACXY ∨ ACYX.

THEOREM 15. 10. *Proof of* 15 *from* G, 13, 16.

This follows from Theorem 15. 4, by aid of Theorem 12. 2.

THEOREM 15. 11. *Proof of* 15 *from* 13, 17.

To prove: ABCX . ABCY . ⊃ . ACXY ∨ ACYX.
By 17, ABCX . ABCY . ⊃ . ABXY ∨ ACYX.
But if ABXY, then by 13, ABCX . ABXY . ⊃ . ACXY.
Therefore ACXY ∨ ACYX.

THEOREM 15. 12. *Proof of* 15 *from* G, 13, 18.

THEOREM 15. 13. *Proof of* 15 *from* G, 13, 19.

These theorems follow from Theorems 15. 6 and 15. 7 by aid of Theorem 12. 2.

THEOREM 15. 14. *Proof of* 15 *from* H, 17.

To prove: ABCX . ABCY . ⊃ . ACXY ∨ ACYX.
Suppose both ACXY and ACYX are false.
By 17, ABCX . ABCY . ⊃ . ABXY ∨ ACYX. Therefore ABXY.
By 17, ABCY . ABCX . ⊃ . ABYX ∨ ACXY. Therefore ABYX.
But ABXY . vs . ABYX, by H. Therefore ACXY ∨ ACYX.

THEOREM 15. 15. *Proof of* 15 *from* G, H, 18.

To prove: ABCX . ABCY . ⊃ . ACXY ∨ ACYX.
By 18, ABCX . ABCY . ⊃ . ABXY ∨ BCYX.

Case 1. Suppose ABXY. Then by G, YABX. By G, ABCY . ⊃ . YABC.
By 18, YABC . YABX . ⊃ . YACX ∨ ABXC, whence by G, ACXY ∨ ABXC.
But ABXC . vs . ABCX, by H. Therefore ACXY.

Case 2. Suppose BCYX. By G, ABCY . ⊃ . BCYA.
By G, ABCX . ⊃ . BCXA.
By 18, BCYA . BCYX . ⊃ . BCAX ∨ CYXA, whence by G, BCAX ∨ ACYX.
But BCAX . vs . BCXA, by H. Therefore ACYX.
Therefore ACXY ∨ ACYX.

THEOREM 15. 16. *Proof of* 15 *from* H, 19.

To prove: ABCX . ABCY . ⊃ . ACXY ∨ ACYX.
Suppose both ACXY and ACYX are false.
By 19, ABCX . ABCY . ⊃ . ACXY ∨ BCYX. Therefore BCYX.
By 19, ABCY . ABCX . ⊃ . ACYX ∨ BCXY. Therefore BCXY.
But BCYX . vs . BCXY, by H. Therefore ACXY ∨ ACYX.

THEOREM 15. 17. *Proof of* 15 *from* G, 10, 11, 14, 16.

To prove: ABCX . ABCY . ⊃ . ACXY ∨ ACYX.
By G, ABCX . ⊃ . BCXA. By G, ABCY . ⊃ . BCYA.
By 14, ABCX . ABCY . ⊃ . ABXY ∨ ABYX.
By 16, ABCX . ABCY . ⊃ . BCXY ∨ BCYX.
By 10, BCXA . Y . ⊃ . BYXA ∨ BCXY, whence by G, ABYX ∨ BCXY.
By 10, BCYA . X . ⊃ . BXYA ∨ BCYX, whence by G, ABXY ∨ BCYX.

Case 1. Suppose both ABXY and BCXY are true. Then by G, XYAB and XYBC.
By 11, $XYAB . XYBC . \supset . XYAC$, whence by G, ACXY.

Case 2. Suppose either ABXY or BCXY is false. Then ABYX and BCYX are true, whence by G, YXAB and YXBC are true.
By 11, $YXAB . YXBC . \supset . YXAC$, whence by G, ACYX.
Therefore $ACXY \vee ACYX$.

THEOREM 15. 18. *Proof of* 15 *from* G, 11, 14, 16, 18.

To prove: $ABCX . ABCY . \supset . ACXY \vee ACYX$.
Suppose $ACXY = 0$ and $ACYX = 0$.
Then by G, $XYAC = 0$ and $YXAC = 0$.
By 14, $ABCX . ABCY . \supset . ABXY \vee ABYX$, whence by G, $XYAB \vee YXAB$.
By 18, $ABCX . ABCY . \supset . ABXY \vee BCYX$, whence by G, $XYAB \vee YXBC$.
By 18, $ABCY . ABCX . \supset . ABYX \vee BCXY$, whence by G, $YXAB \vee XYBC$.
By 16, $ABCY . ABCX . \supset . BCYX \vee BCXY$, whence by G, $YXBC \vee XYBC$.

Case 1. Suppose XYAB is false. Then YXAB and YXBC are true.
Then by 11, $YXAB . YXBC . \supset . YXAC$, which is false.

Case 2. Suppose XYBC is false. Then YXAB and YXBC are true.
Then by 11, $YXAB . YXBC . \supset . YXAC$, which is false.
Therefore XYAB and XYBC are true.
By 11, $XYAB . XYBC . \supset . XYAC$, whence by G, ACXY, which is false.
Therefore $ACXY \vee ACYX$.

THEOREM 15. 19. *Proof of* 15 *from* G, 11, 14, 19.

To prove: $ABCX . ABCY . \supset . ACXY \vee ACYX$.
Suppose both ACXY and ACYX are false.
By 19, $ABCX . ABCY . \supset . ACXY \vee BCYX$. Then BCYX; by G, YXBC.
By 19, $ABCY . ABCX . \supset . ACYX \vee BCXY$. Then BCXY; by G, XYBC.
By 14, $ABCX . ABCY . \supset . ABXY \vee ABYX$, whence by G, $XYAB \vee YXAB$.
If XYAB, then by 11, $XYAB . XYBC . \supset . XYAC$; by G, ACXY.
If YXAB, then by 11, $YXAB . YXBC . \supset . YXAC$; by G, ACYX.
Therefore $ACXY \vee ACYX$.

THEOREM 15. 20. *Proof of* 15 *from* G, 11, 16, 17.

To prove: $ABCX . ABCY . \supset . ACXY \vee ACYX$.
Suppose both ACXY and ACYX are false.
By 17, $ABCX . ABCY . \supset . ABXY \vee ACYX$. Then ABXY; by G, XYAB.

By 17, ABCY . ABCX . ⊃ . ABYX ∨ ACXY. Then ABYX; by G, YXAB.
By 16, ABCX . ABCY . ⊃ . BCXY ∨ BCYX; by G, XYBC ∨ YXBC.
If XYBC, then by 11, XYAB . XYBC . ⊃ . XYAC; by G, ACXY.
If YXBC, then by 11, YXAB . YXBC . ⊃ . YXAC; by G, ACYX.
Therefore ACXY ∨ ACYX.

THEOREM 15. 21. *Proof of* 15 *from* G, 11, 17, 19.

To prove: ABCX . ABCY . ⊃ . ACXY ∨ ACYX.
Suppose ACYX is false.
By 17, ABCX . ABCY . ⊃ . ABXY ∨ ACYX. Then ABXY; by G, XYAB.
By 19, ABCY . ABCX . ⊃ . ACYX ∨ BCXY. Then BCXY; by G, XYBC.
By 11, XYAB . XYBC . ⊃ . XYAC, whence by G, ACXY.
Therefore ACXY ∨ ACYX.

THEOREM 15. 22. *Proof of* 15 *from* H, 10, 14.

To prove: ABCX . ABCY . ⊃ . ACXY ∨ ACYX.
By 14, ABCX . ABCY . ⊃ . ABXY ∨ ABYX.

Case 1. If ABXY, then by 10, ABXY . C . ⊃ . ACXY ∨ ABXC.
But ABXC . VS . ABCX, by H. Therefore ACXY.

Case 2. If ABYX, then by 10, ABYX . C . ⊃ . ACYX ∨ ABYC.
But ABYC . VS . ABCY, by H. Therefore ACYX.
Therefore ACXY ∨ ACYX.

THEOREM 15. 23. *Proof of* 15 *from* H, 10, 18.

To prove: ABCX . ABCY . ⊃ . ACXY ∨ ACYX.
By 18, ABCX . ABCY . ⊃ . ABXY ∨ BCYX.
By 18, ABCY . ABCX . ⊃ . ABYX ∨ BCXY.

Case 1. If ABXY, then by 10, ABXY . C . ⊃ . ACXY ∨ ABXC.
But ABXC . VS . ABCX, by H. Therefore ACXY.

Case 2. If BCYX, then by H, BCXY is false. Then ABYX is true.
Then by 10, ABYX . C . ⊃ . ACYX ∨ ABYC.
But ABYC . VS . ABCY, by H. Therefore ACYX.
Therefore ACXY ∨ ACYX.

THEOREM 15. 24. *Proof of* 15 *from* H, 13, 18.

To prove: ABCX . ABCY . ⊃ . ACXY ∨ ACYX.
By 18, ABCX . ABCY . ⊃ . ABXY ∨ BCYX.
By 18, ABCY . ABCX . ⊃ . ABYX ∨ BCXY.

Case 1. If ABXY, then by 13, ABCX . ABXY . ⊃ . ACXY.

Case 2. If BCYX, then by H, BCXY is false. Then ABYX is true.
Then by 13, ABCY . ABYX . ⊃ . ACYX.
Therefore ACXY ∨ ACYX.

THEOREM 15. 25. *Proof of* 15 *from* R, G, 11, 14.

To prove: ABCX . ABCY . ⊃ . ACXY ∨ ACYX.
By R, G, ABCX . ⊃ . CBAX. By R, G, ABCY . ⊃ . CBAY.
By 14, ABCX . ABCY . ⊃ . ABXY ∨ ABYX.
By 14, CBAX . CBAY . ⊃ . CBXY ∨ CBYX.

Case 1. If ABXY and CBYX, by R, G, XYAB and XYBC.
By 11, XYAB . XYBC . ⊃ . XYAC. By G, ACXY.

Case 2. If ABYX and CBXY, by R, G, YXAB and YXBC.
By 11, YXAB . YXBC . ⊃ . YXAC. By G, ACYX.

Case 3. If ABXY and CBXY, by R, YXBA and YXBC.
By 14, YXBA. YXBC . ⊃ . YXAC ∨ YXCA . By R, G, ACYX ∨ ACXY.
Case 4. If ABYX and CBYX, by R, XYBA and XYBC.
By 14, XYBA . XYBC . ⊃ . XYAC ∨ XYCA. By R, G, ACXY ∨ ACYX.
Therefore ACXY ∨ ACYX.

THEOREM 15. 26. *Proof of* 15 *from* R, G, 11, 16.

Proof: 14 from R, G, 16, by Theorem 14. 20; then 15 from R, G, 11, 14 by Theorem 15. 25.

THEOREM 15. 27. *Proof of* 15 *from* R, G, 11, 17.

Proof: 14 from G, 11, 17 by Theorem 14. 15; then 15 from R, G, 11, 14 by Theorem 15. 25.

THEOREM 15. 28. *Proof of* 15 *from* R, G, 11, 19.

Proof: 14 from R, G, 11, 19 by Theorem 14. 21; then 15 from R, G, 11, 14 by Theorem 15. 25.

THEOREM 15. 29. *Proof of* 15 *from* R, G, 14, 17.

To prove: ABCX . ABCY . ⊃ . ACXY ∨ ACYX.
Suppose ACXY = 0 and ACYX = 0.
By 17, ABCX . ABCY . ⊃ . ABXY ∨ ACYX. Hence ABXY. By R, YXBA.
By R, G, ABCX . ⊃ . CBAX. By R, G, ABCY . ⊃ . CBAY.

By 17, CBAX . CBAY . ⊃ . CBXY ∨ CAYX. By R, G, YXBC ∨ ACXY. Hence YXBC.
By 14, YXBA . YXBC . ⊃ . YXAC ∨ YXCA. By R, G, ACYX ∨ ACXY.
Therefore ACXY ∨ ACYX.

THEOREM 15. 30. *Proof of* 15 *from* R, G, 14, 19.

Proof: 17 from R, G, 19 by Theorem 17. 27 (below); then 15 from R, G, 14, 17 by Theorem 15. 29.

THEOREM 15. 31. *Proof of* 15 *from* R, G, 16, 17.

Proof: 14 from R, G, 16 by Theorem 14. 20; then 15 from R, G, 14, 17 by Theorem 15. 29.

THEOREM 15. 32. *Proof of* 15 *from* R, G, 16, 19.

Proof: 17 from R, G, 19 by Theorem 17. 27 (below); then 15 from R, G, 16, 17 by Theorem 15. 31.

THEOREM 15. 33. *Proof of* 15 *from* R, G, H, 14.

To prove: ABCX . ABCY . ⊃ . ACXY ∨ ACYX.
By G, ABCY . ⊃ . YABC. By G, ABCX . ⊃ . XABC.
By R, G, ABCX . ⊃ . AXCB. By R, G, ABCY . ⊃ . AYCB.
Suppose ACXY = 0 and ACYX = 0.
By 14, ABCX . ABCY . ⊃ . ABXY ∨ ABYX.

Case 1. If ABXY, then by G, YABX.
By 14, YABX . YABC . ⊃ . YAXC ∨ YACX. By G, AXCY ∨ ACXY. Hence AXCY.
By 14, AXCY . AXCB . ⊃ . AXYB ∨ AXBY. By R, G, ABYX ∨ YAXB.
But ABYX . vs . ABXY by H; and YAXB . vs . YABX, by H.

Case 2. If ABYX, then by G, XABY.
By 14, XABY . XABC . ⊃ . XAYC ∨ XACY. By G, AYCX ∨ ACYX. Hence AYCX.
By 14, AYCX . AYCB . ⊃ . AYXB ∨ AYBX. By R, G, ABXY ∨ XAYB.
But ABXY . vs . ABYX, by H; and XAYB . vs . XABY, by H.
Therefore ACXY ∨ ACYX.

THEOREM 15. 34. *Proof of* 15 *from* R, G, H, 16.

Proof: 14 from R, G, 16 by Theorem 14. 20; then 15 from R, G, H, 14. by Theorem 15. 33.

Theorem 15. 35. *Proof of* 15 *from* R, F, G, 12.

Proof: 14 from R, F, G, 12 by Theorem 14. 26; then 15 from G, 12, 14 by Theorem 15. 3.

Theorem 15. 36. *Proof of* 15 *from* R, F, G, 13.

Proof: 12 from G, 13 by Theorem 12. 2; hence 15 from R, F, G, 12, by Theorem 15. 35.

Theorem 15. 37. *Proof of* 15 *from* R, H, 11, 18.

To prove: ABCX . ABCY ⊃ . ACXY ∨ ACYX.
By 18, ABCX . ABCY . ⊃ . ABXY ∨ BCYX.
By 18, ABCY . ABCX . ⊃ . ABYX ∨ BCXY.

Case 1. If ABXY is true, then ABYX = 0, by H. Hence BCXY.
By R, ABXY . ⊃ . YXBA. By R, BCXY . ⊃ . YXCB.
By 11, YXCB . YXBA . ⊃ . YXCA. By R, ACXY.

Case 2. If BCYX is true, then BCXY = 0, by H. Hence ABYX.
By R, BCYX . ⊃ . XYCB. By R, ABYX . ⊃ . XYBA.
By 11, XYCB . XYBA . ⊃ . XYCA. By R, ACYX.
Therefore ACXY ∨ ACYX.

Theorem 15. 38. *Proof of* 15 *from* R, 11, 12, 14.

To prove: ABCX . ABCY . ⊃ . ACXY ∨ ACYX.
By 14, ABCX . ABCY . ⊃ . ABXY ∨ ABYX.

Case 1. If ABXY, then by 12, ABCX . ABXY . ⊃ . BCXY. By R, YXCB.
By R, ABXY . ⊃ . YXBA.
By 11, YXCB . YXBA . ⊃ . YXCA. By R, ACXY.

Case 2. If ABYX, then by 12, ABCY . ABYX . ⊃ . BCYX. By R, XYCB.
By R, ABYX . ⊃ . XYBA.
By 11, XYCB . XYBA . ⊃ . XYCA. By R, ACYX.
Therefore ACXY ∨ ACYX.

Theorem 15. 39. *Proof of* 15 *from* R, 11, 12, 17.

To prove: ABCX . ABCY . ⊃ . ACXY ∨ ACYX.
Suppose ACYX = 0.
By 17, ABCX . ABCY . ⊃ . ABXY ∨ ACYX. Hence ABXY. By R, YXBA.
By 12, ABCX . ABXY . ⊃ . BCXY. By R, YXCB.
By 11, YXCB . YXBA . ⊃ . YXCA. By R, ACXY.
Therefore ACXY ∨ ACYX.

THEOREM 15. 40. *Proof of* 15 *from* R, 11, 17, 19.

To prove: ABCX . ABCY . ⊃ . ACXY ∨ ACYX.
Suppose ACYX = 0.
By 17, ABCX . ABCY . ⊃ . ABXY ∨ ACYX. Hence ABXY. By R, YXBA.
By 19, ABCY . ABCX . ⊃ . ACYX ∨ BCXY. Hence BCXY. By R, YXCB.
By 11, YXCB . XYBA . ⊃ . YXCA. By R, ACXY.
Therefore ACXY ∨ ACYX.

THEOREM 15. 41. *Proof of* 15 *from* R, 11, 14, 19.

To prove: ABCX . ABCY . ⊃ . ACXY ∨ ACYX.
Suppose ACXY = 0 and ACYX = 0.
By 19, ABCX . ABCY . ⊃ . ACXY ∨ BCYX. Hence BCYX. By R, XYCB.
By 19, ABCY . ABCX . ⊃ . ACYX ∨ BCXY. Hence BCXY. By R, YXCB.
By 14, ABCX . ABCY . ⊃ . ABXY ∨ ABYX. By R, YXBA ∨ XYBA.
If YXBA, by 11, YXCB . YXBA . ⊃ . YXCA; by R, ACXY.
If XYBA, by 11, XYCB . XYBA . ⊃ . XYCA; by R, ACYX.
Therefore ACXY ∨ ACYX.

THEOREM 15. 42. *Proof of* 15 *from* R, 11, 16, 17.

To prove: ABCX . ABCY . ⊃ . ACXY ∨ ACYX.
Suppose ACXY = 0 and ACYX = 0.
By 17, ABCX . ABCY . ⊃ . ABXY ∨ ACYX. Hence ABXY. By R, YXBA.
By 17, ABCY . ABCX . ⊃ . ABYX ∨ ACXY. Hence ABYX. By R, XYBA.
By 16, ABCX . ABCY . ⊃ . BCXY ∨ BCYX. By R, YXCB ∨ XYCB.
If YXCB, by 11, YXCB . YXBA . ⊃ . YXCA. By R, ACXY.
If XYCB, by 11, XYCB . XYBA . ⊃ . XYCA. By R, ACYX.
Therefore ACXY ∨ ACYX.

THEOREM 15. 43. *Proof of* 15 *from* R, 10, 12, 17.

Proof: Postulate 11 follows from R, 10, 12, 17, by Theorem 11. 6; then 15 from R, 11, 12, 17, by Theorem 15. 39.

THEOREM 15. 44. *Proof of* 15 *from* R, H, 10, 12, 16.

Proof: Postulate 19 follows from R, H, 10, 12, 16, by Theorem 19. 37 (below); then 15 follows from H, 19, by Theorem 15. 16.

THEOREM 15. 45. *Proof of* 15 *from* R, 10, 12, 14.

The proof of this theorem is left to the reader.

Thirty-One Proofs of Postulate 16.

Theorem 16. 1. *Proof of* 16 *from* G, H, 10.

To prove: ABCX . ABCY . $\supset$. BCXY $\cup$ BCYX.

By G, ABCX . $\supset$. BCXA; and by G, ABCY . $\supset$. BCYA.

Suppose both BCXY and BCYX are false.

By 10, BCXA . Y . $\supset$. BYXA $\cup$ BCXY. Hence BYXA. By G, ABYX.

By 10, BCYA . X . $\supset$. BXYA $\cup$ BCYX. Hence BXYA. By G, ABXY.

But ABYX . vs . ABXY, by H.

Therefore BCXY $\cup$ BCYX.

Theorem 16. 2. *Proof of* 16 *from* G, 10, 12.

To prove: ABCX . ABCY . $\supset$. BCXY $\cup$ BCYX.

By G, ABCY . $\supset$. BCYA.

By 10, BCYA . X . $\supset$. BXYA $\cup$ BCYX.

If BXYA, then by G, ABXY. By 12, ABCX . ABXY . $\supset$. BCXY.

Therefore BCXY $\cup$ BCYX.

Theorem 16. 3. *Proof of* 16 *from* 12, 14.

To prove: ABCX . ABCY . $\supset$. BCXY $\cup$ BCYX.

By 14, ABCX . ABCY . $\supset$. ABXY $\cup$ ABYX.

If ABXY, then by 12, ABCX . ABXY . $\supset$. BCXY.

If ABYX, then by 12, ABCY . ABYX . $\supset$. BCYX.

Therefore BCXY $\cup$ BCYX.

Theorem 16. 4. *Proof of* 16 *from* G, 12, 15.

To prove: ABCX . ABCY . $\supset$. BCXY $\cup$ BCYX.

By G, ABCX . $\supset$. CXAB. By G, ABCY . $\supset$. CYAB.

By 15, ABCX . ABCY . $\supset$. ACXY $\cup$ ACYX. By G, CXYA $\cup$ CYXA.

Case 1. If CXYA, by 12, CXYA . CXAB . $\supset$. XYAB. By G, ABXY.
By 12, ABCX . ABXY . $\supset$. BCXY.

Case 2. If CYXA, by 12, CYXA . CYAB . $\supset$. YXAB. By G, ABYX.
By 12, ABCY . ABYX . $\supset$. BCYX.
Therefore BCXY $\cup$ BCYX.

Theorem 16. 5. *Proof of* 16 *from* G, 12, 17.

To prove: ABCX . ABCY . $\supset$. BCXY $\cup$ BCYX.

By 17, ABCX . ABCY . $\supset$. ABXY $\cup$ ACYX.

Case 1. If ABXY, by 12, ABCX . ABXY . ⊃ . BCXY.

Case 2. If ACYX, by G, CYXA. By G, ABCY . ⊃ . CYAB.
By 12, CYXA . CYAB . ⊃ . YXAB. By G, ABYX.
By 12, ABCY . ABYX . ⊃ . BCYX.
Therefore BCXY ∪ BCYX.

THEOREM 16. 6. *Proof of* 16 *from* 12, 18.

To prove: ABCX . ABCY . ⊃ . BCXY ∪ BCYX.
By 18, ABCX . ABCY . ⊃ . ABXY ∪ BCYX.
If ABXY, by 12, ABCX . ABXY . ⊃ . BCXY.
Therefore BCXY ∪ BCYX.

THEOREM 16. 7. *Proof of* 16 *from* G, 12, 19.

To prove: ABCX . ABCY . ⊃ . BCXY ∪ BCYX.
Suppose BCYX is false.
By 19, ABCX . ABCY . ⊃ . ACXY ∪ BCYX. Hence ACXY. By G, CXYA.
By G, ABCX . ⊃ . CXAB.
By 12, CXYA . CXAB . ⊃ . XYAB. By G, ABXY.
By 12, ABCX . ABXY . ⊃ . BCXY.
Therefore BCYX ∪ BCXY.

THEOREM 16. 8. *Proof of* 16 *from* G, 10, 13.

THEOREM 16. 9. *Proof of* 16 *from* G, 13, 14.

THEOREM 16. 10. *Proof of* 16 *from* G, 13, 15.

THEOREM 16. 11. *Proof of* 16 *from* G, 13, 17.

THEOREM 16. 12. *Proof of* 16 *from* G, 13, 18.

THEOREM 16. 13. *Proof of* 16 *from* G, 13, 19.

These six theorems follow from Theorems 16. 2–16. 7, by the aid of Theorem 12. 2 (which says that 12 follows from G, 13).

THEOREM 16. 14. *Proof of* 16 *from* G, 11, 15.

To prove: ABCX . ABCY . BCXY ∪ BCYX.
By G, ABCX . ⊃ . CXAB. By G, ABCY . ⊃ . CYAB.
By 15, ABCX . ABCY . ⊃ . ACXY ∪ ACYX. By G, CXYA ∪ CYXA.

Case 1. If CXYA, then by 11, CXYA . CXAB . ⊃ . CXYB. By G, BCXY.

Case 2. If CYXA, then by 11, CYXA . CYAB . ⊃ . CYXB. By G, BCYX. Therefore BCXY ∨ BCYX.

THEOREM 16. 15. *Proof of* 16 *from* G, 11, 19.

To prove: ABCX . ABCY . ⊃ . BCXY ∨ BCYX.
Suppose BCYX is false. By G, ABCX . ⊃ . CXAB.
By 19, ABCX . ABCY . ⊃ . ACXY ∨ BCYX. Hence ACXY. By G, CXYA.
By 11, CXYA . CXAB . ⊃ . CXYB. By G, BCXY.
Therefore BCXY ∨ BCYX.

THEOREM 16. 16. *Proof of* 16 *from* G, H, 14.

To prove: ABCX . ABCY . ⊃ . BCXY ∨ BCYX.
By G, ABCY . ⊃ . YABC. By G, ABCX . ⊃ . XABC.
By G, ABCY . ⊃ . CYAB. By G, ABCY . ⊃ . BCYA.
Suppose BCXY = O and BCYX = O. By G, CYXB = O and XYBC = O.
By 14, ABCX . ABCY . ⊃ . ABXY ∨ ABYX. By G, YABX ∨ XABY.
If YABX, then by 14, YABX . YABC . ⊃ . YAXC ∨ YACX.
If XABY, then by 14, XABY . XABC . ⊃ . XAYC ∨ XACY.

Case 1. Suppose YABX and YAXC.
By G, YAXC . ⊃ . CYAX. By G, YAXC . ⊃ . XCYA.
By 14, CYAX . CYAB . ⊃ . CYXB ∨ CYBX. Hence CYBX. By G, XCYB.
By 14, XCYB . XCYA . ⊃ . XCBA ∨ XCAB. By G, AXCB ∨ ABXC.
But ABXC . VS . ABCX, by H. Hence AXCB.
By G, YAXC . ⊃ . AXCY. By G, YABX . ⊃ . XYAB.
By 14, AXCY . AXCB . ⊃ . AXYB ∨ AXBY. By G, XYBA ∨ YAXB.
But XYBA . VS . XYAB, by H; and YAXB . VS . YABX, by H.
Therefore Case 1 is impossible.

Case 2. Suppose YABX and YACX.
By G, YABX . ⊃ . XYAB. By G, YACX . ⊃ . XYAC.
By G, YABX . ⊃ . BXYA.
By 14, XYAB . XYAC . ⊃ . XYBC ∨ XYCB. Hence XYCB. By G, BXYC.
By 14, BXYC . BXYA . ⊃ . BXCA ∨ BXAC. By G, ABXC ∨ XACB.
But ABXC . VS . ABCX, by H; and XACB . VS . XABC, by H.
Therefore Case 2 is impossible.

Case 3. Suppose XABY and XAYC.
By interchanging X and Y in Case 1, we see that Case 3 is impossible.

Case 4. Suppose XABY and XACY.
By interchanging X and Y in Case 2, we see that Case 4 is impossible.
Therefore BCXY ∨ BCYX.

THEOREM 16. 17. *Proof of* 16 *from* G, H, 17.

To prove: ABCX . ABCY . ⊃ . BCXY ∨ BCYX.
By 17, ABCX . ABCY . ⊃ . ABXY ∨ ACYX. By G, XYAB ∨ YXAC.
By 17, ABCY . ABCX . ⊃ . ABYX ∨ ACXY. By G, YXAB ∨ XYAC.

Case 1. Suppose XYAB and XYAC. By G, ABCX . ⊃ . XABC.
By 17, XYAB . XYAC . ⊃ . XYBC ∨ XACB.
But XACB . vs . XABC, by H. Hence XYBC. By G, BCXY.

Case 2. Suppose YXAB and YXAC. By G, ABCY . ⊃ . YABC.
By 17, YXAB . YXAC . ⊃ . YXBC ∨ YACB.
But YACB . vs . YABC, by H. Hence YXBC. By G, BCYX.

Case 3. Suppose XYAB and YXAB. By G, ABXY and ABYX.
But ABXY . vs . ABYX, by H.

Case 4. Suppose YXAC and XYAC. By G, ACYX and ACXY.
But ACYX . vs . ACXY, by H.
Therefore BCXY ∨ BCYX.

THEOREM 16. 18. *Proof of* 16 *from* H, 18.

To prove: ABCX . ABCY . ⊃ . BCXY ∨ BCYX.
Suppose both BCXY and BCYX are false.
By 18, ABCX . ABCY . ⊃ . ABXY ∨ BCYX. Hence ABXY.
By 18, ABCY . ABCX . ⊃ . ABYX ∨ BCXY. Hence ABYX.
But ABXY . vs . ABYX, by H.
Therefore BCXY ∨ BCYX.

THEOREM 16. 19. *Proof of* 16 *from* H, 19.

To prove: ABCX . ABCY . ⊃ . BCXY ∨ BCYX.
Suppose both BCXY and BCYX are false.
By 19, ABCX . ABCY . ⊃ . ACXY ∨ BCYX. Hence ACXY.
By 19, ABCY . ABCX . ⊃ . ACYX ∨ BCXY. Hence ACYX.
But ACXY . vs . ACYX, by H.
Therefore BCXY ∨ BCYX.

THEOREM 16. 20. *Proof of* 16 *from* H, 12, 17.

To prove: ABCX . ABCY . ⊃ . BCXY ∨ BCYX.
By 17, ABCX . ABCY . ⊃ . ABXY ∨ ACYX.
By 17, ABCY . ABCX . ⊃ . ABYX ∨ ACXY.

Case 1. If ABXY, by 12, ABCX . ABXY . ⊃ . BCXY.

Case 2. If ABYX, by 12, ABCY . ABYX . $\supset$. BCYX.

Case 3. If both ABXY and ABYX are false, then ACYX and ACXY, which is impossible, by H.

Therefore BCXY $\vee$ BCYX.

THEOREM 16. 21. *Proof of* 16 *from* R, G, 14.

To prove: ABCX . ABCY . $\supset$. BCXY $\vee$ BCYX.
By R, G, ABCX . $\supset$. CBAX. By R, G, ABCY . $\supset$. CBAY.
By 14, CBAX . CBAY . $\supset$. CBXY $\vee$ CBYX.
If CBXY, then by R, G, BCYX; and if CBYX, by R, G, BCXY.
Therefore BCXY $\vee$ BCYX.

THEOREM 16. 22. *Proof of* 16 *from* R, G, 11, 17.

Proof by Theorems 19. 26 and 16. 15.

THEOREM 16. 23. *Proof of* 16 *from* R, G, 10, 15, 17.

THEOREM 16. 24. *Proof of* 16 *from* R, G, 10, 15, 19.

THEOREM 16. 25. *Proof of* 16 *from* R, G, 15, 17, 18.

THEOREM 16. 26. *Proof of* 16 *from* R, G, 15, 18, 19.

THEOREM 16. 27. *Proof of* 16 *from* R, F, G, 12.

THEOREM 16. 28. *Proof of* 16 *from* R, F, G, 13.

These six theorems follow from Theorems 14. 21–14. 27 (which give postulate 14), by the aid of Theorem 16. 21.

THEOREM 16. 29. *Proof of* 16 *from* R, 10, 12, 19.

To prove: ABCX . ABCY . $\supset$. BCXY $\vee$ BCYX.
Suppose BCXY $= 0$ and BCYX $= 0$. By R, YXCB $= 0$ and XYCB $= 0$.
By 19, ABCX . ABCY . $\supset$. ACXY $\vee$ BCYX. Then ACXY. By R, YXCA.
By 19, ABCY . ABCX . $\supset$. ACYX $\vee$ BCXY. Then ACYX. By R, XYCA.
By 10, YXCA . B . $\supset$. YBCA $\vee$ YXCB. Then YBCA. By R, ACBY.
By 10, XYCA . B . $\supset$. XBCA $\vee$ XYCB. Then XBCA. By R, ACBX.
Now ABXY $= 0$ (for if not, by 12, ABCX . ABXY . $\supset$. BCXY). By R, YXBA $= 0$.

Also, ABYX = o (for if not, by 12, ABCY . ABYX . ⊃ . BCYX). By R, XYBA = o.
By R, ABCX . ⊃ . XCBA.
By 10, XCBA . Y . ⊃ . XYBA ∨ XCBY. Then XCBY. By R, YBCX.
By R, ABCY . ⊃ . YCBA.
By 10, YCBA . X . ⊃ . YXBA ∨ YCBX. Then YCBX. By R, XBCY.
By 19, ACBX . ACBY . ⊃ . ABXY ∨ CBYX. Then CBYX. By R, XYBC.
By 19, ACBY . ACBX . ⊃ . ABYX ∨ CBXY. Then CBXY. By R, YXBC.
By 10, XYBC . A . ⊃ . XABC ∨ XYBA. Then XABC. By R, CBAX.
By 10, YXBC . A . ⊃ . YABC ∨ YXBA. Then YABC. By R, CBAY.
By 10, ACXY . B . ⊃ . ABXY ∨ ACXB. Then ACXB.
By 10, ACYX . B . ⊃ . ABYX ∨ ACYB. Then ACYB.
By 12, ACXB . ACBY . ⊃ . CXBY.
By 12, ACYX . ACXB . ⊃ . CYXB.
By 12, ACXY . ACYB . ⊃ . CXYB.
Now CXBA = o (for if not, by 12, CXYB . CXBA . ⊃ . XYBA).
Also, CYBA = o (for if not, by 12, CYXB . CYBA . ⊃ . YXBA).
By 10, CXBY . A . ⊃ . CABY ∨ CXBA. Then CABY. By R, YBAC.
By 12, YBAC . YBCX . ⊃ . BACX.
By 12, CBAX . CBXY . ⊃ . BAXY.
By 12, CBAY . CBYX . ⊃ . BAYX.
By 10, BAXY . C . ⊃ . BCXY ∨ BAXC. Then BAXC.
By 10, BAYX . C . ⊃ . BCYX ∨ BAYC. Then BAYC.
By 12, BAYC . BACX . ⊃ . AYCX. By R, XCYA.
By 12, BAYX . BAXC . ⊃ . AYXC.
By R, ABCX . ⊃ . XCBA.
Now, XCYB = o (for if not, by 12, XCYB . XCBA . ⊃ . CYBA).
By 10, XCYA . B . ⊃ . XBYA ∨ XCYB. Then XBYA.
By 12, XBCY . XBYA . ⊃ . BCYA. By R, AYCB.
By 12, AYXC . AYCB . ⊃ . YXCB, which is false.
Therefore BCXY ∨ BCYX.

THEOREM 16. 30. *Proof of* 16 *from* R, 10, 12, 15.

Proof: 19 from R, 10, 12, 15, by Theorem 19. 38 (below) then 16 from R, 10, 12, 19, by Theorem 16. 29.

THEOREM 16. 31. *Proof of* 16 *from* R, 10, 12, 17.

Proof: 15 from R, 10, 12, 17, by Theorem 15. 43; then 16 from R, 10, 12, 15 by Theorem 16. 30.

Thirty-Nine Proofs of Postulate 17.

THEOREM 17. 1. *Proof of* 17 *from* G, H, 10.

To prove: $ABCX . ABCY . \supset . ABXY \vee ACYX$.
By G, $ABCX . \supset . BCXA$. By G, $ABCY . \supset . BCYA$.
Suppose $ABXY$ is false. By G, $BXYA$ is false.
By 10, $BCYA . X . \supset . BXYA \vee BCYX$. Hence $BCYX$. By G, $CYXB$.
By 10, $CYXB . A . \supset . CAXB \vee CYXA$. By G, $BCAX \vee ACYX$.
But $BCAX . \text{VS} . BCXA$, by H. Hence $ACYX$.
Therefore $ABXY \vee ACYX$.

THEOREM 17. 2. *Proof of* 17 *from* G, 10, 12.

To prove: $ABCX . ABCY . \supset . ABXY \vee ACYX$.
By G, $ABCX . \supset . BCXA$. By G, $ABCY . \supset . BCYA$.
Suppose $ABXY$ is false. Then by G, $BXYA$ is false.
By 10, $BCYA . X . \supset . BXYA \vee BCYX$. Hence $BCYX$.
By 12, $BCYX . BCXA . \supset . CYXA$. By G, $ACYX$.
Therefore $ABXY \vee ACYX$.

THEOREM 17. 3. *Proof of* 17 *from* G, 12, 14.

To prove: $ABCX . ABCY . \supset . ABXY \vee ACYX$.
By 14, $ABCX . ABCY . \supset . ABXY \vee ABYX$.
Suppose $ABXY$ is false. Then $ABYX$.
By 12, $ABCY . ABYX . \supset . BCYX$. By G, $ABCX . \supset . BCXA$.
By 12, $BCYX . BCXA . \supset . CYXA$. By G, $ACYX$.
Therefore $ABXY \vee ACYX$.

THEOREM 17. 4. *Proof of* 17 *from* G, 12, 15.

To prove: $ABCX . ABCY . \supset . ABXY \vee ACYX$.
By 15, $ABCX . ABCY . \supset . ACXY \vee ACYX$.
Suppose $ACYX$ is false. Hence $ACXY$. By G, $CXYA$.
By G, $ABCX . \supset . CXAB$.
By 12, $CXYA . CXAB . \supset . XYAB$. By G, $ABXY$.
Therefore $ABXY \vee ACYX$.

THEOREM 17. 5. *Proof of* 17 *from* G, 12, 16.

To prove: $ABCX . ABCY . \supset . ABXY \vee ACYX$.
By 16, $ABCX . ABCY . \supset . BCXY \vee BCYX$. By G, $ABCX . \supset . BCXA$.
If $BCYX$ is true, then by 12, $BCYX . BCXA . \supset . CYXA$, whence by G, $ACYX$.
By G, $ABCY . \supset . BCYA$.

If BCXY is true, then by 12, BCXY . BCYA . ⊃ . CXYA.
By G, ABCX . ⊃ . CXAB.
By 12, CXYA . CXAB . ⊃ . XYAB. By G, XYAB . ⊃ . ABXY.
Therefore ABXY ∨ ACYX.

Theorem 17. 6. *Proof of* 17 *from* G, 12, 18.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ ACYX.
Suppose ABXY is false. By G, ABCX . ⊃ . BCXA.
By 18, ABCX . ABCY . ⊃ . ABXY ∨ BCYX. Hence BCYX.
By 12, BCYX . BCXA . ⊃ . CYXA. By G, ACYX.
Therefore ABXY ∨ ACYX.

Theorem 17. 7. *Proof of* 17 *from* G, 12, 19.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ ACYX.
Suppose ACYX is false.
By 19, ABCY . ABCX . ⊃ . ACYX ∨ BCXY. Hence BCXY.
By G, ABCY . ⊃ . BCYA. By G, ABCX . ⊃ . CXAB.
By 12, BCXY . BCYA . ⊃ . CXYA.
By 12, CXYA . CXAB . ⊃ . XYAB. By G, ABXY.
Therefore ABXY ∨ ACYX.

Theorem 17. 8. *Proof of* 17 *from* G, 10, 13.

This theorem follows from Theorem 17. 2 by aid of Theorem 12. 2.

Theorem 17. 9. *Proof of* 17 *from* 13, 14.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ ACYX.
By 14, ABCX . ABCY . ⊃ . ABXY ∨ ABYX.
If ABYX, then by 13, ABCY . ABYX . ⊃ . ACYX.
Therefore ABXY ∨ ACYX.

Theorem 17. 10. *Proof of* 17 *from* G, 13, 15.

Theorem 17. 11. *Proof of* 17 *from* G, 13, 16.

Theorem 17. 12. *Proof of* 17 *from* G, 13, 18.

Theorem 17. 13. *Proof of* 17 *from* G, 13, 19.

These four theorems follow from Theorems 17. 4–17. 7 by aid of Theorem 12. 2.

THEOREM 17. 14. *Proof of 17 from* G, 11, 15.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ ACYX.
Suppose ACYX is false. By G, ABCY . ⊃ . YABC.
By 15, ABCX . ABCY . ⊃ . ACXY ∨ ACYX. Hence ACXY. By G, YACX.
By 11, YABC . YACX . ⊃ . YABX. By G, ABXY.
Therefore ABXY ∨ ACYX.

THEOREM 17. 15. *Proof of 17 from* G, 10, 11, 14.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ ACYX.
By G, ABCY . ⊃ . BCYA.
By 14, ABCX . ABCY . ⊃ . ABXY ∨ ABYX. By G, ABXY ∨ YXAB.
By 10, BCYA . X . ⊃ . BXYA ∨ BCYX. By G, ABXY ∨ YXBC.
Suppose ABXY is false. By 11, YXAB . YXBC . ⊃ . YXAC. By G, ACYX.
Therefore ABXY ∨ ACYX.

THEOREM 17. 16. *Proof of 17 from* G, 11, 14, 18.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ ACYX.
Suppose ABXY is false.
By 14, ABCX . ABCY . ⊃ . ABXY ∨ ABYX. Hence ABYX. By G, YXAB.
By 18, ABCX . ABCY . ⊃ . ABXY ∨ BCYX. Hence BCYX. By G, YXBC.
By 11, YXAB . YXBC . ⊃ . YXAC. By G, ACYX.
Therefore ABXY ∨ ACYX.

THEOREM 17. 17. *Proof of 17 from* G, 11, 14, 19.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ ACYX.
Suppose ABXY is false. By G, ABCY . ⊃ . YABC.
By 14, ABCX . ABCY . ⊃ . ABXY ∨ ABYX. Hence ABYX. By G, YXAB.
By 19, ABCX . ABCY . ⊃ . ACXY ∨ BCYX. By G, YACX ∨ YXBC.

Case 1. If YACX, by 11, YABC . YACX . ⊃ . YABX. By G, ABXY, which is false.

Case 2. If YXBC, by 11, YXAB . YXBC . ⊃ . YXAC. By G, ACYX.
Therefore ABXY ∨ ACYX.

THEOREM 17. 18. *Proof of 17 from* G, H, 14, 15.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ ACYX.
Suppose ABXY = 0 and ACYX = 0. By G, ABCX . ⊃ . XABC.
By 14, ABCX . ABCY . ⊃ . ABXY ∨ ABYX. Hence ABYX. By G, XABY.

By 15, ABCX . ABCY . ⊃ . ACXY ∨ ACYX. Hence ACXY. By G, CXYA.
By 14, XABC . XABY . ⊃ . XACY ∨ XAYC. By G, ACYX ∨ CXAY.
But ACYX = 0; and CXAY . vs . CXYA, by H.
Therefore ABXY ∨ ACYX.

THEOREM 17. 19. *Proof of* 17 *from* G, H, 15, 16.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ ACYX.
Suppose ABXY = 0 and ACYX = 0. By G, XYAB = 0 and CYXA = 0.
By 15, ABCX . ABCY . ⊃ . ACXY ∨ ACYX. Hence ACXY. By G, CXYA.
By 16, ABCX . ABCY . ⊃ . BCXY ∨ BCYX.

Case 1. Suppose BCXY. By G, CXYB. By G, XYBC.
By G, ABCY . ⊃ . BCYA. By G, ABCY . ⊃ . CYAB.
By 16, CXYB . CXYA . ⊃ . XYBA ∨ XYAB. Hence XYBA.
By 16, XYBC . XYBA . ⊃ . YBCA ∨ YBAC . By G, BCAY ∨ CYBA.
But BCAY . vs . BCYA, by H; and CYBA . vs . CYAB, by H.

Case 2. Suppose BCYX. By G, ABCY . ⊃ . BCYA. By G, CYAB.
By G, ABCX . ⊃ . BCXA.
By 16, BCYX . BCYA . ⊃ . CYXA ∨ CYAX. Hence CYAX.
By 15, CYAX . CYAB . ⊃ . CAXB ∨ CABX. By G, BCAX ∨ ABXC.
But BCAX . vs . BCXA by H; and ABXC vs. ABCX, by H.
Therefore ABXY ∨ ACYX.

THEOREM 17. 20. *Proof of* 17 *from* G, H, 18.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ ACYX.
Suppose ABXY is false. By G, ABCY . ⊃ . BCYA. By G, ABCX . ⊃ . BCXA.
By 18, ABCX . ABCY . ⊃ . ABXY ∨ BCYX. Hence BCYX.
By 18, BCYA . BCYX . ⊃ . BCAX ∨ CYXA.
But BCAX . vs . BCXA. Hence CYXA. By G, ACYX.
Therefore ABXY ∨ ACYX.

THEOREM 17. 21. *Proof of* 17 *from* G, H, 19.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ ACYX.
Suppose ABXY = 0 and ACYX = 0. By G, BXYA = 0.
By G, ABCX . ⊃ . BCXA. By G, CXAB. By G, ABCY . ⊃ . BCYA.
By 19, ABCY . ABCX . ⊃ . ACYX ∨ BCXY. Hence BCXY. By G, XYBC.
By 19, BCXY . BCXA . ⊃ . BXYA ∨ CXAY. Hence CXAY. By G, XAYC.
By 19, CXAB . CXAY . ⊃ . CABY ∨ XAYB. By G, ABYC ∨ XAYB.
But ABYC . vs . ABCY, by H. Hence XAYB.

By 19, XAYC . XAYB . ⊃ . XYCB ∨ AYBC. By G, XYCB ∨ BCAY.
But XYCB . vs . XYBC, by H; and BCAY . vs . BCYA, by H.
Therefore ABXY ∨ ACYX.

THEOREM 17. 22. *Proof of* 17 *from* H, 10, 14.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ ACYX.
Suppose ABXY is false.
By 14, ABCX . ABCY . ⊃ . ABXY ∨ ABYX. Hence ABYX.
By 10, ABYX . C . ⊃ . ACYX ∨ ABYC.
But ABYC . vs . ABCY, by H. Hence ACYX.
Therefore ABXY ∨ ACYX.

THEOREM 17. 23. *Proof of* 17 *from* H, 10, 18.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ ACYX.
Suppose ABXY is false.
By 18, ABCX . ABCY . ⊃ . ABXY ∨ BCYX. Hence BCYX.
By 18, ABCY . ABCX . ⊃ . ABYX ∨ BCXY.
But BCXY . vs . BCYX, by H. Hence ABYX.
By 10, ABYX . C . ⊃ . ACYX ∨ ABYC.
But ABYC . vs . ABCY, by H. Hence ACYX.
Therefore ABXY ∨ ACYX.

THEOREM 17. 24. *Proof of* 17 *from* H, 18, 19.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ ACYX.
Suppose both ABXY and ACYX are false.
By 18, ABCX . ABCY . ⊃ . ABXY ∨ BCYX. Hence BCYX.
By 19, ABCY . ABCX . ⊃ . ACYX ∨ BCXY. Hence BCXY.
But BCXY . vs . BCYX by H.
Therefore ABXY ∨ ACYX.

THEOREM 17. 25. *Proof of* 17 *from* H, 13, 18.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ ACYX.
Suppose ABXY is false.
By 18, ABCX . ABCY . ⊃ . ABXY ∨ BCYX. Hence BCYX.
By 18, ABCY . ABCX . ⊃ . ABYX ∨ BCXY.
But BCXY . vs . BCYX, by H. Hence ABYX.
By 13, ABCY . ABYX . ⊃ . ACYX.
Therefore ABXY ∨ ACYX.

Theorem 17. 26. *Proof of* 17 *from* H, 12, 14, 19.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ ACYX.
Suppose both ABXY and ACYX are false.
By 14, ABCX . ABCY . ⊃ . ABXY ∨ ABYX. Hence ABYX.
By 19, ABCY . ABCX . ⊃ . ACYX ∨ BCXY. Hence BCXY.
By 12, ABCY . ABYX . ⊃ . BCYX. But BCYX . VS . BCXY, by H.
Therefore ABXY ∨ ACYX.

Theorem 17. 27. *Proof of* 17 *from* R, G, 19.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ ACYX.
By R, G, ABCX . ⊃ . CBAX. By R, G, ABCY . ⊃ . CBAY.
By 19, CBAX . CBAY . ⊃ . CAXY ∨ BAYX.
But if CAXY, by R, G, ACYX; and if BAYX, by R, G, ABXY.
Therefore ABXY ∨ ACYX.

Theorem 17. 28. *Proof of* 17 *from* R, G, 11, 14.

Proof: 15 from R, G, 11, 14, by Theorem 15. 25; then 17 from G, 11, 15 by Theorem 17. 4.

Theorem 17. 29. *Proof of* 17 *from* R, G, 11, 16.

Proof: 15 from R, G, 11, 16 by Theorem 15. 16; then 17 from G, 11, 15 by Theorem 17. 14.

Theorem 17. 30. *Proof of* 17 *from* R, G, H, 14.

Proof: 15 from R, G, H, 14 by Theorem 15. 33; then 17 from G, H, 14, 15 by Theorem 17. 18.

Theorem 17. 31. *Proof of* 17 *from* R, G, H, 16.

Proof: 15 from R, G, H, 16 by Theorem 15. 34; then 17 from G, H, 15, 16 by Theorem 17. 19.

Theorem 17. 32. *Proof of* 17 *from* R, F, G, 12.

Proof: 15 from R, F, G, 12 by Theorem 15. 35; then 17 from G, 12, 15 by Theorem 17. 4.

Theorem 17. 33. *Proof of* 17 *from* R, F, G, 13.

Proof: 12 from G, 13 by Theorem 12. 2; then 17 from R, F, G, 12 by Theorem 17. 32.

Theorem 17.34. *Proof of* 17 *from* R, H, 11, 18.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ ACYX.
Suppose ABXY = 0.
By 18, ABCX . ABCY . ⊃ . ABXY ∨ BCYX. Hence BCYX. By R, XYCB.
By 18, ABCY . ABCX . ⊃ . ABYX ∨ BCXY. But BCXY . vs . BCYX, by H.
Hence ABYX. By R, XYBA.
By 11, XYCB . XYBA . ⊃ . XYCA. By R, ACYX.
Therefore ABXY ∨ ACYX.

Theorem 17.35. *Proof of* 17 *from* R, 11, 12, 14.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ ACYX.
Suppose ABXY = 0.
By 14, ABCX . ABCY . ⊃ . ABXY ∨ ABYX. Then ABYX. By R, XYBA.
By 12, ABCY . ABYX . ⊃ . BCYX. By R, XYCB.
By 11, XYCB . XYBA . ⊃ . XYCA. By R, ACYX.
Therefore ABXY ∨ ACYX.

Theorem 17.36. *Proof of* 17 *from* R, 11, 14, 18.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ ACYX.
Suppose ABXY = 0.
By 14, ABCX . ABCY . ⊃ . ABXY ∨ ABYX. Then ABYX. By R, XYBA.
By 18, ABCX . ABCY . ⊃ . ABXY ∨ BCYX. Then BCYX. By R, XYCB.
By 11, XYCB . XYBA . ⊃ . XYCA. By R, ACYX.
Therefore ABXY ∨ ACYX.

Theorem 17.37. *Proof of* 17 *from* R, 10, 11, 13, 15, 18.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ ACYX.
Suppose ABXY = 0 and ACYX = 0. By R, YXBA = 0 and XYCA = 0.
By 15, ABCX . ABCY . ⊃ . ACXY ∨ ACYX. Then ACXY. By R, YXCA.
By 18, ABCX . ABCY . ⊃ . ABXY ∨ BCYX. Then BCYX. By R, XYCB.
Now ABYX = 0 (for if not, by 13, ABCY . ABYX . ⊃ . ACYX).
By 18, ABCY . ABCX . ⊃ . ABYX ∨ BCXY. Then BCXY. By R, YXCB.
By 18, YXCB . YXCA . ⊃ . YXBA ∨ XCAB. Then XCAB. By R, BACX.
By 10, ACXY . B . ⊃ . ABXY ∨ ACXB. Then ACXB. By R, BXCA.
By 10, XYCB . A . ⊃ . XACB ∨ XYCA. Then XACB. By R, BCAX.
By R, ABCY . ⊃ . YCBA.
By 10, YCBA . X . ⊃ . YXBA ∨ YCBX. Then YCBX.
By 11, BCAX . BCXY . ⊃ . BCAY. By R, YACB.
By 13, BCAX . BCXY . ⊃ . BAXY.

By 11, BACX . BAXY . ⊃ . BACY. By R, YCAB.
By 13, YCAB . YCBX . ⊃ . YABX.
By 11, YACB . YABX . ⊃ . YACX.
Now YCXB = 0 (for if not, by 13, YCXB . YCBA . ⊃ . YXBA). By R, BXCY = 0.
By 10, BXCA . Y . ⊃ . BYCA ∨ BXCY. Then BYCA. By R, ACYB.
By 18, YACX . YACB . ⊃ . YAXB ∨ ACBX.
But if YAXB, then by R, BXAY, and by 11, BXCA . BXAY . ⊃ . BXCY, which is false, and if ACBX, then by 11, ACYB . ACBX . ⊃ . ACYX, which is false.
Therefore ABXY ∨ ACYX.

THEOREM 17. 38. *Proof of* 17 *from* R, 10, 11, 12, 15, 18.

Proof: 13 from R, 11, 12, by Theorem 13. 12; then 17 from R, 10, 11, 13, 15, 18, by Theorem 17. 37.

THEOREM 17. 39. *Proof of* 17 *from* R, 10, 12, 13, 15, 18.

Proof: 11 from R, 12, 13, by Theorem 11. 4; then 17 from R, 10, 11, 13, 15, 18, by Theorem 17. 37.

THIRTY-ONE PROOFS OF POSTULATE 18.

THEOREM 18. 1. *Proof of* 18 *from* G, 10.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ BCYX.
By G, ABCY . ⊃ . BCYA.
By 10, BCYA . X . ⊃ . BXYA ∨ BCYX.
Hence by G, ABXY ∨ BCYX.

THEOREM 18. 2. *Proof of* 18 *from* 12, 14.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ BCYX.
Suppose ABXY is false.
By 14, ABCX . ABCY . ⊃ . ABXY ∨ ABYX. Hence ABYX.
By 12, ABCY . ABYX . ⊃ . BCYX.
Therefore ABXY ∨ BCYX.

THEOREM 18. 3. *Proof of* 18 *from* G, 12, 15.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ BCYX.
By G, ABCX . ⊃ . CXAB. By G, ABCY . ⊃ . CYAB.
By 15, ABCX . ABCY . ⊃ . ACXY ∨ ACYX. By G, CXYA ∨ CYXA.

Case 1. If CXYA, by 12, CXYA . CXAB . ⊃ . XYAB. By G, ABXY.

Case 2. If CYXA, by 12, CYXA . CYAB . ⊃ . YXAB. By G, ABYX.
By 12, ABCY . ABYX . ⊃ . BCYX.
Therefore ABXY ∨ BCYX.

THEOREM 18. 4. *Proof of* 18 *from* G, 12, 16.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ BCYX.
Suppose BCYX is false.
By 16, ABCX . ABCY . ⊃ . BCXY ∨ BCYX. Hence BCXY.
By G, ABCY . ⊃ . BCYA. By G, ABCX . ⊃ . CXAB.
By 12, BCXY . BCYA . ⊃ . CXYA.
By 12, CXYA . CXAB . ⊃ . XYAB. By G, ABXY.
Therefore ABXY ∨ BCYX.

THEOREM 18. 5. *Proof of* 18 *from* G, 12, 17.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ BCYX.
Suppose ABXY is false.
By 17, ABCX . ABCY . ⊃ . ABXY ∨ ACYX. Hence ACYX. By G, CYXA.
By G, ABCY . ⊃ . CYAB.
By 12, CYXA . CYAB . ⊃ . YXAB. By G, ABYX.
By 12, ABCY . ABYX . ⊃ . BCYX.
Therefore ABXY ∨ BCYX.

THEOREM 18. 6. *Proof of* 18 *from* G, 12, 19.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ BCYX.
Suppose BCYX is false. By G, ABCX . ⊃ . CXAB.
By 19, ABCX . ABCY . ⊃ . ACXY ∨ BCYX. Hence ACXY. By G, CXYA.
By 12, CXYA . CXAB . ⊃ . XYAB. By G, ABXY.
Therefore ABXY ∨ BCYX.

THEOREM 18. 7. *Proof of* 18 *from* G, 13, 14.

THEOREM 18. 8. *Proof of* 18 *from* G, 13, 15.

THEOREM 18. 9. *Proof of* 18 *from* G, 13, 16.

THEOREM 18. 10. *Proof of* 18 *from* G, 13, 17.

Theorem 18. 11. *Proof of* 18 *from* G, 13, 19.

These five theorems follow from Theorems 18. 2 to 18. 6, by aid of Theorem 12. 2.

Theorem 18. 12. *Proof of* 18 *from* G, 11, 15.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ BCYX.
By 15, ABCX . ABCY . ⊃ . ACXY ∨ ACYX.

Case 1. If ACXY, by G, YACX. By G, ABCY . ⊃ . YABC.
By 11, YABC . YACX . ⊃ . YABX. By G, ABXY.

Case 2. If ACYX, by G, CYXA. By G, ABCY . ⊃ . CYAB.
By 11, CYXA . CYAB . ⊃ . CYXB. By G, BCYX.
Therefore ABXY ∨ BCYX.

Theorem 18. 13. *Proof of* 18 *from* G, 11, 17.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ BCYX.
Suppose ABXY is false. By G, ABCY . ⊃ . CYAB.
By 17, ABCX . ABCY . ⊃ . ABXY ∨ ACYX. Hence ACYX. By G, CYXA.
By 11, CYXA . CYAB . ⊃ . CYXB. By G, BCYX.
Therefore ABXY ∨ BCYX.

Theorem 18. 14. *Proof of* 18 *from* G, 11, 19.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ BCYX.
Suppose BCYX is false. By G, ABCY . ⊃ . YABC.
By 19, ABCX . ABCY . ⊃ . ACXY ∨ BCYX. Hence ACXY. By G, YACX.
By 11, YABC . YACX . ⊃ . YABX. By G, ABXY.
Therefore ABXY ∨ BCYX.

Theorem 18. 15. *Proof of* 18 *from* G, H, 14, 15.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ BCYX.
Suppose ABXY = 0 and BCYX = 0. By G, ABCY . ⊃ . YABC.
By 14, ABCX . ABCY . ⊃ . ABXY ∨ ABYX. Hence ABYX. By G, XABY.
By G, ABCX . ⊃ . XABC.
By 14, XABY . XABC . ⊃ . XAYC ∨ XACY. By G, CXAY ∨ ACYX.
By 15, ABCX . ABCY . ⊃ . ACXY ∨ ACYX. By G, CXYA ∨ ACYX.
Now if ACYX = 0, then CXAY and CXYA, which is impossible, by H.
Hence ACYX. By G, YXAC. By G, ABYX . ⊃ . YXAB.
By 14, YXAB . YXAC . ⊃ . YXBC ∨ YXCB. By G, BCYX ∨ BYXC.
But BCYX = 0. Hence BYXC. By G, ABYX . ⊃ . BYXA.

By 14, BYXC . BYXA . ⊃ . BYCA ∨ BYAC. By G, ABYC ∨ YACB.
But ABYC . vs . ABCY, by H; and YACB . vs . YABC, by H.
Therefore ABXY ∨ BCYX.

THEOREM 18. 16. *Proof of* 18 *from* G, H, 15, 16.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ BCYX.
Suppose ABXY = 0 and BCYX = 0.
By 16, ABCX . ABCY . ⊃ . BCXY ∨ BCYX. Hence BCXY.
By G, ABCX . ⊃ . BCXA.
By 16, BCXY . BCXA . ⊃ . CXYA ∨ CXAY. By G, ACXY ∨ XAYC.
By 15, ABCX . ABCY . ⊃ . ACXY ∨ ACYX. By G, ACXY ∨ XACY.
Now if ACXY = 0, then XAYC and XACY, which is impossible, by H.
Hence ACXY. By G, CXYA. By G, BCXY . ⊃ . CXYB.
By 16, CXYA . CXYB . ⊃ . XYAB ∨ XYBA. By G, ABXY ∨ XYBA.
But ABXY = 0. Hence XYBA. By G, BCXY . ⊃ . XYBC.
By G, ABCX . ⊃ . CXAB. By G, ABCX . ⊃ . BCAX.
By 15, XYBA . XYBC . ⊃ . XBAC ∨ XBCA. By G, CXBA ∨ BCAX.
But CXBA . vs . CXAB, by H, and BCAX . vs . BCXA, by H.
Therefore ABXY ∨ BCYX.

THEOREM 18. 17. *Proof of* 18 *from* G, H, 17.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ BCYX.

Suppose ABXY is false. By G, ABCY . ⊃ . YABC.
By 17, ABCX . ABCY . ⊃ . ABXY ∨ ACYX. Hence ACYX. By G, YXAC.
By 17, ABCY . ABCX . ⊃ . ABYX ∨ ACXY.
But ACXY . vs . ACYX, by H. Hence ABYX. By G, YXAB.
By 17, YXAB . YXAC . ⊃ . YXBC ∨ YACB.
But YACB . vs . YABC, by H. Hence YXBC. By G, BCYX.
Therefore ABXY ∨ BCYX.

THEOREM 18. 18. *Proof of* 18 *from* G, H, 19.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ BCYX.
Suppose BCYX is false. By G, ABCY . ⊃ . CYAB.
By 19, ABCX . ABCY . ⊃ . ACXY ∨ BCYX. Hence ACXY. By G, CXYA.
By 19, ABCY . ABCX . ⊃ . ACYX ∨ BCXY.
But ACYX . vs . ACXY, by H. Hence BCXY. By G, CXYB.
By 19, CXYB . CXYA . ⊃ . CYBA ∨ XYAB.
But CYBA . vs . CYAB, by H. Hence XYAB. By G, ABXY.
Therefore ABXY ∨ BCYX.

THEOREM 18. 19. *Proof of* 18 *from* H, 10, 14, 19.

To prove: ABCX . ABCY . ⊃ . ABXY ⌣ BCYX.
Suppose ABXY = 0 and BCYX = 0.
By 14, ABCX . ABCY . ⊃ . ABXY ⌣ ABYX. Hence ABYX.
By 19, ABCX . ABCY . ⊃ . ACXY ⌣ BCYX. Hence ACXY.
By 10, ABYX . C . ⊃ . ACYX ⌣ ABYC.
But ACYX . vs . ACXY, by H; and ABYC . vs . ABCY, by H.
Therefore ABXY ⌣ BCYX.

THEOREM 18. 20. *Proof of* 18 *from* H, 13, 14, 19.

To prove: ABCX . ABCY . ⊃ . ABXY ⌣ BCYX.
Suppose ABXY = 0 and BCYX = 0.
By 14, ABCX . ABCY . ⊃ . ABXY ⌣ ABYX. Hence ABYX.
By 19, ABCX . ABCY . ⊃ . ACXY ⌣ BCYX. Hence ACXY.
By 13, ABCY . ABYX . ⊃ . ACYX. But ACYX . vs . ACXY.
Therefore ABXY ⌣ BCYX.

THEOREM 18. 21. *Proof of* 18 *from* H, 17, 19.

To prove: ABCX . ABCY . ⊃ . ABXY ⌣ BCYX.
Suppose ABXY = 0 and BCYX = 0.
By 17, ABCX . ABCY . ⊃ . ABXY ⌣ ACYX. Hence ACYX.
By 19, ABCX . ABCY . ⊃ . ACXY ⌣ BCYX. Hence ACXY.
But ACYX . vs . ACXY, by H.
Therefore ABXY ⌣ BCYX.

THEOREM 18. 22. *Proof of* 18 *from* H, 12, 17.

To prove: ABCX . ABCY . ⊃ . ABXY ⌣ BCYX.
Suppose ABXY is false.
By 17, ABCX . ABCY . ⊃ . ABXY ⌣ ACYX. Hence ACYX.
By 17, ABCY . ABCX . ⊃ . ABYX ⌣ ACXY. But ACXY . vs . ACYX, by H.
Hence ABYX. By 12, ABCY . ABYX . ⊃ . BCYX.
Therefore ABXY ⌣ BCYX.

THEOREM 18. 23. *Proof of* 18 *from* R, G, 11, 14.

THEOREM 18. 24. *Proof of* 18 *from* R, G, 11, 16.

THEOREM 18. 25. *Proof of* 18 *from* R, G, H, 14.

THEOREM 18. 26. *Proof of* 18 *from* R, G, H, 16.

THEOREM 18. 27. *Proof of* 18 *from* R, F, G, 12.

THEOREM 18. 28. *Proof of* 18 *from* R, F, G, 13.

These six theorems follow from Theorems 15. 25 to 15. 26 and 15. 33 to 15. 36 (which give Postulate 15), by the aid of Theorem 18. 12.

THEOREM 18. 29. *Proof of* 18 *from* R, 10, 13, 14, 19.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ BCYX.
Suppose ABXY = 0 and BCYX = 0. By R, XYCB = 0.
By 14, ABCX . ABCY ⊃ ABXY ∨ ABYX. Hence ABYX.
By 19, ABCX . ABCY . ⊃ . ACXY ∨ BCYX. Hence ACXY.
By 13, ABCY . ABYX . ⊃ . ACYX. By R, XYCA.
By 10, XYCA . B . ⊃ . XBCA ∨ XYCB. Hence XBCA. By R, ACBX.
By 13, ACBX . ACXY . ⊃ . ABXY, which is false.
Therefore ABXY ∨ BCYX.

THEOREM 18. 30. *Proof of* 18 *from* R, 10, 12, 17.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ BCYX.
In view of Theorem 11. 6, Theorem 16. 31, and Theorem 19. 6, we may make use of Postulates 11, 16, and 19, as well as R, 10, 12, 17, in this proof.
Suppose ABXY = 0 and BCYX = 0. By R, YXBA = 0 and XYCB = 0.
By 17, ABCX . ABCY . ⊃ . ABXY ∨ ACYX. Hence ACYX. By R, XYCA.
By 19, ABCX . ABCY . ⊃ . ACXY ∨ BCYX. Hence ACXY. By R, YXCA.
By 10, XYCA . B . ⊃ . XBCA ∨ XYCB. Hence XBCA. By R, ACBX.
By 11, ACBX . ACXY . ⊃ . ACBY.
By 16, ACBX . ACBY . ⊃ . CBXY ∨ CBYX. By R, YXBC ∨ XYBC.
But if YXBC, then by 11, YXBC . YXCA . ⊃ . YXBA, which is false. Hence XYBC.
By 11, XYBC . XYCA . ⊃ . XYBA. By R, ABYX.
By 12, ABCY . ABYX . ⊃ . BCYX, which is false.
Therefore ABXY ∨ BCYX.

THEOREM 18. 31. *Proof of* 18 *from* R, 10, 13, 17, 19.

To prove: ABCX . ABCY . ⊃ . ABXY ∨ BCYX.
Suppose ABXY = 0 and BCYX = 0.
By R, XYCB = 0.

By 17, $ABCX . ABCY . \supset . ABXY \vee ACYX$. But $ABXY = 0$. Hence $ACYX$. By R, $XYCA$.

By 10, $XYCA . B . \supset . XBCA \vee XYCB$. But $XYCB = 0$. Hence $XBCA$. By R, $ACBX$.

By 13, if $ACXY$, then $ACBX . ACXY . \supset . ABXY$, which $= 0$. Hence $ACXY = 0$.

By 19, $ABCX . ABCY . \supset . ACXY \vee BCYX$. But $ACXY = 0$ and $BCYX = 0$. Hence the supposition is false. Therefore $ABXY \vee BCYX$.

Forty Proofs of Postulate 19.

Theorem 19. 1. *Proof of* 19 *from* G, H, 10.

To prove: $ABCX . ABCY . \supset . ACXY \vee BCYX$.
Suppose $BCYX$ is false. By G, $ABCY . \supset . BCYA$.
By 10, $BCYA . X . \supset . BXYA \vee BCYX$. Hence $BXYA$. By G, $ABXY$.
By 10, $ABXY . C . \supset . ACXY \vee ABXC$.
But $ABXC . \mathrm{vs} . ABCX$, by H. Hence $ACXY$.
Therefore $ACXY \vee BCYX$.

Theorem 19. 2. *Proof of* 19 *from* G, 10, 12.

To prove: $ABCX . ABCY . \supset . ACXY \vee BCYX$.
By G, $ABCX . \supset . BCXA$. By G, $ABCY . \supset . BCYA$.
By 10, $BCXA . Y . \supset . BYXA \vee BCXY$. By G, $ABYX \vee BCXY$.
If $ABYX$, then by 12, $ABCY . ABYX . \supset . BCYX$.
If $BCXY$, then by 12, $BCXY . BCYA . \supset . CXYA$. By G, $ACXY$.
Therefore $ACXY \vee BCYX$.

Theorem 19. 3. *Proof of* 19 *from* G, 12, 14.

To prove: $ABCX . ABCY . \supset . ACXY \vee BCYX$.
By G, $ABCY . \supset . BCYA$.
By 14, $ABCX . ABCY . \supset . ABXY \vee ABYX$.

Case 1. If $ABXY$, by 12, $ABCX . ABXY . \supset . BCXY$.
By 12, $BCXY . BCYA . \supset . CXYA$. By G, $ACXY$.

Case 2. If $ABYX$, by 12, $ABCY . ABYX . \supset . BCYX$.
Therefore $ACXY \vee BCYX$.

Theorem 19. 4. *Proof of* 19 *from* G, 12, 15.

To prove: $ABCX . ABCY . \supset . ACXY \vee BCYX$.
Suppose $ACXY$ is false. By G, $ABCY . \supset . CYAB$.

By 15, ABCX . ABCY . ⊃ . ACXY ⌣ ACYX. Hence ACYX. By G, CYXA.
By 12, CYXA . CYAB . ⊃ . YXAB. By G, ABYX.
By 12, ABCY . ABYX . ⊃ . BCYX.
Therefore ACXY ⌣ BCYX.

THEOREM 19. 5. *Proof of* 19 *from* G, 12, 16.

To prove: ABCX . ABCY . ⊃ . ACXY ⌣ BCYX.
Suppose BCYX is false. By G, ABCY . ⊃ . BCYA.
By 16, ABCX . ABCY . ⊃ . BCXY ⌣ BCYX. Hence BCXY.
By 12, BCXY . BCYA . ⊃ . CXYA. By G, ACXY.
Therefore ACXY ⌣ BCYX.

THEOREM 19. 6. *Proof of* 19 *from* 12, 17.

To prove: ABCX . ABCY . ⊃ . ACXY ⌣ BCYX.
By 17, ABCY . ABCX . ⊃ . ABYX ⌣ ACXY.
If ABYX, by 12, ABCY . ABYX . ⊃ . BCYX.
Therefore ACXY ⌣ BCYX.

THEOREM 19. 7. *Proof of* 19 *from* G, 12, 18.

To prove: ABCX . ABCY . ⊃ . ACXY ⌣ BCYX.
By G, ABCY . ⊃ . BCYA.
By 18, ABCY . ABCX . ⊃ . ABYX ⌣ BCXY.
If ABYX, then by 12, ABCY . ABYX . ⊃ . BCYX.
If BCXY, then by 12, BCXY . BCYA . ⊃ . CXYA. By G, ACXY.
Therefore ACXY ⌣ BCYX.

THEOREM 19. 8. *Proof of* 19 *from* G, 10, 13.

THEOREM 19. 9. *Proof of* 19 *from* G, 13, 14.

THEOREM 19. 10. *Proof of* 19 *from* G, 13, 15.

THEOREM 19. 11. *Proof of* 19 *from* G, 13, 16.

THEOREM 19. 12. *Proof of* 19 *from* G, 13, 17.

These five theorems follow from Theorems 19. 2 to 19. 6 by aid of Theorem 12. 2.

Theorem 19. 13. *Proof of* 19 *from* 13, 18.

To prove: ABCX . ABCY . ⊃ . ACXY ∨ BCYX.
By 13, ABCX . ABCY . ⊃ . ABXY ∨ BCYX.
If ABXY, then by 13, ABCX . ABXY . ⊃ . ACXY.
Therefore ACXY ∨ BCYX.

Theorem 19. 14. *Proof of* 19 *from* G, 11, 15.

To prove: ABCX . ABCY . ⊃ . ACXY ∨ BCYX.
Suppose ACXY is false. By G, ABCY . ⊃ . CYAB.
By 15, ABCX . ABCY . ⊃ . ACXY ∨ ACYX. Hence ACYX. By G, CYXA.
By 11, CYXA . CYAB . ⊃ . CYXB. By G, BCYX.
Therefore ACXY ∨ BCYX.

Theorem 19. 15. *Proof of* 19 *from* G, 10, 11, 16.

To prove: ABCX . ABCY . ⊃ . ACXY ∨ BCYX.
Suppose BCYX is false. By G, ABCY . ⊃ . BCYA.
By 16, ABCX . ABCY . ⊃ . BCXY ∨ BCYX. Hence BCXY. By G, XYBC.
By 10, BCYA . X . ⊃ . BXYA ∨ BCYX. Hence BXYA. By G, XYAB.
By 11, XYAB . XYBC . ⊃ . XYAC. By G, ACXY.
Therefore ACXY ∨ BCYX.

Theorem 19. 16. *Proof of* 19 *from* G, 11, 16, 17.

To prove: ABCX . ABCY . ⊃ . ACXY ∨ BCYX.
Suppose ACXY = 0, and BCYX = 0. By G, ABCY . ⊃ . CYAB.
By 16, ABCX . ABCY . ⊃ . BCXY ∨ BCYX. Hence BCXY. By G, XYBC.
By 17, ABCX . ABCY . ⊃ . ABXY ∨ ACYX. By G, XYAB ∨ CYXA.

Case 1. If XYAB, by 11, XYAB . XYBC . ⊃ . XYAC. By G, ACXY.

Case 2. If CYXA, by 11, CYXA . CYAB . ⊃ . CYXB. By G, BCYX.
Therefore ACXY ∨ BCYX.

Theorem 19. 17. *Proof of* 19 *from* G, 11, 16, 18.

To prove: ABCX . ABCY . ⊃ . ACXY ∨ BCYX.
Suppose BCYX = 0. By 16, ABCX . ABCY . ⊃ . BCXY ∨ BCYX. Hence BCXY. By G, XYBC.
By 18, ABCX . ABCY . ⊃ . ABXY ∨ BCYX. Hence ABXY. By G, XYAB.
By 11, XYAB . XYBC . ⊃ . XYAC. By G, ACXY.
Therefore ACXY ∨ BCYX.

THEOREM 19. 18. *Proof of* 19 *from* G, H, 14, 15.

To prove: ABCX . ABCY . ⊃ . ACXY ∨ BCYX.
Suppose ACXY = 0 and BCYX = 0. By G, ABCY . ⊃ . YABC.
By 15, ABCX . ABCY . ⊃ . ACXY ∨ ACYX. Hence ACYX.
By G, CYXA. By G, YXAC.
By 14, ABCX . ABCY . ⊃ . ABXY ∨ ABYX. By G, YABX ∨ YXAB.

Case 1. Suppose YABX. By 14, YABX . YABC . ⊃ . YAXC ∨ YACX.
By G, CYAX ∨ ACXY. But CYAX . vs . CYXA by H; and ACXY = 0.

Case 2. Suppose YXAB. By 14, YXAB . YXAC . ⊃ . YXBC ∨ YXCB.
By G, BCYX ∨ BYXC. But BCYX = 0. Hence BYXC.
By G, YXAB . ⊃ . BYXA. By G, ABCX . ⊃ . XABC.
By 15, BYXC . BYXA . ⊃ . BXCA ∨ BXAC. By G, ABXC ∨ XACB.
But ABXC . vs . ABCX, by H; and XACB . vs . XABC by H.
Therefore ACXY ∨ BCYX.

THEOREM 19. 19. *Proof of* 19 *from* G, H, 15, 16.

To prove: ABCX . ABCY . ⊃ . ACXY ∨ BCYX.
Suppose ACXY = 0 and BCYX = 0. By G, ABCX . ⊃ . BCXA.
By 15, ABCX . ABCY . ⊃ . ACXY ∨ ACYX. Hence ACYX. By G, XACY.
By 16, ABCX . ABCY . ⊃ . BCXY ∨ BCYX. Hence BCXY.
By 16, BCXA . BCXY . ⊃ . CXAY ∨ CXYA. By G, XAYC ∨ ACXY.
But XAYC . vs . XACY, by H; and ACXY = 0.
Therefore ACXY ∨ BCYX.

THEOREM 19. 20. *Proof of* 19 *from* G, H, 17.

To prove: ABCX . ABCY . ⊃ . ACXY ∨ BCYX.
Suppose ACXY = 0 and BCYX = 0. By G, ABCX . ⊃ . XABC.
By 17, ABCY . ABCX . ⊃ . ABYX ∨ ACXY. Hence ABYX. By G, XABY.
By 17, XABY . XABC . ⊃ . XAYC ∨ XBCY. By G, AYCX ∨ BCYX.
Hence AYCX. By G, CXAY. By G, ABCX . ⊃ . CXAB.
By 17, CXAY . CXAB . ⊃ . CXYB ∨ CABY. By G, BCXY ∨ ABYC.
But ABYC . vs . ABCY, by H. Hence BCXY.
By G, ABCX . ⊃ . BCXA. By G, ABCY . ⊃ . BCYA.
By 17, BCXA . BCXY . ⊃ . BCAY ∨ BXYA.
But BCAY . vs . BCYA, by H. Hence BXYA. By G, ABXY.
But ABXY . vs . ABYX by H.
Therefore ACXY ∨ BCYX.

THEOREM 19. 21. *Proof of* 19 *from* G, H, 18.

To prove: $ABCX . ABCY . \supset . ACXY \vee BCYX$.
Suppose $BCYX = 0$. By G, $ABCY . \supset . YABC$.
By 18, $ABCX . ABCY . \supset . ABXY \vee BCYX$. Hence $ABXY$. By G, $YABX$.
By 18, $YABC . YABX . \supset . YACX \vee ABXC$.
But $ABXC . vs . ABCX$, by H. Hence $YACX$. By G, $ACXY$.
Therefore $ACXY \vee BCYX$.

THEOREM 19. 22. *Proof of* 19 *from* H, 10, 12, 14.

To prove: $ABCX . ABCY . \supset . ACXY \vee BCYX$.
By 14, $ABCX . ABCY . \supset . ABXY \vee ABYX$.

Case 1. If $ABXY$, then by 10, $ABXY . C . \supset . ACXY \vee ABXC$.
But $ABXC . vs . ABCX$, by H. Hence $ACXY$.

Case 2. If $ABYX$, then by 12, $ABCY . ABYX . \supset . BCYX$.
Therefore $ACXY \vee BCYX$.

THEOREM 19. 23. *Proof of* 19 *from* H, 10, 18.

To prove: $ABCX . ABCY . \supset . ACXY \vee BCYX$.
Suppose $BCYX = 0$.
By 18, $ABCX . ABCY . \supset . ABXY \vee BCYX$. Hence $ABXY$.
By 10, $ABXY . C . \supset . ACXY \vee ABYC$.
But $ABYC . vs . ABCY$, by H. Hence $ACXY$.
Therefore $ACXY \vee BCYX$.

THEOREM 19. 24. *Proof of* 19 *from* H, 17, 18.

To prove: $ABCX . ABCY . \supset . ACXY \vee BCYX$.
Suppose $ACXY = 0$ and $BCYX = 0$.
By 18, $ABCX . ABCY . \supset . ABXY \vee BCYX$. Hence $ABXY$.
By 17, $ABCY . ABCX . \supset . ABYX \vee ACXY$. Hence $ABYX$.
But $ABXY . vs . ABYX$, by H.
Therefore $ACXY \vee BCYX$.

THEOREM 19. 25. *Proof of* 19 *from* 12, 13, 14.

To prove: $ABCX . ABCY . \supset . ACXY \vee BCYX$.
By 14, $ABCX . ABCY . \supset . ABXY \vee ABYX$.
If $ABXY$, then by 13, $ABCX . ABXY . \supset . ACXY$.
If $ABYX$, then by 12, $ABCY . ABYX . \supset . BCYX$.
Therefore $ACXY \vee BCYX$.

THEOREM 19. 26. *Proof of* 19 *from* R, G, 17.

To prove: ABCX . ABCY . ⊃ . ACXY ∨ BCYX.
By R, G, ABCX . ⊃ . CBAX. By R, G, ABCY . ⊃ . CBAY.
By 17, CBAX . CBAY . ⊃ . CBXY ∨ CAYX. By R, G, BCYX ∨ ACXY.
Therefore ACXY ∨ BCYX.

THEOREM 19. 27. *Proof of* 19 *from* R, G, 11, 14.

THEOREM 19. 28. *Proof of* 19 *from* R, G, 11, 16.

THEOREM 19. 29. *Proof of* 19 *from* R, G, H, 14.

THEOREM 19. 30. *Proof of* 19 *from* R, G, H, 16.

THEOREM 19. 31. *Proof of* 19 *from* R, F, G, 12.

THEOREM 19. 32. *Proof of* 19 *from* R, F, G, 13.

These six theorems follow from Theorems 15. 25 to 15. 26 and 15. 33 to 15. 36 (which give Postulate 15), by the aid of Theorems 19. 14, 19. 18, 19. 19, 19. 4 and 19. 10.

THEOREM 19. 33. *Proof of* 19 *from* R, H, 11, 18.

Proof: 13 from R, H, 11, 18, by Theorem 13. 11; then 19 from 13, 18, by Theorem 19. 13.

THEOREM 19. 34. *Proof of* 19 *from* R, 11, 12, 14.

Proof: 17 from R, 11, 12, 14 by Theorem 17. 35; then 19 from 12, 17, by Theorem 19. 6.

THEOREM 19. 35. *Proof of* 19 *from* R, 11, 12, 18.

Proof: 13 from R, 11, 12, by Theorem 13. 12; then 19 from 13, 18, by Theorem 19. 13.

THEOREM 19. 36. *Proof of* 19 *from* R, 11, 16, 18.

To prove: ABCX . ABCY . ⊃ . ACXY ∨ BCYX.
Suppose BCYX = 0.
By 16, ABCX . ABCY . ⊃ . BCXY ∨ BCYX. Hence BCXY. By R, YXCB.
By 18, ABCX . ABCY . ⊃ . ABXY ∨ BCYX. Hence ABXY. By R, YXBA.
By 11, YXCB . YXBA . ⊃ . YXCA. By R, ACXY.
Therefore ACXY ∨ BCYX.

THEOREM 19. 37. *Proof of* 19 *from* R, H, 10, 12, 16.

To prove: ABCX . ABCY . ⊃ . ACXY ∨ BCYX.
Suppose ACXY = 0 and BCYX = 0. By R, YXCA = 0 and XYCB = 0.
By 16, ABCX . ABCY . ⊃ . BCXY ∨ BCYX. Hence BCXY. By R, YXCB.
By 10, YXCB . A . ⊃ . YACB ∨ YXCA. Hence YACB. By R, BCAY.
By R, ABCX . ⊃ . XCBA. By R, ABCY . ⊃ . YCBA.
Suppose ABYX; then, by 13, ABCY . ABYX . ⊃ . BCYX. Hence ABYX = 0.
By R, XYBA = 0.
By 10, XCBA . Y . ⊃ . XYBA ∨ XCBY. Hence XCBY.
By H, ABCX . ABXC = 0.
Suppose ABXY; then by 10, ABXY . C . ⊃ . ACXY ∨ ABXC.
Hence ABXY = 0.
By R, YXBA = 0.
By 10, YCBA . X . ⊃ . YXBA ∨ YCBX. Hence YCBX. By R, XBCY.
By H, YCBA . VS . YCAB. By R, BACY = 0.
Suppose XBAC; then by 12, XBAC . XBCY . ⊃ . BACY. Hence XBAC = 0.
By H, BCAY . VS . BCYA. By H, YACB . VS . YABC. By R, CBAY = 0.
Suppose XBYA; then by 12, XBCY . XBYA . ⊃ . BCYA. Hence XBYA = 0.
By R, AYBX = 0.
By 16, XCBA . XCBY . ⊃ . CBAY ∨ CBYA. Hence CBYA. By R, AYBC.
By 10, AYBC . X . ⊃ . AXBC ∨ AYBX. Hence AXBC. By R, CBXA.
By H, YXCB . ⊃ . YXBC = 0. By R, CBXY = 0.
By 10, CBXA . Y . ⊃ . CYXA ∨ CBXY. Hence CYXA. By R, AXYC.
Suppose AXCB; then by 12, AXYC . AXCB . ⊃ . XYCB. Hence AXCB = 0.
By R, BCXA = 0.
By 10, BCXY . A . ⊃ . BAXY ∨ BCXA. Hence BAXY. By R, YXAB.
By H, CBXA . ⊃ . CBAX = 0.
Suppose YCAX; then by 12, YCBA . YCAX . ⊃ . CBAX. Hence YCAX = 0.
By R, XACY = 0.
By H, YCBA . ⊃ . YCAB = 0.
By 10, YXAB . C . ⊃ . YCAB ∨ YXAC. Hence YXAC.
By 12, YXAC . YXCB . ⊃ . XACB.
By 10, XACB . Y . ⊃ . XYCB ∨ XACY. But XACY = 0, and XYCB = 0.
Therefore ACXY ∨ BCYX.

THEOREM 19. 38. *Proof of* 19 *from* R, 10, 12, 15.

To prove: ABCX . ABCY . ⊃ . ACXY ∨ BCYX.
Suppose ACXY = 0 and BCYX = 0. By R, YXCA = 0 and XYCB = 0.
By 15; ABCX . ABCY . ⊃ . ACXY ∨ ACYX. Hence ACYX. By R, XYCA.
By 10, XYCA . B . ⊃ . XBCA ∨ XYCB. Hence XBCA. By R, ACBX.

By R, ABCX . ⊃ . XCBA. By R, ABCY . ⊃ . YCBA.
Now, ABYX = 0 (for if not, by 12, ABCY . ABYX . ⊃ . BCYX). By R, XYBA = 0.
By 10, XCBA . Y . ⊃ . XYBA ∨ XCBY. Hence XCBY. By R, YBCX.
By 10, ACYX . B . ⊃ . ABYX ∨ ACYB. Hence ACYB. By R, BYCA.
Now BYXC = 0 (for if not, by 12, BYXC . BYCA . ⊃ . YXCA). By R, CXYB = 0.
By 12, ACYB . ACBX . ⊃ . CYBX. By R, XBYC.

Case 1. Suppose BYCX. By R, XCYB.
By 12, XCYB . XCBA . ⊃ . CYBA. By R, ABYC.
By 10, ABYC . X . ⊃ . AXYC ∨ ABYX. Hence AXYC. By R, CYXA.
By 12, CYBX . CYXA . ⊃ . YBXA.
By 12, YBCX . YBXA . ⊃ . BCXA. By R, AXCB.
By 12, AXYC . AXCB . ⊃ . XYCB. But XYCB = 0.
Therefore BYCX = 0. By R, XCYB = 0.
By 10, BYCA . X . ⊃ . BXCA ∨ BYCX. Hence BXCA. By R, ACXB.
By 10, ACXB . Y . ⊃ . AYXB ∨ ACXY. Hence AYXB. By R, BXYA.
By 12, ACYX . ACXB . ⊃ . CYXB.
Now ABYC = 0 (for if not, by 12, ABYC . ABCX . ⊃ . BYCX). By R, CYBA = 0.
By 10, CYBX . A . ⊃ . CABX ∨ CYBA. Hence CABX. By R, XBAC.

Case 2. Suppose ACBY.
By 12, ACBY . ACYX . ⊃ . CBYX. By R, XYBC.
By 12, XYBC . XYCA . ⊃ . YBCA. By R, ACBY.
By 15, ACBX . ACBY . ⊃ . ABXY ∨ ABYX. Hence ABXY.
By 10, ABXY . C . ⊃ . ACXY ∨ ABXC. Hence ABXC.
By 10, XYBC . A . ⊃ . XABC ∨ XYBA. Hence XABC. By R, CBAX.
By 12, ABXC . ABCY . ⊃ . BXCY.
By 12, BXCY . BXYA . ⊃ . XCYA.
By 12, XCBY . XCYA . ⊃ . CBYA.
By 12, CBYA . CBAX . ⊃ . BYAX. By R, XAYB.
By 10, XAYB . C . ⊃ . XCYB ∨ XAYC. Hence XAYC. By R, CYAX.
By 12, CYAX . CYXB . ⊃ . YAXB. By R, BXAY.
By 12, BXCA . BXAY . ⊃ . XCAY.
By 12, XCBA . XCAY . ⊃ . CBAY. By R, YABC.
By 12, YAXB . YABC . ⊃ . AXBC.
Now CYXA is either true or false.
If CYXA, then by R, AXYC,
and by 12, CYBX . CYXA . ⊃ . YBXA;

then, by 12, $YBCX . YBXA . \supset . BCXA$; then by R, $AXCB$;
then by 12, $AXYC . AXCB . \supset . XYCB$, which is false.
If $CYXA = 0$, then by R, $AXYC = 0$;
then by 15, $ABXY . ABXC . \supset . AXYC \vee AXCY$; then $AXCY$;
whence by R, $YCXA$;
then by 12, $AXBC . AXCY . \supset . XBCY$; then by R, $YCBX$;
then by 12, $YCBX . YCXA . \supset . CBXA$;
then by 12, $CBYX . CBXA . \supset . BYXA$; then by R, $AXYB$;
then by 12, $AXCY . AXYB . \supset . XCYB$, which is false.
Therefore $ACBY = 0$. By R, $YBCA = 0$.
By 10, $ACBX . Y . \supset . ACBY \vee AYBX$. Hence $AYBX$. By R, $XBYA$.
By 10, $YBCX . A . \supset . YACX \vee YBCA$. Hence $YACX$. By R, $XCAY$.
Now $XYBC = 0$ (for if not, by 12, $XYBC . XYCA . \supset . YBCA$).
By 12, $XCBA . XCAY . \supset . CBAY$.
By 12, $XBYA . XBAC . \supset . BYAC$. By R, $CAYB$.
By 10, $CAYB . X . \supset . CXYB \vee CAYX$. Hence $CAYX$.
Now $CABY = 0$ (for if not, by 12, $CABY . CAYX . \supset . ABYX$). By R, $YBAC$.
Then $XCAB = 0$ (for if not, by 12, $XCAB . XCBY . \supset . CABY$). By R, $BACX = 0$.
By 10, $XCAY . B . \supset . XBAY \vee XCAB$. Hence $XBAY$.
By 12, $XBCA . XBAY . \supset . BCAY$. By R, $YACB$.
By 12, $XBAY . XBYC . \supset . BAYC$.
Now $BAYX = 0$ (for if not, by 15, $BAYX . BAYC . \supset . BYXC \vee BYCX$).

Case 3. Suppose $XBCY = 0$. By R, $YCBX = 0$.
By 10, $YCBA . X . \supset . YXBA \vee YCBX$. Hence $YXBA$.
By 15, $YACX . YACB . \supset . YCXB \vee YCBX$. Hence $YCXB$. By R, $BXCY$.
By 12, $BXCY . BXYA . \supset . XCYA$.
By 12, $XCBY . XCYA . \supset . CBYA$. By R, $AYBC$.
By 12, $AYXB . AYBC . \supset . YXBC$.
By 15, $YXBA . YXBC . \supset . YBAC \vee YBCA$.
But $YBAC = 0$ and $YBCA = 0$.
Therefore $XBCY$.
By 12, $XBCY . XBYA . \supset . BCYA$.
By 12, $XBAC . XBCY . \supset . BACY$.
By 10, $BACY . X . \supset . BXCY \vee BACX$. Hence $BXCY$.

Case 4. Suppose $CXAB = 0$.
By 10, $CYAB . X . \supset . CXAB \vee CYAX$. Hence $CYAX$. By R, $XAYC$.
By 10, $BYAC . X . \supset . BXAC \vee BYAX$.

If BYAX, then by R, XAYB;
then by 15, XAYB . XAYC . ⊃ . XYBC ∨ XYCB;
where XYBC = 0 and XYCB = 0.
Hence BXAC.
By 12, BXAC . BXCY . ⊃ . XACY; by R, YCAX.
By 12, YCBA . YCAX . ⊃ . CBAX. By R, XABC.
By 10, XABC . Y . ⊃ . XYBC ∨ XABY. Hence XABY.
By 12, XABY . XAYC . ⊃ . ABYC, which is false.
Therefore, CXAB. By R, BAXC.
By 10, BAXC . Y . ⊃ . BYXC ∨ BAXY. Hence BAXY. By R, YXAB.
By 12, BAXY . BAYC . ⊃ . AXYC.
By 10, AXYC . B . ⊃ . ABYC ∨ AXYB. Hence AXYB. By R, BYXA.
By 12, BYXA . BYAC . ⊃ . YXAC. By R, CAXY.
By 12, CABY . CAXY . ⊃ . ABXY.
By 12, ABCX . ABXY . ⊃ . BCXY.
By 12, BCXY . BCYA . ⊃ . CXYA.
By 12, CXYA . CXAB . ⊃ . XYAB. By R, BAYX.
But BAYX = 0.
Therefore ACXY ∨ BCYX.

THEOREM 19. 39. *Proof of* 19 *from* R, 10, 12, 14.

Proof: Postulate 15 follows from R, 10, 12, 14, by Theorem 15. 45.
Then 19 follows from R, 10, 12, 15, by Theorem 19. 38.
Or thus: Postulate 18 follows from 12 and 14 by Theorem 18. 2. Then 19 follows from R, 10, 12, 18 by Theorem 19. 40 (below).

THEOREM 19. 40. *Proof of* 19 *from* R, 10, 12, 18.

To prove: ABCX . ABCY . ⊃ . ACXY ∨ BCYX.
Suppose both ACXY = 0 and BCYX = 0. By R, YXCA = 0, XYCB = 0.
By R, ABCX, XCBA; ABCY, YCBA.
By 18, ABCX . ABCY . ⊃ . ABXY ∨ BCYX. But BCYX = 0. Hence ABXY. By R, YXBA.
By 12, ABCX . ABXY . ⊃ . BCXY. By R, YXCB.
By 10, YXCB . A . ⊃ . YACB ∨ YXCA. But YXCA = 0. Hence YACB. By R, BCAY.
By 10, ABXY . C . ⊃ . ACXY ∨ ABXC. But ACXY = 0. Hence ABXC. By R, CXBA.
By 12, ABXC . ABCY . ⊃ . BXCY. By R, YCXB.
By 12, if ABYX, then ABCY . ABYX . ⊃ . BCYX, which = 0.
Hence ABYX = 0. By R, XYBA = 0.

By 12, if CXYB, then CXYB . CXBA . ⊃ . XYBA, which = 0. Hence CXYB = 0. By R, BYXC = 0.

By 10, XCBA . Y . ⊃ . XYBA ⌣ XCBY. But XYBA = 0. Hence XCBY. By R, YBCX.

By 10, BCAY . X . ⊃ . BXAY ⌣ BCAX.

By 10, YCXB . A . ⊃ . YAXB ⌣ YCXA. By R, BXAY ⌣ YCXA.

Hence BXAY ⌣ (BCAX . YCXA).

Supposition 1. Suppose BXAY = 0. By R, YAXB = 0.

Then BCAX and YCXA. By R, XACB and AXCY.

By 10, XACB . Y . ⊃ . XYCB ⌣ XACY. But XYCB = 0. Hence XACY. By R, YCAX.

By 12, YCBA . YCAX . ⊃ . CBAX.

By 12, YCAX . YCXB . ⊃ . CAXB. By R, BXAC.

By 10, BXAC . Y . ⊃ . BYAC ⌣ BXAY. But BXAY = 0. Hence BYAC. By R, CAYB.

By 18, BCAY . BCAX . ⊃ . BCYX ⌣ CAXY. But BCYX = 0. Hence CAXY.

By 12, CAXY . CAYB . ⊃ . AXYB. By R, BYXA.

By 12, AXCY . AXYB . ⊃ . XCYB.

By 12, XCYB . XCBA . ⊃ . CYBA. By R, ABYC.

By 10, ABYC . X . ⊃ . AXYC ⌣ ABYX. But ABYX = 0. Hence AXYC.

By 10, BYXA . C . ⊃ . BCXA ⌣ BYXC. But BYXC = 0. Hence BCXA. By R, AXCB.

By 12, AXYC . AXCB . ⊃ . XYCB, which = 0.

Hence Supposition 1 is false. Therefore, BXAY, and by R, YAXB.

Supposition 2. Suppose XCAY = 0. By R, YACX = 0.

By 12, if BXCA, then BXCA . BXAY . ⊃ . XCAY, which = 0. Hence BXCA = 0.

By 10, YBCX . A . ⊃ . YACX ⌣ YBCA. But YACX = 0. Hence YBCA. By R, ACBY.

By 12, if YBXC, then YBXC . YBCA . ⊃ . BXCA, which = 0. Hence YBXC = 0. By R, CXBY = 0.

By 10, CXBA . Y . ⊃ . CYBA ⌣ CXBY. But CXBY = 0. Hence CYBA. By R, ABYC.

By 12, ABXY . ABYC . ⊃ . BXYC.

By 12, BXAY . BXYC . ⊃ . XAYC. By R, CYAX.

By 12, CYBA . CYAX . ⊃ . YBAX.

By 10, ABYC . X . ⊃ . AXYC ⌣ ABYX. But ABYX = 0. Hence AXYC.

By 12, if AXCB, then AXYC . AXCB . ⊃ . XYCB, which = 0. Hence AXCB = 0. By R, BCXA = 0.

By 10, BCXY . A . ⊃ . BAXY ⌣ BCXA. But BCXA = 0. Hence BAXY.

By 12, if BACX, then BACX . BAXY . ⊃ . ACXY, which = 0. Hence BACX = 0.

By 12, if YBAC, then YBAC . YBCX . ⊃ . BACX, which = 0. Hence YBAC = 0.

By 12, if YBXA, then YBCX . YBXA . ⊃ . BCXA, which = 0. Hence YBXA = 0. By R, AXBY = 0.

By 12, if CYBX, then CYBX . CYXA . ⊃ . YBXA, which = 0. Hence CYBX = 0. By R, XBYC = 0.

By 10, ACBY . X . ⊃ . AXBY ∨ ACBX. But AXBY = 0. Hence ACBX.

By 12, if ACYB, then ACYB . ACBX . ⊃ . CYBX, which = 0. Hence ACYB = 0.

By 10, XAYC . B . ⊃ . XBYC ∨ XAYB. But XBYC = 0. Hence XAYB.

By 12, if XACY, then XACY . XAYB . ⊃ . ACYB, which = 0. Hence XACY = 0. By R, YCAX = 0.

By 10, YBAX . C . ⊃ . YCAX ∨ YBAC, both of which = 0.

Hence Supposition 2 is false. Hence XCAY. By R, YACX.

We have BXAY and XCAY. By R, YAXB and YACX.

By 12, YACX . YAXB . ⊃ . ACXB.

By 10, ACXB . Y . ⊃ . AYXB ∨ ACXY. But ACXY = 0. Hence AYXB. By R, BXYA.

By 12, BXCY . BXYA . ⊃ . XCYA.

By 12, XCBY . XCYA . ⊃ . CBYA. By R, AYBC.

By 12, AYXB . AYBC . ⊃ . YXBC. By R, CBXY.

By 18, YXBC . YXBA . ⊃ . YXCA ∨ XBAC. But YXCA = 0. Hence XBAC. By R, CABX.

Supposition 3. Suppose BAXY = 0. By R, YXAB = 0.

By 12, if CBAX, then CBAX . CBXY . ⊃ . BAXY, which = 0. Hence CBAX = 0.

By 12, if YCAX, then YCBA . YCAX . ⊃ . CBAX, which = 0. Hence YCAX = 0. By R, XACY = 0.

By 12, if BXAC, then BXAC . BXCY . ⊃ . XACY, which = 0. Hence BXAC = 0. By R, CAXB = 0.

By 10, BCXY . A . ⊃ . BAXY ∨ BCXA. But BAXY = 0. Hence BCXA. By R, AXCB.

By 12, if AXYC, then AXYC . AXCB . ⊃ . XYCB, which = 0. Hence AXYC = 0. By R, CYXA = 0.

By 10, if XACB, then XACB . Y . ⊃ . XYCB ∨ XACY. But XYCB = 0 and XACY = 0. Hence XACB = 0.

By 12, if YXAC, then YXAC . YXCB . ⊃ . XACB, which = 0. Hence YXAC = 0.

By 18, YXBA . YXBC . ⊃ . YXAC ∨ XBCA. But YXAC = O. Hence XBCA.

By 10, if YCAB, then YCAB . X . ⊃ . YXAB ∨ YCAX. But YXAB = O and YCAX = O. Hence YCAB = O. By R, BACY = O.

By 12, if XBCY, then XBAC . XBCY . ⊃ . BACY, which = O. Hence XBCY = O.

By 10, XBCA . Y . ⊃ . XYCA ∨ XBCY. But XBCY = O. Hence XYCA. By R, ACYX.

By 12, ACYX . ACXB . ⊃ . CYXB.

By 10, CYXB . A . ⊃ . CAXB ∨ CYXA, both of which = O.

Hence Supposition 3 is false. Hence BAXY. By R, YXAB.

The items used in the remainder of the proof are the following, with their reverses: ACXY = O, XYCB = O; XCBA, ABCY; BCXY; YACB; YCXB; ABYX = O; BYXC = O; YBCX; BXAY; XCAY; CBYA, YXBC; CABX; BAXY.

By 12, if BACX, then BACX . BAXY . ⊃ . ACXY, which = O. Hence BACX = O. By R, XCAB = O.

By 10, XCAY . B . ⊃ . XBAY ∨ XCAB. But XCAB = O. Hence XBAY. By R, YABX.

By 12, if YBAC, then YBAC . YBCX . ⊃ . BACX, which = O. Hence YBAC = O. By R, CABY = O.

By 10, CABX . Y . ⊃ . CYBX ∨ CABY. But CABY = O. Hence CYBX. By R, XBYC.

By 12, XBAY . XBYC . ⊃ . BAYC.

By 12, BAXY . BAYC . ⊃ . AXYC.

By 12, YXAB . YXBC . ⊃ . XABC. By R, CBAX.

By 12, CBYA . CBAX . ⊃ . BYAX.

By 12, YACB . YABX . ⊃ . ACBX. By R, XBCA.

By 12, XBYC . XBCA . ⊃ . BYCA.

By 12, BYCA . BYAX . ⊃ . YCAX.

By 12, YCAX . YCXB . ⊃ . CAXB.

By 12, if AXCB, then AXYC . AXCB . ⊃ . XYCB, which = O. Hence AXCB = O. By R, BCXA = O.

By 10, if BYXA, then BYXA . C . ⊃ . BCXA ∨ BYXC, both of which = O. Hence BYXA = O. By R, AXYB = O.

By 18, if CAXY, then CAXB . CAXY . ⊃ . CABY ∨ AXYB, both of which = O. Hence CAXY = O. By R, YXAC = O.

By 12, if BCAX, then BCAX . BCXY . ⊃ . CAXY, which = O. Hence BCAX = O.

By 10, BYAX . C . ⊃ . BCAX ∨ BYAC. But BCAX = O. Hence BYAC.

By 12, if BYXA, then BYXA . BYAC . ⊃ . YXAC, which = O. Hence BYXA = O.

By 12, XCBA . XCAY . ⊃ . CBAY. By R, YABC.
By 12, YAXB . YABC . ⊃ . AXBC. By R, CBXA.
By 12, if CBYX, then CBYX . CBXA . ⊃ . BYXA, which = 0. Hence CBYX = 0. By R, XYBC = 0.
By 12, YXAB . YXBC . ⊃ . XABC. By R, CBAX.
By 12, CBYA . CBAX . ⊃ . BYAX.
By 10, XCAY . B . ⊃ . XBAY ∨ XCAB. But XCAB = 0. Hence XBAY. By R, YABX.
By 12, YACB . YABX . ⊃ . ACBX. By R, XBCA.
By 12, XBYC . XBCA . ⊃ . BYCA.
By 12, BYCA . BYAX . ⊃ . YCAX.
By 12, YCAX . YCXB . ⊃ . CAXB.
By 10, CAXB . Y . ⊃ . CYXB ∨ CAXY. But CAXY = 0. Hence CYXB. By R, BXYC.
By 12, BXAY . BXYC . ⊃ . XAYC.
By 10, XABC . Y . ⊃ . XYBC ∨ XABY. But XYBC = 0. Hence XABY.
By 12, XABY . XAYC . ⊃ . ABYC.
By 10, ABYC . X . ⊃ . AXYC ∨ ABYX. But ABYX = 0. Hence AXYC. By R, CYXA.
By 10, CABX . Y . ⊃ . CYBX ∨ CABY. But CABY = 0. Hence CYBX.
By 12, CYBX . CYXA . ⊃ . YBXA.
By 12, YBCX . YBXA . ⊃ . BCXA. By R, AXCB.
By 12, AXYC . AXCB . ⊃ . XYCB, which = 0.
Hence the original supposition is false. Hence ACXY ∨ BCYX.

Construction of Examples of Systems (K, R).

In the following examples of systems (K, R), the class K consists, in each case, of five elements, 1, 2, 3, 4, 5, and the relation R (A, B, C, D) is defined by exhibiting, in each case, the tetrads which are supposed to be true in that case. (There are 120 possible tetrads formed from the five elements, 1, 2, 3, 4, 5.)

For brevity, the notation 1234G denotes not only the tetrad 1234, but also all the tetrads which can be derived from it by the use of Postulate G; that is, 1234G stands for the four tetrads, 1234, 2341, 3412, 4123.

Similarly, 1234R denotes not only 1234 but also 4321 (which is derived from it by the use of Postulate R).

So 1234GR stands for all eight of the tetrads, 1234, 2341, 3412, 4123; 3214, 2143, 1432, 4321.

The numbering of the examples begins with 200 to avoid confusion with earlier lists.

The properties which these systems possess will be set forth in the tables below.

Ex. 200. 1234GR, 1235GR, 1245GR, 1345GR, 2345GR. ("Reversible order on a closed line.")
Ex. 201. All tetrads false.
Ex. 202. 1234G, 1235G, 1245G, 1345G, 2345G. ("Cyclic order.")
Ex. 203. 1234GR.
Ex. 204. 1234R, 1254R, 1324R, 1354R, 1524R, 1534R.
Ex. 205. 1234GR, 1235GR, 1243GR, 1245GR, 1253GR, 1254GR.
Ex. 206. All the ninety-six tetrads which include the element 1.
Ex. 207. 1234R, 1254R, 1324R, 1354R, 1524R, 1534R, 2135R, 2145R, 2315R, 2345R, 2415R, 2435R.
Ex. 208. All tetrads true.
Ex. 209. 1234R, 1235, 1245, 1345, 2145, 2345, 4521.
Ex. 210. 1234GR; 1235G, 1254G, 1324G, 1325G, 1354G, 1425G, 1435G.
Ex. 211. 1234GR, 1235G, 1245G, 1345G, 2345G.
Ex. 212. 1234R, 1235R, 1245R, 1345R, 2345R. ("Betweenness.")
Ex. 213. 1235GR, 1243GR, 1354GR, 1425GR, 2345GR.
Ex. 214. 1235GR, 1243GR, 1245GR, 1345GR, 2354GR.
Ex. 215. All the 120 possible tetrads *except* 2435GR.
Ex. 216. All except 1245R, 1354R, 2135R, 2453R, 3124R.
Ex. 217. 1234, 1245, 1254, 1324, 1325, 1345, 1354, 1524, 1534, 3125, 3145, 3215, 3245, 3415, 3425, with all their reverses.
Ex. 218. 1235R, 1243R, 1423R, 1425R, 1435R, 1543R, 4235R.
Ex. 219. 1234R, 1245R, 1354R, 2345R, 3125R.
Ex. 220. 1234R, 1245, 1345, 1534, 2145, 2345, 4521, 5213, 5413.
Ex. 221. 1234GR, 1235GR, 1243GR, 1245GR, 1253GR, 1254GR, 1345GR, 1354GR, 2345GR, 2354GR.
Ex. 222. 1234GR, 1235G, 1254G, 2345G, 3415G.
Ex. 223. All tetrads *except* 2135R, 2145R, 2315R, 2345R, 2415R, 2435R.
Ex. 224. 1234, 1235, 1245, 1324, 1325, 1345, 1423, 1425, 1435, 1523, 1542, 1543, 2143, 2153, 2154, 2314, 2315, 2354, 2413, 2453, 2513, 2514, 2543, 3154, 3245, 3514, 4215, 4235, 4315, 4325, with all their reverses.
Ex. 225. 1235R, 1243, 1254, 1354, 1435, 1543, 2145, 2345, 5421.

Ex. 226. 1234R, 1235R, 1254R, 1345R, 2145R, 2345R.

Ex. 227. 1234, 1235, 1243, 1245, 1253, 1254, 1345R, 1453R, 1543R, 2143, 2341, 2345, 2351, 2354, 2431, 2435, 2451, 2453, 2531, 2534, 2541, 2543, 3241, 3251, 4231, 4251, 4351, 4531, 5231, 5241, 5341.

Ex. 228. 1235, 1243, 1245, 1453, 2435R.

Ex. 229. 1235GR, 1243GR, 1345GR, 1425GR, 2345GR.

Ex. 230. 1345GR, 2345GR.

Ex. 231. 1234GR, 1235G, 4215G, 4315G, 5324G.

Ex. 232. 1234G, 1235GR, 1324GR, 1425GR, 1435GR, 2315G, 2345G, 2354GR, 4215G, 4315G.

Ex. 233. 1234G, 1235G, 1342G, 1345G, 1352G, 1354GR, 1423G, 1425GR, 1435GR, 1452G, 1523G, 1542G, 2345G, 2354G.

Ex. 234. All tetrads *except* 1245GR, 1254GR.

Ex. 235. 1234GR, 1235GR, 1324GR, 1325GR, 1425GR, 1435GR, 2345GR 2354GR.

Ex. 236. 1234GR, 1235GR, 1243GR, 1324GR, 1345GR, 1425GR, 2345GR, 2354GR, 2435GR.

Ex. 237. 1234GR, 1235GR, 1243GR, 1325GR, 1345GR, 1354GR, 1425GR, 2435GR.

Ex. 238. 1234, 1235, 1342, 1352, 1354, 1432, 1452, 1453, 1532, 1542, 2143, 2145, 2153, 2354, 2453, 3145, 3214, 3215, 3245, 4125, 4135, 4215, 4235, 4315, 4325, and their reverses.

Ex. 239. 1234, 1235, 1342, 1345, 1352, 1354, 1452, 1542, 2134, 2135, 2345, 2354, 2453, 2543, and their reverses.

Ex. 240. 1234R, 1235, 1342, 1345, 1352, 1354, 1452, 1542, 2145, 2154, 2345, 2354, 3142, 3152, 3452, 3542, 4521.

Ex. 241. 1234, 1235, 1324, 1325, 1423, 1432, 1452, 1453, 1523, 1532, 1542, 1543, 2145, 2154, 2314, 2315, 2345, 2354, 2415, 2514, 3145, 3154, 3214, 3215, 3245, 3254, 3415, 3514, 4235, 4325, and their reverses.

Ex. 242. All tetrads *except* 1243R, 1245R, 1253R, 1254R, 1324R, 1325R, 1423R, 1425R, 1523R, 1524R, 3124R, 3125R, 3245R, 3254R, 3425R, 3524R.

Ex. 243. 1234, 1235, 1243, 1325, 1342, 1345, 1352, 1423, 1425, 1432, 1435, 1452, 1523, 1543, 2134, 2135, 2145, 2154, 2314, 2315, 2345, 2354, 2415, 3124, 3145, 3214, 3215, 3245, 3415, 3425, 3514, 3524, 4125, 4135, 4215, 4235, 4325, and their reverses.

Ex. 244. 1234, 1235, 1342, 1345, 1352, 1354, 1452, 1542, 2134, 2135, 2145, 2154, 2314, 2315, 2345, 2354, 2413, 2453, 2513, 2543, 3145, 3154, 3214, 3215, 4125, 4135, 4215, 4235, 4315, 4325, and their reverses.

Ex. 245. 1234G, 1235G, 1243G, 1245G, 1253G, 1254G, 1425GR, 1435GR, 2314G, 2315G, 2345G, 2354G, 2435GR, 4315G, 5314G.

Ex. 246. 1234GR, 1235G, 1243G, 1245G, 1253GR, 1254GR, 1435GR, 2435GR, 4315G, 4325G.

Ex. 247. All tetrads *except* 1243GR, 1245GR, 1325GR, 1345GR, 1354GR, 2345GR.

Ex. 248. 1234, 1235, 3214, 3215, 4125, 4135, 4215, 4235, 4315, 4325, and their reverses.

Ex. 249. 1234, 1235, 1324, 1325, 1425, 1432, 1435, 1523, 1524, 1534, 2143, 2145, 2315, 2345, 2413, 2415, 2513, 2514, 2543, 3145, 3214, 3254, 3415, 3514, 4235, 4325, and their reverses.

Ex. 250. 1234R, 1235R, 1245R, 1435R, 2345R.

Ex. 251. 1234R, 1235R, 1245R, 2354R, 3415R.

Ex. 252. 1234R, 1235R, 1435R, 1542R, 2345R.

Ex. 253. 1234, 1235, 1245, 1453, 2345R.

Ex. 254. 1234R, 1235, 1435, 2145, 2345, 4521.

Ex. 255. 1234R, 1235R, 1245R, 1254R, 1435R, 2345R, 2354R.

Ex. 256. 1234R, 1235R, 1243R, 1245R, 1253R, 1254R, 1543R, 2143R, 2543R, 3415R, 3425R.

Ex. 257. 1234R, 1235, 1243, 1245, 1253, 1254, 2341, 2345, 2351, 2354, 2431, 2435, 2451, 2453, 2531, 2534, 2541, 2543, 3241, 3251, 3451, 3541, 4231, 4251, 4351, 4521, 4531, 5231, 5241, 5341, 5431.

Ex. 258. All tetrads *except* 1345R, 1354R, 1324R, 2134R.

Ex. 259. 1234G, 1235G, 1243GR, 1245G, 1253G, 1254G, 1324G, 1325G, 1345G, 1354GR, 1425G, 1435GR, 2435GR, 2453G, 2543G.

Ex. 260. 1234R, 1235R, 1345R, 2453R, 4125R.

Ex. 261. 1234, 1235, 1245, 1324, 1325, 1345, 4125, 4135, 4215, 4235, 4315, 4325, and their reverses.

Ex. 262. 1234R, 1235, 1342, 1345, 1423, 1425, 2153, 2453, 4521.

Ex. 263. 1234R, 1235, 1342, 1345, 1352, 1354, 1452, 1542, 2315, 2415, 3142, 3152, 3452, 3542, 4521.

Ex. 264. 1234, 1235, 1245, 1254, 1324, 1325, 1345, 1354, 2143, 2153, 2413, 2453, 2513, 2543, and their reverses.

Ex. 265. 1234R, 1235R, 1345R, 1354R, 2514R, 3524R.
Ex. 266. All tetrads *except* 1245G and 1354G.
Ex. 267. All tetrads *except* 1245GR, 1325GR, 1354GR, 2435GR.
Ex. 268. 1234R, 1235R, 1254R, 1345R, 2354R.
Ex. 269. 1234, 1235, 1253, 1254, 1325, 1342, 1345, 1425R, 1534R, 2134, 2341, 2345, 2351, 2354, 2451, 2531, 2534, 2541, 3125, 3241, 3251, 3421, 3425, 3451, 3541, 4251, 5341.
Ex. 270. 1234R, 1235, 1342, 1345, 1352, 1452, 2145, 2154, 2345, 2354, 3152, 3452, 4521.
Ex. 271. 1234R, 1235R, 1345R, 1425R, 2134R, 2345R, 2354R.
Ex. 272. All tetrads *except* 1243R, 1245R, 1253R, 1254R, 1324R, 1352R, 1354R, 1423R, 1452R, 1453R, 1523R, 1524R, 1542R, 1543R, 2453R, 2513R, 2543R, 3124R, 3154R, 3245R, 3254R, 3524R.
Ex. 273. All tetrads *except* 1243R, 1245R, 1352R, 1354R, 1423R, 1425R, 1532R, 1534R, 2354R, 2534R, 3245R, 3425R.
Ex. 274. All tetrads *except* 1245GR, 1324GR, 2354GR.
Ex. 275. 1234R, 1235, 1254R, 1342, 1345, 1352, 1354, 1452, 1542, 2145, 2345, 3142, 3152, 3452, 3542.
Ex. 276. 1234R, 1235R, 1254R, 1345R, 1354R, 2135R, 2145R, 2345R.
Ex. 277. All tetrads *except* 1243R, 1245R, 1253R, 1423R, 1453R, 1523R, 1524R, 1532R, 1534R, 1543R, 2143R, 2153R, 2154R, 2354R, 2534R, 3245R, 3514R, 3524R.
Ex. 278. 1234, 1235, 1342, 1345, 1352, 1354, 1452, 1542, 2145, 2345, 3142R, 3152R, 3452, 3542.
Ex. 279. 1234R, 1235R, 1345R, 1354R, 1425R, 2135R, 2345R.
Ex. 280. All tetrads *except* 1345G and 2354G.
Ex. 281. 1234R, 1235, 1324, 1352, 1354, 1524, 2145, 2345, 3124, 3524, 4521.
Ex. 282. 1234R, 1235R, 1354R, 1425R, 2345R.
Ex. 283. 1234R, 1235, 1243, 1245, 1342, 1352, 1354, 1435, 1542, 2135, 2145, 2345, 2435, 3142, 3542, 4521.
Ex. 284. 1234R, 1235, 1243, 1245, 1253, 1254R, 1354, 1435, 1534, 2134, 2135, 2143, 2145, 2153, 2154, 2345, 2435, 2534.
Ex. 285. All tetrads *except* 1345R and 2354R.
Ex. 286. All tetrads *except* 1342R, 1345R, 1532R, 1542R, 2134R, 2154R, 2315R, 2345R, 2354R, 4315R.
Ex. 287. 1234, 1235, 1324, 1325, 2145, 2314, 2315, 2345, 3145, 3214, 3215, 3245, 4125, 4135, 4215, 4235, 4315, 4325, and their reverses.

Table of Theorems.

The following table of theorems gives several items of information. The use of the table may best be explained by a concrete illustration.

Line 35, for instance, tells us what can be done with Postulates G, H, 19. Thus:

In the first place, the *italic figures* within parentheses tell us that Postulates G, H, 19 imply Postulates 14, 17, and 18, as proved in Theorem 14.19, Theorem 17.21, and Theorem 18.18 respectively.

In the second place, the *empty brackets* tell us that Postulates G, H, 19 imply also Postulates 15 and 16, but that not all of the Postulates G, H, 19 are required for the proof. (It will be found from line 107 that Postulates 15 and 16 follow from H and 19 alone.)

In the third place, the *dots* tell us that Postulates F, R, 10, 11, 12, 13 are not deducible from Postulates G, H, 19.

Finally, the *Roman numeral* IX at the end of the line shows that the theorems on this line are useful in connection with the "Set IX" of independent postulates. All the theorems required for Set IX (that is, all the theorems required for deducing the remaining postulates from Postulates F, G, H, R, 11, 19) are listed in the lines marked IX. (The lines marked with IX in parentheses contain alternative theorems which may be used in connection with Set IX but are not required for that purpose.)

Line	F′	R′	F	G	H	R	10	11	12	13	14	15	16	17	18	19	Set
1	(F′)	.	F	.	.	.	.	.	.	.	.	.	.	.	.	.	
2	.	(R′)	.	.	.	R	.	.	.	.	.	.	.	.	.	.	
3	F′	.	(F1)	G	H	.	10	( )	( )	( )	( )	( )	( )	( )	( )	( )	X
4	F′	.	(F2)	G	.	.	10	( )	12	( )	( )	( )	( )	( )	( )	( )	
5	F′	.	(F3)	G	.	.	10	( )	( )	13	( )	( )	( )	( )	( )	( )	
6	F′	.	(F4)	G	.	.	10	.	.	.	14	.	.	.	( )	.	
7	F′	.	(F5)	G	.	.	10	.	.	.	.	15	.	.	( )	.	
8	F′	.	(F6)	G	.	.	10	.	.	.	.	.	16	.	( )	.	
9	F′	.	(F7)	G	.	.	10	11	.	.	( )	.	.	17	( )	.	
10	F′	.	(F8)	G	.	.	10	11	.	.	.	.	( )	.	( )	19	
11	.	R′	.	G	H	(R1)	10	( )	( )	( )	( )	( )	( )	( )	( )	( )	X

Line	F G H R	10	11	12	13	14	15	16	17	18	19	Set
12	F G · R	(10·1)	( )	12	( )	(14·26)	(15·35)	(16·27)	(17·32)	(18·27)	(19·31)	II
13	F G · R	(10·2)	( )	( )	13	(14·27)	(15·36)	(16·28)	(17·33)	(18·28)	(19·32)	III
14	F G · R	(10·3)	11	·	·	14	( )	( )	( )	( )	( )	IV
15	F G · R	(10·4)	11	·	·	( )	15	( )	( )	( )	( )	V
16	F G · R	(10·5)	11	·	·	( )	( )	16	( )	( )	( )	VI
17	F G · R	(10·6)	11	·	·	( )	( )	( )	17	( )	( )	VII
18	F G · R	(10·7)	11	·	·	·	·	·	·	18	·	VIII
19	F G · R	(10·8)	11	·	·	( )	( )	( )	( )	( )	19	IX
20	· G H R	·	·	·	·	14	(15·33)	( )	(17·30)	(18·25)	(19·29)	IV
21	· G H R	·	·	·	·	( )	(15·34)	16	(17·31)	(18·26)	(19·30)	VI
22	· G H R	·	11	(12·10)	(13·9)	14	( )	( )	( )	( )	( )	IV
23	· G H R	·	11	(12·11)	(13·10)	( )	( )	16	( )	( )	( )	VI
24	· G H ·	10	(11·1)	(12·1)	( )	(14·1)	(15·1)	(16·1)	(17·1)	( )	(19·1)	I, X
25	· G H ·	·	11	(12·3)	(13·3)	( )	15	( )	( )	( )	( )	V
26	· G H ·	·	11	(12·4)	( )	( )	( )	( )	17	( )	( )	VII
27	· G H ·	·	11	( )	(13·5)	( )	( )	( )	( )	18	( )	VIII
28	· G H ·	·	11	(12·6)	(13·6)	( )	( )	( )	( )	( )	19	IX
29	· G H ·	·	·	·	·	14	·	(16·16)	·	·	·	IV
30	· G H ·	·	·	·	·	14	15	( )	(17·18)	(18·15)	(19·18)	
31	· G H ·	·	·	·	·	( )	15	16	(17·19)	(18·16)	(19·19)	
32	· G H ·	·	·	·	·	(14·16)	( )	16	·	·	·	VI
33	· G H ·	·	·	·	·	( )	( )	(16·17)	17	(18·17)	(19·20)	VII
34	· G H ·	·	·	·	·	( )	(15·15)	( )	(17·20)	18	(19·21)	VIII
35	· G H ·	·	·	·	·	(14·19)	( )	( )	(17·21)	(18·18)	19	IX
36	· G · R	10	·	·	·	(14·22)	15	(16·23)	17	( )	( )	
37	· G · R	10	·	·	·	(14·23)	15	(16·24)	·	·	19	
38	· G · R	·	11	·	·	14	(15·25)	( )	(17·28)	(18·23)	(19·27)	(IV)
39	· G · R	·	11	·	·	( )	(15·26)	16	(17·29)	(18·24)	(19·28)	(VI)
40	· G · R	·	11	·	·	( )	(15·27)	(16·22)	17	( )	( )	(VII)
41	· G · R	·	11	·	·	(14·21)	(15·28)	( )	( )	( )	19	(IX)
42	· G · R	·	·	·	·	14	·	(16·21)	·	·	·	(IV)
43	· G · R	·	·	·	·	14	(15·29)	( )	17	·	( )	
44	· G · R	·	·	·	·	14	(15·30)	( )	( )	·	19	

Line	F G H R	10	11	12	13	14	15	16	17	18	19	Set
45	· G · R	·	·	·	·	*(14·24)*	**15**	*(16·25)*	**17**	**18**	( )	
46	· G · R	·	·	·	·	*(14·25)*	**15**	*(16·26)*	( )	**18**	**19**	
47	· G · R	·	·	·	·	*(14·20)*	·	**16**	·	·	·	(VI)
48	· G · R	·	·	·	·	( )	*(15·31)*	**16**	**17**	·	( )	
49	· G · R	·	·	·	·	( )	*(15·32)*	**16**	( )	·	**19**	
50	· G · R	·	·	·	·	·	·	·	**17**	·	*(19·26)*	VII
51	· G · R	·	·	·	·	·	·	·	*(17·27)*	·	**19**	IX
52	· G · ·	10	·	·	·	·	·	·	·	*(18·1)*	·	I, X
53	· G · ·	10	11	·	·	**14**	·	·	*(17·15)*	( )	·	
54	· G · ·	10	11	·	·	**14**	*(15·17)*	**16**	( )	( )	( )	
55	· G · ·	10	11	·	·	·	·	**16**	·	( )	*(19·15)*	
56	· G · ·	10	( )	12	( )	*(14·2)*	*(15·2)*	*(16·2)*	*(17·2)*	( )	*(19·2)*	
57	· G · ·	10	( )	( )	13	*(14·8)*	*(15·8)*	*(16·8)*	*(17·8)*	( )	*(19·8)*	
58	· G · ·	·	11	·	·	**14**	*(15·18)*	**16**	( )	**18**	( )	
59	· G · ·	·	11	·	·	**14**	·	·	*(17·16)*	**18**	·	
60	· G · ·	·	11	·	·	**14**	*(15·19)*	( )	*(17·17)*	( )	**19**	
61	· G · ·	·	11	·	·	*(14·14)*	**15**	*(16·14)*	*(17·14)*	*(18·12)*	*(19·14)*	V
62	· G · ·	·	11	·	·	( )	*(15·20)*	**16**	**17**	( )	*(19·16)*	
63	· G · ·	·	11	·	·	·	·	**16**	·	**18**	*(19·17)*	
64	· G · ·	·	11	·	·	*(14·15)*	·	·	**17**	*(18·13)*	·	VII
65	· G · ·	·	11	·	·	( )	*(15·21)*	( )	**17**	( )	**19**	
66	· G · ·	·	11	·	·	·	·	*(16·15)*	·	*(18·14)*	**19**	IX
67	· G · ·	·	*(11·2)*	12	*(13·2)*	·	·	·	·	·	·	II
68	· G · ·	·	( )	12	( )	**14**	*(15·3)*	( )	*(17·3)*	( )	*(19·3)*	
69	· G · ·	·	( )	12	( )	*(14·3)*	**15**	*(16·4)*	*(17·4)*	*(18·3)*	*(19·4)*	
70	· G · ·	·	( )	12	( )	*(14·4)*	*(15·4)*	**16**	*(17·5)*	*(18·4)*	*(19·5)*	
71	· G · ·	·	( )	12	( )	*(14·5)*	*(15·5)*	*(16·5)*	**17**	*(18·5)*	( )	
72	· G · ·	·	( )	12	( )	*(14·6)*	*(15·6)*	( )	*(17·6)*	**18**	*(19·7)*	
73	· G · ·	·	( )	12	( )	*(14·7)*	*(15·7)*	*(16·7)*	*(17·7)*	*(18·6)*	**19**	
74	· G · ·	·	*(11·3)*	*(12·2)*	13	·	·	·	·	·	· ·	III
75	· G · ·	·	( )	( )	13	**14**	( )	*(16·9)*	( )	*(18·7)*	*(19·9)*	
76	· G · ·	·	( )	( )	13	*(14·9)*	**15**	*(16·10)*	*(17·10)*	*(18·8)*	*(19·10)*	
77	· G · ·	·	( )	( )	13	*(14·10)*	*(15·10)*	**16**	*(17·11)*	*(18·9)*	*(19·11)*	

Line	F G H R	10	11	12	13	14	15	16	17	18	19	Set
78	· G · ·	·	( )	( )	13	(14·11)	( )	(16·11)	17	(18·10)	(19·12)	
79	· G · ·	·	( )	( )	13	(14·12)	(15·12)	(16·12)	(17·12)	18	( )	
80	· G · ·	·	( )	( )	13	(14·13)	(15·13)	(16·13)	(17·13)	(18·11)	19	
81	· · H R	10	(11·5)	·	( )	·	·	·	·	·	·	
82	· · H R	10	( )	12	( )	·	(15·44)	16	·	·	(19·37)	
83	· · H R	10	( )	(12·12)	( )	( )	( )	( )	( )	18	( )	
84	· · H R	10	( )	(12·13)	( )	·	( )	( )	·	·	19	
85	· · H R	·	11	( )	(13·11)	( )	(15·37)	( )	(17·34)	18	(19·33)	
86	· · H ·	10	·	·	(13·1)	·	·	·	·	·	·	I, X
87	· · H ·	10	11	(12·7)	( )	·	( )	( )	·	·	19	
88	· · H ·	10	·	12	( )	14	( )	( )	( )	( )	(19·22)	
89	· · H ·	10	·	·	( )	14	(15·22)	·	(17·22)	·	·	
90	· · H ·	10	·	·	( )	14	( )	( )	( )	(18·19)	19	
91	· · H ·	10	·	·	( )	( )	(15·23)	( )	(17·23)	18	(19·23)	
92	· · H ·	·	11	12	(13·7)	·	( )	( )	·	·	19	
93	· · H ·	·	11	(12·8)	13	·	( )	( )	·	·	19	
94	· · H ·	·	11	·	(13·4)	( )	( )	·	17	·	·	VII
95	· · H ·	·	11	(12·9)	( )	( )	( )	( )	17	( )	19	
96	· · H ·	·	11	(12·5)	·	( )	·	( )	·	18	·	VIII
97	· · H ·	·	11	( )	(13·8)	( )	( )	( )	( )	18	19	
98	· · H ·	·	·	12	·	14	( )	( )	(17·26)	( )	19	
99	· · H ·	·	·	12	·	( )	( )	(16·20)	17	(18·22)	( )	
100	· · H ·	·	·	·	13	14	( )	( )	( )	(18·20)	19	
101	· · H ·	·	·	·	13	( )	(15·24)	( )	(17·25)	18	( )	
102	· · H ·	·	·	·	·	(14·17)	(15·14)	·	17	·	·	VII
103	· · H ·	·	·	·	·	( )	( )	( )	17	18	(19·24)	
104	· · H ·	·	·	·	·	( )	( )	( )	17	(18·21)	19	
105	· · H ·	·	·	·	·	(14·18)	·	(16·18)	·	18	·	VIII
106	· · H ·	·	·	·	·	( )	( )	( )	(17·24)	18	19	
107	· · H ·	·	·	·	·	·	(15·16)	(16·19)	·	·	19	IX
108	· · · R	10	11	12	( )	( )	15	( )	(17·38)	18	( )	
109	· · · R	10	11	·	13	(14·29)	15	·	(17·37)	18	( )	
110	· · · R	10	( )	12	13	( )	15	( )	(17·39)	18	( )	

Line	F G H R	10	11	12	13	14	15	16	17	18	19	Set
111	. . . R	10	.	12	.	14	*(15·45)*	( )	.	( )	*(19·39)*	
112	. . . R	10	.	12	.	.	15	*(16·30)*	.	.	*(19·38)*	
113	. . . R	10	.	12	.	*(14·31)*	15	( )	.	18	( )	
114	. . . R	10	*(11·6)*	12	*(13·13)*	*(14·28)*	*(15·43)*	*(16·31)*	17	*(18·30)*	( )	
115	. . . R	10	.	12	.	.	.	( )	.	18	*(19·40)*	
116	. . . R	10	.	12	.	.	.	*(16·29)*	.	.	19	
117	. . . R	10	.	.	13	*(14·30)*	( )	.	17	18	( )	
118	. . . R	10	.	.	13	14	( )	.	( )	*(18·29)*	19	
119	. . . R	10	.	.	13	*(14·32)*	( )	.	17	*(18·31)*	19	
120	. . . R	.	11	12	*(13·12)*	.	.	.	.	.	.	
121	. . . R	.	11	12	( )	14	*(15·38)*	( )	*(17·35)*	( )	*(19·34)*	
122	. . . R	.	11	12	( )	.	*(15·39)*	.	17	.	( )	
123	. . . R	.	11	12	( )	.	.	( )	.	18	*(19·35)*	
124	. . . R	.	11	.	.	14	.	.	*(17·36)*	18	.	
125	. . . R	.	11	.	.	14	*(15·41)*	.	.	.	19	
126	. . . R	.	11	.	.	.	*(15·42)*	16	17	.	.	
127	. . . R	.	11	.	.	.	.	16	.	18	*(19·36)*	
128	. . . R	.	11	.	.	.	*(15·40)*	.	17	.	19	
129	. . . R	.	*(11·4)*	12	13	.	.	.	.	.	.	
130	. . . .	.	.	12	13	14	( )	( )	( )	( )	*(19·25)*	
131	. . . .	.	.	12	.	14	.	*(16·3)*	.	*(18·2)*	.	
132	. . . .	.	.	12	.	.	.	.	17	.	*(19·6)*	
133	. . . .	.	.	12	.	.	.	*(16·6)*	.	18	.	
134	. . . .	.	.	.	13	14	*(15·9)*	.	*(17·9)*	.	.	
135	. . . .	.	.	.	13	.	*(15·11)*	.	17	.	.	
136	. . . .	.	.	.	13	.	.	.	.	18	*(19·13)*	

The ten sets of independent postulates may be exhibited in the same tabular form, as follows:

	F′	R′	F	G	H	R	10	11	12	13	14	15	16	17	18	19
I	( )	( )	F	G	H	R	10	( )	( )	( )	( )	( )	( )	( )	( )	( )
II	( )	( )	F	G	H	R	( )	( )	12	( )	( )	( )	( )	( )	( )	( )
III	( )	( )	F	G	H	R	( )	( )	( )	13	( )	( )	( )	( )	( )	( )
IV	( )	( )	F	G	H	R	( )	11	( )	( )	14	( )	( )	( )	( )	( )
V	( )	( )	F	G	H	R	( )	11	( )	( )	( )	15	( )	( )	( )	( )
VI	( )	( )	F	G	H	R	( )	11	( )	( )	( )	( )	16	( )	( )	( )
VII	( )	( )	F	G	H	R	( )	11	( )	( )	( )	( )	( )	17	( )	( )
VIII	( )	( )	F	G	H	R	( )	11	( )	( )	( )	( )	( )	( )	18	( )
IX	( )	( )	F	G	H	R	( )	11	( )	( )	( )	( )	( )	( )	( )	19
X	F′	R′	( )	G	H	( )	10	( )	( )	( )	( )	( )	( )	( )	( )	( )

TABLE OF EXAMPLES.

The following table states which postulates are satisfied by each of the eighty-eight systems (K, R) which were constructed above. The numbering of the examples begins with 200 to avoid confusion with earlier lists.

Either a dot or a dash indicates that the system fails to satisfy the postulate in question. For instance, line 235 of the table shows that Example 235 satisfies Postulates F,′ R,′ F, G, R, 10, 11, 18, but fails to satisfy Postulates H, 12, 13, 14, 15, 16, 17, 19.

The 154 dashes in the table give 154 *theorems on non-deducibility*. For instance, line 235 contains five such theorems on non-deducibility, since it shows that Postulates 14, 15, 16, 17, and 19 cannot be deduced from Postulates F′, R′, F, G, R, 10, 11, 18. The dots in line 235 are to be interpreted as follows: Postulates H, 12, and 13 are not deducible from F′, R′, F, G, R, 10, 11, 18, but in the case of these three postulates stronger examples will be found elsewhere in the table. (Thus, Example 208 is a stronger example for H; and Example 221 is a stronger example for 12 and 13.)

If we wish to find all the theorems on non-deducibility which concern any particular postulate, we have merely to pick out all the dashes which occur in the column corresponding to that postulate.

These 154 theorems on non-deducibility, together with the 267 theorems on deducibility given in the previous table, will be found to be exhaustive; that is, the two tables together enable us to determine at once whether any given postulate of our basic list is or is not deducible from any given combination of the postulates of the list.

For instance, if we ask whether Postulate 15 is deducible from G, 10, 11, 14, 16, 18, the answer is Yes, since line 54 of the table shows that 15 follows from G, 10, 11, 14, 16 even without the use of 18. Again, if we ask whether Postulate 15 is deducible from G, 10, 11, 18, 19, the answer is No, since line 232 of the table shows that 15 is not deducible from G, 10, 11, 18, 19, even with the aid of F′, R′, F, and 16. In a similar manner, every question of this kind can be answered at once, by inspection of the tables.

Ex.	F′	R′	F	G	H	R	10	11	12	13	14	15	16	17	18	19
200	F′	R′	F	G	H	R	10	11	12	13	14	15	16	17	18	19
201	–	R′	–	G	H	R	10	11	12	13	14	15	16	17	18	19
202	F′	–	F	G	H	–	10	11	12	13	14	15	16	17	18	19
203	F′	R′	–	G	H	R	–	11	12	13	14	15	16	17	18	19
204	F′	R′	–	·	H	R	10	11	12	13	14	15	16	17	18	19
205	F′	R′	–	G	·	R	10	11	·	·	·	·	·	·	18	·
206	F′	R′	–	G	·	R	10	·	·	·	·	·	·	17	18	19
207	F′	R′	F	–	H	R	10	11	12	13	14	15	16	17	18	19
208	F′	R′	F	G	–	R	10	11	12	13	14	15	16	17	18	19
209	F′	R′	F	·	H	–	10	11	12	13	14	15	16	17	18	19
210	F′	R′	F	G	·	–	10	11	12	13	14	15	16	17	18	19
211	F′	R′	F	G	H	–	–	11	12	13	14	15	16	17	18	19
212	F′	R′	F	·	H	R	–	11	12	13	14	15	16	17	18	19
213	F′	R′	F	G	H	R	–	–	–	–	14	15	16	17	18	19
214	F′	R′	F	G	H	R	–	11	–	–	–	–	–	–	–	–
215	F′	R′	F	G	·	R	10	–	·	·	14	15	16	17	18	19
216	F′	R′	F	·	·	R	10	–	12	–	14	15	16	–	18	19
217	F′	R′	F	·	·	R	10	–	·	13	14	15	16	17	18	19
218	F′	R′	F	·	H	R	·	–	–	13	14	15	16	17	18	19
219	F′	R′	F	·	H	R	·	–	12	–	14	15	16	17	18	19
220	F′	R′	F	·	H	·	10	–	12	13	14	15	16	17	18	19
221	F′	R′	F	G	·	R	10	11	–	–	14	15	16	17	18	19
222	F′	R′	F	G	H	·	·	11	–	–	14	–	16	–	–	–
223	F′	R′	F	·	·	R	10	11	–	13	14	15	16	17	18	19
224	F′	R′	F	·	H	R	10	11	–	13	14	15	16	17	–	–
225	F′	R′	F	·	H	·	10	·	–	13	14	15	16	17	18	19
226	F′	R′	F	·	H	R	·	11	–	–	14	15	16	–	–	19
227	F′	R′	F	·	·	·	10	11	12	–	14	15	16	17	18	19
228	F′	R′	F	·	H	·	·	11	12	–	14	15	16	–	18	–
229	F′	R′	F	G	H	R	·	·	·	·	–	15	–	–	–	–
230	F′	R′	·	G	H	R	·	11	12	13	–	–	–	–	–	–
231	F′	R′	F	G	H	·	·	11	12	13	–	–	–	–	–	–
232	F′	R′	F	G	·	·	10	11	·	·	–	–	16	–	18	19
233	F′	R′	F	G	·	·	10	·	·	·	–	15	16	17	18	19
234	F′	R′	F	G	·	R	10	·	·	·	–	–	–	17	18	19
235	F′	R′	F	G	·	R	10	11	·	·	–	–	–	–	18	–
236	F′	R′	F	G	·	R	10	·	·	·	–	15	–	·	18	·
237	F′	R′	F	G	·	R	·	·	·	·	–	15	–	17	·	19
238	F′	R′	F	·	H	R	10	11	12	13	–	15	16	–	–	19
239	F′	R′	F	·	·	R	·	11	12	13	–	15	16	17	18	19
240	F′	R′	F	·	·	·	10	11	12	13	–	15	16	17	18	19
241	F′	R′	F	·	·	R	10	11	12	13	–	–	16	–	18	19
242	F′	R′	F	·	·	R	10	11	·	·	–	15	16	17	18	19
243	F′	R′	F	·	·	R	10	·	·	13	–	15	16	–	18	19
244	F′	R′	F	·	·	R	10	11	·	13	–	15	16	17	·	·
245	F′	R′	F	G	·	·	10	·	·	·	14	–	16	17	18	19
246	F′	R′	F	G	·	·	10	11	·	·	14	–	–	17	18	–
247	F′	R′	F	G	·	R	10	·	·	·	14	–	16	·	18	·
248	F′	R′	F	·	H	R	10	11	12	13	·	–	–	·	·	–
249	F′	R′	F	·	H	R	10	11	·	13	·	–	16	·	·	·
250	F′	R′	F	·	H	R	·	·	12	·	14	–	16	·	18	·
251	F′	R′	F	·	H	R	·	11	·	·	14	–	16	·	·	·

Ex.	F′	R′	F	G	H	R	10	11	12	13	14	15	16	17	18	19
252	F′	R′	F	·	H	R	·	11	12	13	·	–	16	·	·	·
253	F′	R′	F	·	H	·	·	11	12	·	14	–	16	·	18	·
254	F′	R′	F	·	H	·	10	11	12	13	·	–	16	·	·	·
255	F′	R′	F	·	·	R	·	·	12	·	14	–	16	17	18	19
256	F′	R′	F	·	·	R	10	11	·	·	14	–	·	17	18	·
257	F′	R′	F	·	·	·	10	11	12	·	14	–	16	17	18	19
258	F′	R′	F	·	·	R	10	·	·	·	14	–	16	17	18	19
259	F′	R′	F	G	·	·	10	·	·	·	14	15	–	17	18	19
260	F′	R′	F	·	H	R	·	11	12	13	·	15	–	·	·	·
261	F′	R′	F	·	H	R	10	11	·	13	14	15	–	17	·	·
262	F′	R′	F	·	H	·	10	11	12	13	·	15	–	·	·	·
263	F′	R′	F	·	·	·	10	11	12	13	·	15	–	17	·	19
264	F′	R′	F	·	·	R	10	11	·	13	14	15	–	17	18	19
265	F′	R′	F	·	·	R	·	11	12	13	·	15	–	17	·	19
266	F′	R′	F	G	·	·	10	·	·	·	14	15	16	–	18	19
267	F′	R′	F	G	·	R	10	·	·	·	14	15	16	–	18	–
268	F′	R′	F	·	H	R	·	·	12	·	14	15	16	–	18	–
269	F′	R′	F	·	·	·	10	11	12	·	14	15	16	–	18	19
270	F′	R′	F	·	·	·	10	11	12	13	·	15	16	–	18	19
271	F′	R′	F	·	·	R	·	11	12	13	·	15	16	–	18	19
272	F′	R′	F	·	·	R	10	11	·	·	·	15	16	–	18	19
273	F′	R′	F	·	·	R	10	11	·	·	14	15	16	–	·	19
274	F′	R′	F	G	·	R	·	·	·	·	14	15	16	17	–	19
275	F′	R′	F	·	·	·	10	11	·	13	14	15	16	17	–	19
276	F′	R′	F	·	·	R	·	11	·	13	14	15	16	17	–	19
277	F′	R′	F	·	·	R	10	11	·	·	14	15	16	17	–	19
278	F′	R′	F	·	·	·	10	11	12	13	·	15	16	17	–	19
279	F′	R′	F	·	·	R	·	11	12	13	·	15	16	17	–	19
280	F′	R′	F	G	·	·	10	·	·	·	14	15	16	17	18	–
281	F′	R′	F	·	H	·	10	11	12	13	·	15	16	·	·	–
282	F′	R′	F	·	H	R	·	11	12	13	·	15	16	·	·	–
283	F′	R′	F	·	·	·	10	11	12	·	14	15	16	·	18	–
284	F′	R′	F	·	·	·	10	11	·	·	14	15	16	17	18	–
285	F′	R′	F	·	·	R	10	·	·	·	14	15	16	17	18	–
286	F′	R′	F	·	·	R	10	11	·	·	14	15	·	17	18	–
287	F′	R′	F	·	·	R	10	11	12	13	·	·	16	·	·	–

These two tables, which summarize the conclusions of the paper, are complete, and free from redundancies.

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BACTERIAL DETOXIFICATION.

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BACTERIAL DETOXIFICATION.

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Theory of Bacterial Detoxification.

Bacterial detoxification may be defined as the neutralization of the toxic products of bacterial cells whereby the toxin is still able to stimulate the production of antibodies but is unable to exert its toxic effect on the body of the animal in which it is contained. Effective germicides and antiseptics combat the attack of bacteria but, if effective, they also circumvent antibody production. Detoxifiers neutralize the attack of bacteria without necessarily killing them, although inhibiting their physiological processes, and allowing the production of antibodies to continue. This method of treating the bacterial infections is, therefore, advantageous for it renders the body immune to reinfection by the organism detoxified.

The Role of Surface Tension in Detoxification.

One of the better bacterial detoxifiers is sodium ricinoleate. In researches with organisms in respect to the effect of sodium ricinoleate on their toxicity it has been observed (57a) that those organisms which normally grow at a low surface tension are relatively insensitive to the action of sodium ricinoleate. On the other hand, those organisms normally accustomed to rather higher surface tensions are quite sensitive to its action. It was this curious and interesting fact which led workers to postulate that detoxification is essentially due to the reduction of the surface tension of the fluid in which the bacteria are living.

The surface tension of a liquid may be regarded as being the seat of a special force. This force, called the surface tension, acting perpendicularly to a section of the surface one centimeter in length is generally measured in dynes. Surface tension exists at the interface between liquid and air. A surface tension also exists at the boundary of two immiscible liquids, between a liquid and a solid, and indeed, wherever a free surface is formed.

Larson and his associates have found ample evidence that surface tension affects bacterial growth. They caused (52) surface growing bacteria to grow throughout the medium by decreasing the surface tension from 59 dynes to 40 dynes. Organisms normally found in the intestine grow abundantly on media of low surface tension. This is to be expected, as the contents of the intestinal tract have a low surface tension due to the presence of the bile salts, soaps and other surface tension depressants. Governed by well-known physical laws, the *surface tension depressants in culture media concentrate in the* surface layers of the media (55). This explains why the colon-typhoid-cholera group selects the surface zone for growth. It is for this reason that pellicle formers grow as they do, at the top of the broth medium where the surface tension depressants are concentrated. If the interfacial tension between liquid and bacterial cells is sufficiently depressed, the organisms grow throughout the medium and even at the bottom. On the other hand, by increasing the surface tension, non-pellicle formers may be made to form pellicles.

Tubercle bacilli, when grown for ten days at 44 dynes, lost their pathogenicity to guinea pigs (62). In explanation of this Larson suggested that when tubercle bacilli are growing diffusely throughout the medium they are "wetted" and that when they are introduced into the animal body they are the more easily penetrated by antibodies or bactericidal substances present, and destroyed. He found that when equal amounts of tubercular sputum and a 2% solution of sodium ricinoleate were mixed and allowed to stand several hours and portions of this injected into guinea pigs, tuberculosis did not appear. All of the controls died of tuberculosis.

Bacteria, like colloid particles, carry electrical charges and wander in an electrical field to one pole or the other. The surfaces of such particles are, therefore, the seat of free electrical energy. They must also be the seat of free mechanical energy arising from the surface tension of the field in which they are suspended. If oppositely charged particles or ions can establish an equilibrium by reducing the amount of free electrical energy by moving to such surfaces, they will do so in obedience to the second law of thermodynamics. If these particles or ions have the power of reducing the amount of free mechanical surface energy by virtue of an ability to lower surface tension, they would also come under the influence of mechanical, as well as electrical, adsorption forces. A detoxifier which does not ionize extensively is not greatly influenced by electrical charges and is to a much greater extent dependent on mechanical adsorption, and therefore, upon its own power of reducing surface tension. Chemicals, in other words, which are not highly ionized, depend largely upon their ability to lower surface tension in order to be adsorbed. It seems agreed that the sodium ricinoleate is adsorbed onto the surfaces of the toxin molecule, "imprisoning" it so that it is not free to react with the tissues. This imprisonment is not permanent, for this would influence its antigenic properties. Rather, the imprisonment lasts for a time long enough to allow the production of antibodies. When the toxin molecule is then freed it is attacked by the antibodies and disease symptoms are prevented, while antibody production presumably continues, thus building up immunity.

Wolf's point (124) that "in the contest for the surface layer those substances which are powerful in reducing the surface tension tend to remove that which is less efficient" may be important and aid in solving the problem. Substances like sodium ricinoleate which lower the surface tension thereby expel the nutrient substances of less surface tension depressing power from the surface of the liquid. The surface then no longer contains rich nutrient substances but has a high concentration of ricinoleate which may be indirectly toxic to bacteria at or near the interfaces.

To sum up, therefore, it may be said that the mechanism of the action of sodium ricinoleate in detoxifying bacteria is as yet only theoretically explainable. It appears that surface tension is not the sole cause of its action, although it seems to be a contributory factor. The other factor of importance is a chemical one having to do with molecular structure.

HISTORICAL.

Dr. H. Vincent of the Pasteur Institute has perhaps the greatest claim to being the "father of detoxification"—or, as he calls it, "cryptotoxication." It was he who in 1907 found that the bile constituents neutralized tetanus toxin and that bile thus has cryptotoxic properties (112). In the same year (113) he said "The antitoxic properties of soaps in very weak concentration appear worth mentioning." He found that cholesterol, lecithin and the bile salts have these properties. In 1909 he stated (115): "The most active (detoxifiers) are soaps, cholesterol and sodium glycocholate." Scattered papers appeared on detoxification between 1909 and 1920. It was at about this time that Larson and others discovered the detoxifying powers of sodium ricinoleate and in 1923, Larson and Montank (62) reported the neutralization of the tubercle bacillus by a 2% sodium ricinoleate solution. Sodium ricinoleate was made still more pure and its detoxifying powers were greatly increased. Many papers on detoxification with ricinoleate have appeared in the journals of this country. These have been summarized in the tables.

KNOWN BACTERIAL DETOXIFIERS.

Ultra-violet light (69, 79, 123), sunlight (32), light and eosin (68), heat (46), heat and formaldehyde (31, 70) are physical agents which detoxify bacteria.

TABLE I.

BACTERIAL DETOXIFICATION BY PHYSICAL AGENCIES.

Bacterial toxin	Agent	Authors	Reference
Diphtheria	Ultra-violet light	Lowenstein	69
General	Formaldehyde and heat	Eisler and Lowenstein	31
	Formaldehyde and heat	Lowenstein	70
Tetanus	Sunlight (15 hours)	Fernie and Pernosie	32
	Ultra-violet light	Megrail and Welch	79
	Ultra-violet light	Welch	123
	Heat (68° C. for 5 min.)	Kitasato	46
	Eosin and light	Lowenstein	68

TABLE II.

BACTERIAL DETOXIFICATION BY INORGANIC AGENCIES.

Bacterial toxin	Agent	Authors	Reference
Diphtheria	Aluminum sulphate	Glenny	38
	Ammonium thiosulphate	Schumacher	99
	Copper	Laubenheimer	65
	Copper chloride	Laubenheimer	65
	Gold salts	Behring	7
		Osman	89

TABLE II—cont.

Bacterial toxin	Agent	Authors	Reference
	Iodine trichloride	Behring	7
	Mineral water	Billard	10
	Potassium aluminum sulphate	Glenny	38
General	Chlorine	Moussu and Goupil	82
	Ozone	Nelis	84
	Sodium fluoride	Berthelot and Ramon	8
	Sodium perborate	Berthelot and Ramon	8
Poliomyelitis	Colloidal aluminum hydroxide	Rhoades	93
Tetanus	Copper	Laubenheimer	65
	Copper chloride	Laubenheimer	65
	Mineral water	Billard and Courtoil	11
	Radium emanation	Ferroux and Mutermilch	34
Typhus	Carbon disulphide	Lowenstein	70
	Mineral water	Ferreyrolles	33

TABLE III.

BACTERIAL DETOXIFICATION BY ORGANIC AGENCIES (EXCEPTING SOAPS).

Bacterial toxin	Agent	Authors	Reference
Botulinus	Cholesterol	Suranyi	102
Colon bacillus	Halogen compounds of salicylic acid	Vincent	121
Diphtheria	Adrenalin	Abramov and Mishennikov	1
	Amino benzoic acid	Nishuira	88
	Cholesterol	Suranyi	102
		Suranyi and Jarno	103
	Hydroxy-benzoic acid	Nishuira	88
	Lecithin	Almagia	5
		Bruschettini and Calcaterra	15
		Larson	54
	Organic acids	Roux and Yersin	97
	Phenol	Nishuira	88
	Pus	Larson	54
	Quinine	Sbarsky and Subkowa	98

TABLE III—cont.

Bacterial toxin	Agent	Authors	Reference
	Quinine bisulphate	Nelis	84
	Quinine dihydrochloride	Nelis	84
	Salicylic acid (halogen compounds)	Vincent	121
	Sodium benzoate	Nishuira	88
	Sodium salicylate	Birkhaug	12
		Nishuira	88
	Triphenylmethane series	Coplans and Green	21
General	Acetic acid	Berthelot and Ramon	8
	Acetone	Douglas and Fleming	27
	Antipyrene	Vincent	117, 118
	Chloramine	Berthelot and Ramon	8
	Cholesterol	Suranyi	102, 103
	Formic acid	Berthelot and Ramon	8
	Pyramidon	Vincent	117, 118
	Sodium salicylate	Vincent	118
	Sodium salts of alkyl resorcinol carboxylic acid	Bleyer	13, 14
	Urotropin	Berthelot and Ramon	8
Pneumococcus	Bile	Grixoni	40
Rabies virus	Cholesterol	Almagia	5
Tetanus	Adrenalin	Abramov and Mishennikov	1
	Amino benzoic acid	Nishuira	88
	Bile	Ninni	87
		Roger	95
		Vincent	110, 113, 114, 115
	Brain lipoids	Loewe	67
	Cerebral emulsion	Marie and Tiffenau	76, 77, 105
	Chloroform	Lowenstein	70
	Cholesterol	Almagia	5
		Suranyi	102
		Suranyi and Jarno	103
		Vincent	112
	Epinephrine	Marie	73, 74, 75
	Ethyl benzoate	Loewe	67
	Gentian violet	Hall and Taber	41

TABLE III—cont.

Bacterial toxin	Agent	Authors	Reference
	Gynocardate	Vincent	118
	Hexylresorcinol	Coleman	20
	Hydroxy benzoic acid	Nishuira	88
	Hydroxy benzoic acid (halogenated)	Vincent and Velluz	120a
	Lecithin	Almagia	5
		Bruschettini and Calcaterra	15
		Larson	59
		Suranyi and Jarno	103
		Vincent	111, 112
	Liver	Vincent	111, 114, 115, 118
	Pancreatic secretions	Vincent	111, 114, 115, 118
	Papain	Marie and Tiffenau	76
	Phenol	Nishuira	88
	Salicylic acid (halogen compounds)	Vincent	121
	Sodium benzoate	Nishuira	88
	Sodium salicylate	Birkhaug	12
		Nishuira	88
		Vincent	119, 120, 121
	Sodium 3, 5-diiodo-o-salicylate	Vincent and Velluz	120a, 120b
	Stomach secretions	Vincent	111, 114, 115, 118
	Strychnine sulphate	Valley	106
	Tetramethyl ammonium chloride	Tiffenau and Marie	105
	Triphenylmethane series	Coplans and Green	21
	Xylene	Lowenstein	70

TABLE IV.

Detoxification of Bacterial Toxins by Soaps (excepting Sodium Ricinoleate).

Bacterial toxin	Agent	Authors	Reference
Colon bacillus	Sodium acyclic acid salts	Vincent	121
	Sodium butyrate	Vincent	121
	Sodium mucate	Vincent	121
	Sodium palmitate	Vincent	116, 117, 118
	Sodium pyruvate	Vincent	121
Diphtheria	Sodium acyclic acid salts	Vincent	121
	Sodium butyrate	Vincent	121
	Sodium mucate	Vincent	121
	Sodium oleate	Nelis	83, 84
		Vincent	118
	Sodium palmitate	Vincent	117
	Sodium pyruvate	Vincent	121
Dysentery	Sodium palmitate	Vincent	117
General	All soaps	Davison	22
Oedematus bacillus	Sodium palmitate	Vincent	117
Para-typhoid B	Sodium palmitate	Vincent	117
Pneumococcus	Sodium cholate	Nicolle and Adil-Bey	86
	Sodium dehydrocholate	Ziegler	125, 125a, 125b
Tetanus	Sodium benzoate	Vincent	120
	Sodium butyrate	Vincent	121
	Sodium glycocholate	Vincent	112
	Sodium guttate	Vincent	115
	Sodium hippurate	Vincent	120
	Sodium mucate	Vincent	120, 121
	Sodium oleate	Vincent	117, 118
	Sodium oleomargarate	Vincent	112
	Sodium palmitate	Vincent	115
	Sodium pyruvate	Vincent	121
	Sodium taurocholate	Vincent	112
	Sodium salts of acyclic acids	Vincent	121
Typhoid	Sodium palmitate	Vincent	120

TABLE V.

Detoxification by Sodium Ricinoleate.

Toxin treated	Detoxifier	Concentration	Subject	Results	Authors	Ref.
Aminata phalloides	Sodium ricinoleate		Guinea pig	Increased toxicity	Green, Stoesser	39
Botulinus	Castor oil soap	100 L. D.	Guinea pig	Increased toxicity	Larson, Nelson	64
Diphtheria	Sodium ricinoleate	1% sol. 0.125 L.	Guinea pig	Detoxified	Larson, Hancock, Eder	60
Diphtheria	Castor oil soap	100 L. D.	Guinea pig	Detoxified	Larson, Nelson	64
Peridontal disease	Sodium ricinoleate		Human	Detoxified	Hopkins	44
Pneumococcus	Sodium ricinoleate	12 L. D. 1% sol.	Rabbit	Detoxified	Netter, Andre, et al.	85
Pneumococcus	Sodium ricinoleate	0.1% sol.	Guinea pig	Detoxified	Larson, Nelson	63
Rattle snake venom	Sodium ricinoleate	5 L. D	Rabbit	Detoxified	Carmichael	16
Ricin	Sodium ricinoleate	2000 L. D.	Rabbit	Detoxified anti-ricin	Carmichael	17
Scarlet fever	Sodium ricinoleate	0.5% sol.	Rabbit	Detoxified agglutinins	Larson, Nelson	63
Scarlet fever	Sodium ricinoleate	15,000 skin test doses	Children	Positive Dicks neg.	Larson, Heunekens, Colby	61
Scarlet fever	Sodium ricinoleate	4000 skin test doses	Adults	Detoxified	Eder	30
Scarlet fever	Sodium ricinoleate		Children	Detoxified	Colby	18, 19
Scarlet fever	Sodium ricinoleate	2% sol.	Guinea pig	Destroyed connect. tis.	Kozlowsky	49
Tetanus	Castor oil soap	100 L. D.	Guinea pigs	Detoxified	Larson, Nelson	64
Tetanus	Sodium ricinoleate	100 L. D.	Guinea pigs	Detoxified	Hartzell, Larson	42
Tubercle	Sodium ricinoleate	2% sol.	Guinea pigs	Detoxified	Larson, Montank	62
Tubercle bacillus	Sodium ricinoleate		Human	Relieved intestinal T.B.	Kline	47
Autointoxication	Sodium ricinoleate	6 gr. capsule	Human	Complete relief	Morris, Dorst	81
Intestinal allergy	Sodium ricinoleate	5 gr. capsule	Human	Relief	Morris, Dorst	81
Polimyelitis	Sodium ricinoleate		Animal	Detoxified	McKinley, Larson	78
Abdominal toxemia	Sodium ricinoleate	2% solution	Human	Aided recovery	Lutterloh, Stroud	72
Burns	Sodium ricinoleate		Human	Aided healing	Lutterloh	71

Aluminum sulphate (38), ammonium thiosulphate (99), carbon disulphide (70), chlorine (82), colloidal aluminum hydroxide (93), copper (65), copper chloride (65), gold salts (7, 89), iodine trichloride (7), mineral water (10, 11, 33), ozone (84), potassium aluminum sulphate (38), radium emanation (34), sodium fluoride (8), and sodium perborate (8) are some of the inorganic compounds which detoxify bacteria.

Acetic acid (8), acetone (27), adrenalin (1), amino benzoic acid (88), antipyrene (117, 118), bile (40, 87, 95, 110, 113, 114, 115), brain lipoids (67), cerebral emulsion (76, 77, 105), chloramine (8), chloroform (70), cholesterol (5, 102, 103, 112), epinephrine (73, 74, 75), ethyl benzoate (67), formic acid (8), gentian violet (41), gynocardate (118), hexylresorcinol (20), hydroxy benzoic acid (88), halogenated hydroxy benzoic acid (120a), lecithin (5, 15, 59, 103, 112), liver (111, 114, 115, 118), organic acids (97), pancreatic secretions (111, 114, 115, 118), papain (76), phenol (88), pus (54), pyramidon (117, 118), quinine (98), quinine bisulphate (84), quinine dihydrochloride (84), sodium benzoate (88), sodium salicylate (12, 88, 118), salicylic acid halogen compounds (121), sodium salts of alkyl resorcinol carboxylic acid (13, 14), sodium 3-5, diiodo-o-salicylate (120a, 120b), stomach secretions (111, 114, 115, 118), strychnine sulphate (106), tetramethyl-ammonium chloride (105), triphenylmethane series (12), urotropin (8) and xylene (70) are organic compounds which are bacterial detoxifiers.

Sodium ricinoleate (16, 17, 18, 19, 30, 39, 42, 44, 47, 49, 60, 61, 62, 63, 64, 71, 72, 78, 81, 85) seems to be the best bacterial detoxifier, perhaps because of its purity (64). It is claimed that all soaps detoxify (22). Sodium butyrate (121), sodium benzoate (120), sodium cholate (86), sodium dehydrocholate (125, 125a, 125b), sodium guttate (115), sodium hippurate (120), sodium mucate (120, 121), sodium oleate (83, 84, 117, 118), sodium oleomargarate (112), sodium palmitate (115, 116, 117, 118), sodium pyruvate (121) and sodium taurocholate (112) are some of the soaps which have been found to be detoxifiers.

It is generally accepted that castor oil soap or its purified form, sodium ricinoleate, is the most rapid detoxifier. The other detoxifiers require many minutes or hours to render toxins impotent, whereas ricinoleate seldom requires more than a few minutes. It is interesting to note that the best detoxifiers are sodium salts of saturated and unsaturated compounds. Vincent (120) found that sodium salts of the lowest acyclic acids have no detoxifying power. Cryptotoxic action starts with sodium butyrate, C_5 is weakly cryptotoxic, C_6 is strongly, C_7 is weakly, C_8 is strongly active but is less so than C_6, C_9 is very feeble, C_{10} is very active, C_{11} acids are very cryptotoxic, C_{12}, C_{14}, C_{16}, C_{18} are feebly cryptotoxic. Only sodium palmitate of the higher members of the series is cryptotoxic, .000002 mg. neutralize one lethal dose of tetanus toxin. The sodium salts of saturated acids of the acyclic series Vincent finds have very uneven cryptotoxic ability. The length of the chain is therefore not the only factor.

Vincent found that there is no absolute relationship between chemical structure, surface tension, and cryptotoxic power, and that salts of unsaturated acids are more active than salts of saturated acids. In this paper we report on the powers of methyl, trienthanolamine, and ephedrine-ricinoleate as detoxifiers.

Kozlowsky reported (49) that sodium ricinoleate destroyed the connective tissues, muscles, blood vessels, and blood of laboratory animals when injected in 2% aqueous solution. These results are sufficiently explained by Larson (53) who claims that Kozlowsky's results were entirely due to the fact that his sodium ricinoleate was prepared through barium. This element is very irritating to the tissues of the body.

Value of Detoxification.

Five doses of toxin, each dose one week apart, have been given in scarlet fever prophylaxis. After two months 95% of those receiving these injections were reported as Dick negative. One injection of ricinoleated toxin has been reported to produce 95% Dick negatives in three weeks (61). The latter procedure is the more simple and more quickly effective. From several sources we have been informed, however, that the effectiveness of the ricinoleate method is not as great as was first indicated. The detoxification method has a great advantage over the old antitoxin method in that it avoids the use of foreign serum proteins and therefore avoids anaphylaxis.

The bite of venomous snakes is usually mortal. *In vivo* injections of sodium ricinoleate have been found (16) to detoxify snake venoms. This is a fertile field for further investigation.

Space will not permit other than mention of Besredka's article (9) on local immunity. Bacteriologists should have read it long before now. If his theory is sound, and it apparently is, detoxification shows promise as a means of treating skin infections. It was with this in mind that the investigation with regard to staphylococcus aureus reported in this paper, was undertaken.

CHEMISTRY OF SODIUM RICINOLEATE.

Sodium ricinoleate is the sodium salt of ricinoleic acid. Castor oil is a tri-glyceride of ricinoleic acid and is therefore the ready source of sodium ricinoleate. Castor oil is readily extracted from pulverized castor bean. In obtaining this oil it is important that one guard against the extremely poisonous protein, ricin, contained in the shells.

For use as a detoxifier, sodium ricinoleate must be very pure. Smallest amounts of impurities will completely destroy its detoxifying powers. Oleic, stearic and hydrostearic acids which are normally present in castor oil must be removed. Phenol, resorcinol and alkaline earth metals in small amounts completely destroy this property. Nonsol or pyrex glassware should be used in preparing the ricinoleate and for keeping it in storage. This glassware should be thoroughly cleansed with soap and rinsed several times with distilled water.

PURPOSE OF THE PREPARATION OF THE OTHER RICINOLEATES.

Sodium ricinoleate is the only ricinoleate which has been tested for its detoxifying powers. If detoxification is not dependent on the cation, it should be possible to detoxify with other ricinoleates than sodium. To prove that such a thing is possible three other ricinoleates were prepared. Methyl ricinoleate was prepared because it represents the most simple organic ricinoleate ester. Triethanolamine ricinoleate is a compound whose cation alone possesses marked surface tension lowering properties. It was hoped that when triethanolamine was chemically combined with the ricinoleate molecule a compound would result which would possess exceptional detoxifying ability. Ephedrine is noted for its bland effect on the mucous membrane. Combination of ephedrine with ricinoleate might result in a detoxifier which is less irritating than sodium ricinoleate and yet equally as effective. From the results we obtained, it is not possible to answer this question but there are indications that ephedrine ricinoleate has these properties.

DETOXIFICATION OF S. AUREUS.

In the literature there is no report on the effect of ricinoleate on S. aureus. If Besredka's theory of local immunization has any value whatsoever and if ricinoleate will detoxify this ordinary pus former, extensive application of the detoxification principle can be made in the treatment of acne and other skin disturbances. The only drawback in such an application would be in its irritating effect on the skin, but this would perhaps be avoided by dilution or mixing with other substances.

PREPARATION OF SODIUM RICINOLEATE.

At the time this work was done the method of Rider (94) was unpublished. Dr. W. P. Larson has kindly furnished us with a procedure for preparing a very pure sodium ricinoleate for detoxification purposes. We publish it here with his permission.

> Saponify cold press castor oil with c. p. sodium hydroxide and isopropyl alcohol in a reflux. Mix with several volumes of distilled water. Treat with hydrochloric acid to free of fatty acids, siphon off the water and place the fatty acids in two volumes of isopropyl alcohol. Stand at −20° C. for one week. Filter off the precipitated fatty acids on a Buchner filter. Saponify with c. p. sodium hydroxide. Pour into large evaporating dish and cut soap formed into small pieces. Let the isopropyl evaporate off and grind to a meal.

PREPARATION OF METHYL RICINOLEATE.

Methyl ricinoleate is prepared by mixing 100 grams of purified ricinoleic acid, 1350 grams absolute methyl alcohol and 150 grams concentrated sulphuric acid and boiling under reflux condensers for 6½ hours. When cool, saturated salt solution is added. Methyl ricinoleate separates completely over-night. Remove the ricinoleate, wash with water until free from chlorides and sulphates and dry with calcium chloride.

PREPARATION OF TRIETHANOLAMINE RICINOLEATE.

Triethanolamine ricinoleate is prepared by saponifying cold press alcohol with c. p. sodium hydroxide in isopropyl alcohol under a reflux condenser. Mix with several volumes of distilled water and treat with hydrochloric acid to free of fatty acids. Siphon off the water and complete the separation in a separatory funnel. The fatty acids are then placed in two volumes of isopropyl alcohol and allowed to stand at −20° C. for one week. This precipitates out the saturated fatty acids. The product is then filtered on a Buchner filter. The filtrate is a solution of pure ricinoleate acid in isopropyl alcohol. One gram of this solution is equivalent to 7.27 cc. sulphuric acid (normal). One gram of triethanolamine requires, then, 7.27/1.176 grams of ricinoleic acid solution to be exactly neutralized. In this preparation 50 grams of triethanolamine were heated on a steam bath for two hours with 309.1 grams of ricinoleic acid solution and the isopropyl alcohol evaporated. The triethanolamine ricinoleate prepared was 99.29% pure.

PREPARATION OF EPHEDRINE RICINOLEATE.

Ricinoleic acid is prepared as in the first part of the triethanolamine ricinoleate procedure. To excess of this the alkaloid is added, heated on a steam bath and then the isopropyl alcohol evaporated off.

DENSITIES AND SURFACE TENSIONS OF AQUEOUS SOLUTIONS OF THE RICINOLEATES.

	Density	Surface tension
Sodium ricinoleate (3%)	1.0010	63.9 dynes per cm.
Triethanolamine ricinoleate (3%)	.9980	57.0 " " "
Ephedrine ricinoleate (2%)		40.0 " " "
Methyl ricinoleate (3%)	.9980	67.0 " " "
Water (distilled)	1.0000	73.0 " " "

Density and surface tension determinations were made under standard conditions, the latter with the use of a Traube stalagmometer.

DETOXIFICATION OF S. AUREUS.

In this test 1 cc. portions of a suspension of S. aureus with the different ricinoleates or castor oil soap were injected subcutaneously into white rats (Wistar) which had been shaved at the site of injection, the back. The culture was obtained from a human case and had been grown on nutrient broth for five days with 24 hour transplants. Each cc. of the injected ricinoleate-aureus mixture was found to contain 5,000,000 bacteria. Dilutions were made of the ricinoleate so that the final solution contains an even percentage. Before dilution the ricinoleates were autoclaved to sterilize and then diluted with sterile distilled water. The material when injected was at 35° C. Separate tests were made to determine the extent of antisepsis from the ricinoleate. Though there was a slight drop in the count it was not significant.

TABLE VI.

Solution injected	Animal No.	Tissue reaction	Pus formation
S. aureus only 5,000,000 per cc.	LA1		abscess
	LA2		abscess
	LA3		abscess
	LA4		abscess
	210		abscess
	211		abscess
	212		abscess
	213		abscess
	214		abscess
	215		abscess
8% Castor oil soap	LB1	slight	
	LB2	vigorous	
	LB3	vigorous	
	LB4	slight	
5% Sodium ricinoleate	LB1	0	
	LB2	0	
	LB3	very slight	
	LB4	0	
3% Sodium ricinoleate	LB5	0	
	LB6	0	
	LB7	0	
	LB8	0	
3% Triethanolamine ricinoleate	222	0	
	223	0	
	224	0	
	225	0	
3% Methyl ricinoleate	216	0	
	217	very slight	
	218	vigorous	
	219	0	
	220	0	
	221	slight	
8% Castor oil soap and aureus	LC5	vigorous	abscess
	LC6	vigorous	0
	LC7	vigorous	abscess
	LC8	vigorous	0

TABLE VI—cont.

Solution injected	Animal No.	Tissue reaction	Pus formation
5% Sodium ricinoleate and aureus	LD1	sores	0
	LD2	sores	0
	LD3	0	0
	LD4	0	0
3% Sodium ricinoleate and aureus	LD5	0	0
	LD6	0	0
	LD7	0	0
	LD8	0	0
	246	0	0
	247	0	0
	248	0	0
	249	0	0
	250	0	0
	251	0	0
3% Triethanolamine ricinoleate and aureus	234	0	0
	235	0	0
	236	0	0
	237	0	slight
	238	0	0
	239	0	0
3% Methyl ricinoleate and aureus	240	slight	abscess
	241	slight	abscess
	242	0	0
	243	slight	0
	244	slight	0
	245	vigorous	abscess

Discussion. The culture used was demonstrated to be virulent enough to cause abscesses. The concentration of castor oil soap used was too high to prevent irritation but too low to detoxify. Both castor oil soap and methyl ricinoleate are poor detoxifiers. Sodium and triethanolamine ricinoleates seem to be of equal detoxifying strength. This would indicate that the ricinoleate part of the molecule is a factor in detoxification (note that all solutions detoxified) but is not the only factor (note the difference in detoxifying power). Detoxification is certainly not due to surface tension alone. Staphylococcus aureus can be detoxified by ricinoleate compounds to an extent sufficient to prevent pus formation.

DETOXIFICATION OF THE TUBERCLE BACILLUS.

The tubercle bacillus produces no true toxins but the bodies of the bacteria themselves cause necrosis of the tissue with subsequent caseation and abscess. Soon after the introduction of the bacilli into the tissues the surrounding tissues show irritation. The connective tissue cells swell and undergo mitotic division and are distinguishable by their large size and pale nuclei. Young guinea pigs are very susceptible to tubercle bacillus injections. When inoculated with a minute quantity of living bacilli they succumb to the disease. Intraperitoneal injection most rapidly produces infection. Death follows the injection of larger doses in ten to twenty days. The omentum clumps of sausage masses contain many bacilli. Serous fluid is to be found in both pleural sacs, the spleen enlarges and becomes covered with tubercles as is also the case with the liver and intestine. The tissues thicken around the site of inoculation. Neighboring lymph nodes become swollen. The mediastinal glands fill with cheesy material rich in organisms.

In the following test, the guinea pigs were individually caged and fed a diet of carrots, cabbage, bran and oats. Each animal weighed about 15 ounces at the commencement of the test. The tuberculosis culture used was that supplied by the Albany Medical College, it having been found the most virulent of a number of cultures tested. Inoculations were made in the animal's groin. The solutions injected were prepared in a manner identical to that in the previous test.

Discussion. Sodium ricinoleate protected two out of eight of the animals, triethanolamine ricinoleate and ephedrine ricinoleate each protected three out of eight animals. This might indicate that sodium ricinoleate is the poorest detoxifier of tubercle bacillus.

It is evident that the concentrations used are too small to completely protect the animals. This test does show, however, that the tubercle bacillus is "detoxified" to some extent and that detoxification should be investigated clinically with the expectancy of success comparable with that of Kline (47) and Morris and Dorst (81) (81a).

These results show detoxification is not due to any one single cation. Detoxification seems to be due to several or all of the following factors: (1) surface tension, (2) solubility of the detoxifier, (3) length of time of contact of detoxifier with toxin, (4) temperature of the

TABLE VII.

Solution injected	Animal no.	Effect on intestine	Effect on spleen	Effect on mediastinals	Lymph glands
3% sodium ricinoleate	505	none	none	none	none
	506	none	none	none	none
3% Triethanolamine ricinoleate	503	none	none	none	none
	504	none	none	none	none
2% Ephedrine ricinoleate	507	none	none	none	none
	508	none	none	none	none
Tubercle bacillus 3,000,000 per cc.	500	ulcered	ulcered	cheesy	enlarged
	501	ulcered	spotted	cheesy	enlarged
	502 (died)	ulcered	spotted	cheesy	enlarged
3% Sodium ricinoleate & T. B.	470	normal	spotted	enlarged	enlarged
	471	normal	spotted	cheesy	enlarged
	472	normal	normal	normal	normal
	473	normal	spotted	enlarged	cheesy
	474	ulcered	spotted	cheesy	cheesy
	475	normal	normal	normal	normal
	476	normal	enlarged	normal	cheesy
	477	normal	enlarged	normal	cheesy
3% Triethanolamine ric. & T. B.	480	normal	cheesy	enlarged	cheesy
	481	normal	normal	normal	normal
	482	normal	normal	normal	normal
	483	normal	spotted	enlarged	cheesy
	484	normal	normal	normal	normal
	485	normal	spotted	enlarged	cheesy
	486	normal	spotted	cheesy	cheesy
	487	normal	spotted	normal	cheesy
2% Ephedrine ricinoleate and T. B.	490	normal	normal	normal	normal
	491	normal	cheesy	cheesy	enlarged
	492	normal	enlarged	normal	enlarged
	493	normal	normal	normal	normal
	494	normal	spotted	normal	cheesy
	495	normal	spotted	cheesy	cheesy
	496	normal	normal	normal	normal
	497	normal	spotted	enlarged	cheesy

mixture, (5) chemistry of the anion or cation (i. e., number of carbon atoms, unsaturated linkages, etc., (6) extent of ionization of the detoxifier, (7) the nature of the organism or toxin being acted upon and (8) adsorption. The presence of alkali earth metals interferes with detoxification.

SUMMARY AND CONCLUSIONS

The theoretical and historical aspects of bacterial detoxification have been summarized.

The authors adhere to the view that detoxification is due not alone to surface tension depressant action of the detoxifier, nor solely to any particular anion or cation, but to the configuration of the molecule as a whole.

S. aureus in a suspension of 5,000,000 organisms per cc. of 3% sodium or tritehanolamine ricinoleate is detoxified sufficiently to prevent abscess formation when injected subcutaneously into albino rats. Methyl ricinoleate was not effective.

Tubercle bacilli in a suspension of 3,000,000 organisms per cc. of 3% sodium, 3% triethanolamine, or 2% ephedrine ricinoleate gave evidence of "detoxification" by the ricinoleates and the protection of some of the animals. The cases are too few to warrant a statistical expression of results.

The protective action against the bacteria tested was in no case due to germicidal action of the detoxifiers.

Further study of bacterial detoxification from a theoretical standpoint, and the development of therapeutic measures founded upon the present state of knowledge concerning this phenomenon are definitely worth while.

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NOTE ON STELLAR PERTURBATIONS OF NEARLY PARABOLIC ORBITS.

BY E. ÖPIK.

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BY E. ÖPIK.

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THE following considerations refer to the question of statistical stability of cometary or meteoric orbits in the solar system, and partly also to the stability of wide double stars as far as perturbations by passing stars are concerned. The probability of ejection of a member out of the solar system, and the probability of getting out of the region of observability of the terrestrial observer will be discussed. Certain simplifications have been introduced, and on this account the estimates are approximate, being supposed to give the order of magnitude only.

An object (which we will call the meteor for the sake of simplicity) of very elongated orbit will spend most of its time near aphelion, and the stellar perturbations are there the strongest and the most frequent. For the order of magnitude it is therefore legitimate to consider only perturbations caused when the meteor is in aphelion. Assuming for the calculation of the frequency of perturbations that the meteor remains *all* the time in aphelion, we slightly overestimate the effect of perturbations, without much affecting the order of magnitude; on the other hand, the probable density of stars in space may be underestimated, because the number of white dwarfs, which may be relatively massive, is not known and is not taken into account here. This underestimate may balance the overestimate mentioned above.

The following considerations are valid when the effective duration of approach of star and meteor, defined later on, is small compared with the period of revolution of the meteor. This condition is easily fulfilled in all cases of practical interest, because of the high average velocity of the stars compared with the aphelion velocity of the meteor. Neglecting very close approaches of stars to the sun, the path of the star, on account of its high velocity, may be assumed to be rectilinear, with respect to the sun as well as to the meteor; the latter, during the short time of passage of the star, may be considered as practically at rest.

By reason of symmetry, after the passage is over, all the gravitational impulses parallel to the path of the star will be cancelled, and the resultant impulse contributed by the star to the meteor will be directed perpendicularly to the rectilinear path, toward the point of closest approach. Let us denote by ξ the distance of closest approach of star to meteor, expressed in astronomical units; by v the velocity of the star relative to the sun, expressed in units of the orbital velocity of the earth; by α the angle between the line joining star and meteor, and the direction of closest approach. Choosing as unit of time $\frac{1}{2\pi}$ years, and assuming the mass of the star equal to the sun's mass (which may be near the average), the gravitational acceleration exerted by the star upon the meteor will be $\frac{\cos^2 \alpha}{\xi^2}$; the component of this force parallel to the line of closest approach is

$$F_\alpha = \frac{\cos^3 \alpha}{\xi^2},$$

and the total impulse contributed to the meteor along this line is

$$\Delta u = 2 \int_0^\infty F_\alpha dt,$$

the time t being reckoned from the moment of closest approach. On the other hand,

$$v = \frac{dx}{dt} \quad \text{where} \quad x = \xi tg\alpha,$$

whence

$$dt = \frac{\xi d\alpha}{v \cos^2 \alpha}.$$

This gives

$$\Delta u = \frac{2}{\xi v} \int_0^{\pi/2} \cos \alpha \, d\alpha$$

or

$$\Delta u = \frac{2}{\xi v}. \tag{1}$$

The geometrical sum of the original velocity of the meteor and of Δu gives the perturbed velocity; the latter determines all the elements of the changed orbit.

The effective duration of approach, t_0, may be defined as the time necessary to produce the whole effect, Δu, by the star placed at the point of closest approach; thus

$$\frac{1}{\xi^2} t_0 = \Delta u,$$

whence

$$t_0 = \frac{2\xi}{v}. \tag{2}$$

One of the conditions of validity of (1) is that the period of revolution of the meteor, P, must be great as compared with t_0. For a very elongated orbit we may put

$$P = 2\pi \cdot \left(\frac{R}{2}\right)^{3/2}$$

in the adopted units of time, whence

$$\frac{P}{t_0} = \frac{\pi v}{2\sqrt{2}} \frac{R^{3/2}}{\xi}. \tag{3}$$

This ratio must be great—at least ten or more. The other condition for the practical validity of our considerations is that the eccentricity be great, that is, greater than 0.9.

For the average velocity of a star relative to the sun, we assume $v = 1 = 30$ km/sec., which is approximately the observed average. In this special case our formulae are

$$\Delta u = \frac{2}{\xi} \tag{1'}$$

and

$$\frac{P}{t_0} = 1,11 \frac{R^{3/2}}{\xi}. \tag{3'}$$

For a highly eccentric orbit the aphelion velocity is very small compared with the velocity of escape at aphelion distance; when considering the probability of the ejection of the meteor out of the solar system, we may neglect altogether the aphelion velocity, assuming it equal to zero. The condition for ejection is, then, that the relative perturbation, or the geometrical difference of the perturbation vectors

Δu_m and Δu_s for meteor and sun, be equal or greater than the velocity of escape, given by

$$v_e = \sqrt{\frac{2}{R}} \tag{4}$$

in the units chosen here. The condition of ejection is

$$\Delta u \geq v_e, \tag{5}$$

Δu being the absolute value of the relative perturbation vector for meteor and sun. It is easy to show that when R is great, it is necessary for ejection that the inequality

$$\frac{\xi_s}{R} \text{ or } \frac{\xi_m}{R} \leq \sqrt{\frac{2}{R}} \tag{6}$$

be fulfilled; this follows when for a given value of ξ_m we consider the case of strongest effect, when the star passes through the line joining m and s between them, and when $\xi_s = R - \xi_m$. Here it is assumed that $\xi_m < \xi_s$. When $\xi_m > \xi_s$, we obtain the same result by interchanging the places of meteor and sun. When $\xi_m > R$ ($\xi_s > R$ also), the absolute sum of the two vectors

$$|\Delta u_m| + |\Delta u_s| = \frac{2}{\xi_m} + \frac{2}{\xi_s} < \frac{4}{R},$$

is smaller than v_e for $R > 8$; thus ejection cannot take place. From (6) we infer that for R exceeding about 100 astronomical units the ratio $\frac{\xi_m}{R}$ is small, whence ξ_m is small compared with R, and thus small relative to ξ_s also, and in (5) we may neglect Δu_s altogether. Ejection may happen only when the passage is so close to one of the two bodies that the relative perturbation of the other may be neglected. This gives the following simple form for the condition of ejection:

$$\Delta u_m = \frac{2}{\xi_m} \geq \sqrt{\frac{2}{R}}, \quad \text{or} \quad \xi_m \leq \sqrt{2R};$$

the effective radius of approach for ejection is thus

$$\xi_e = \sqrt{2R}. \tag{7}$$

The validity ratio (3′) in this case is

$$\frac{P}{t_0} \geq 1,11\frac{R^{3/2}}{\sqrt{2R}} = 0,78R,$$

which is large for all possible values of R that may be of practical interest in our problem.

Assuming an average of twenty-five stars of mass 1 per 1000 cubic parsecs, or 3.10^{-18} per cubic astronomical unit, and taking into account that the total target area for ejection is twice $\pi\xi_e^2$ (because a close approach may happen with respect to the sun as well as with respect to the meteor, and because these two target areas do not overlap when $\xi_e < \frac{1}{2}R$, or when $R > 8$), we find for the probability of ejection within Δt years the value

$$3.10^{-18} \cdot 2\pi\xi_e^2 \cdot 2\pi\Delta t; \tag{8}$$

the last factor, 2π, is the number of astronomical units traveled by the star in one year. After substituting (7), the differential expression (8) leads to the following finite expression for the probable fraction, p_s, of meteors not ejected, having an average aphelion distance R a. u., after an interval of time equal to t years:

$$p_s = e^{-2,4.10^{-16}Rt}. \tag{9}$$

This formula provides us with a means of estimating the maximum possible limit of the solar system. Substituting for t a value near 3.10^9, the probable age of the solar system, we find for the probable fraction of meteors retained since the origin

$$p_s = e^{-10^{-6}R}.$$

The effective limit, corresponding to a fraction $1/e$ retained out of the original number may be put thus at $R_e = 10^6$ a. u. = 5 parsecs. The effective volume of the solar system is about 600 cubic parsecs, or about fifteen times greater than the volume per average solar mass in space.

This result holds for the assumption of a constant density of stars in space; the average density of stars along the path traveled by the sun since the origin of the solar system might have been different from the present density; there may be a variation with time also. That is, if the galactic system has been expanding appreciably, the average density of stars in space must have been greater in the past than it is now, and the effective radius, R_e, may be assumed to have been somewhat smaller. Taking into account this circumstance, we may estimate the possible limit of the solar system at from 2.10^5 to 10^6 astronomical units. The period of revolution for these extreme distances will then be equal to from 3.10^7 to 4.10^8 years; thus, even the most remote present members of the solar system must have had time to complete several revolutions since the time of their origin.

The quantitative conditions of ejection, derived here for nearly parabolic orbits, will not change very much, as far as the order of magnitude is concerned, for less eccentric orbits nor even for nearly circular ones. Thus, the effective limits set here may be regarded as of general validity.

For a star of mass μ times the solar mass, the exponent in formula (9) is to be divided by μ; for the same age, t, the extreme limit of the system R_e is to be multiplied by μ. In this form our considerations may be applied to the question of stability of wide double stars—the so-called pairs with common proper motion. For $\mu = 1$ we find the time of retaining the fraction $1/e$ of companions at an aphelion distance R as follows:

R, a. u.	10^6	10^5	10^4	10^3
t, years	3.10^9	3.10^{10}	3.10^{11}	3.10^{12}

For other values of the mass these intervals of time are to be multiplied by μ. It may be shown that the average velocity of ejection is such that within the time t, for $R < 10^6$, the separation of the two former members of the system becomes much greater than fifty parsecs or 10^7 a. u. so that in most cases an ejection results in practical separation; the former components will not be counted any more as members of the same system.

In the preceding it was assumed that the orbit, or at least the aphelion distance, remains substantially unchanged until a close approach throws the meteor out. This assumption is not exactly true. Less powerful, but more numerous perturbations will have the effect of increasing the major axis and the average aphelion distance, thus making the final ejection more easy. The rate of ejections is therefore somewhat underestimated above; the order of magnitude, however, is not affected. As to the effective limit of the solar system, the estimate need not be changed, because the shifting outward of R will increase the limit and counterbalance the effect of a greater number of ejections. The conclusion is that within intervals comparable with the age of the solar system, meteors or comets remain practically well connected with the sun even at distances as great as the average separation between two stars in space. However, from the standpoint of observability their detection becomes impossible when the perihelion distance exceeds 1 for meteors, and is a small multiple of 1 for comets. The perturbations of the perihelion distance are thus the most important ones from the practical standpoint. The question of persistence of gaseous constituents in meteors, in connection with the appearance of comet tails and with the determination of the age of meteorites from the helium content, also depends chiefly upon the perihelion distance, which determines the maximum temperature attained by the meteor during its revolution around the sun.

The law of conservation of angular velocity gives for a nearly parabolic orbit, when the perihelion velocity is assumed equal to the velocity of escape, $\sqrt{\frac{2}{q}}$,

$$q = \frac{R^2 v_a^2}{2}; \tag{10}$$

here q is the perihelion distance, R the aphelion distance, v_a the aphelion velocity (units same as before). Thus q is very sensitive to changes in v_a, being proportional to the square of the latter. The calculation of the perturbational vector in the present case is more complicated than in the former, because we cannot neglect one of the two impulses, Δu_m and Δu_s. Without going into detail, we may state that the absolute value of the relative impulse meteor *minus* sun is given by

$$\Delta u = \frac{2R \cdot |\sin\gamma|}{\xi_m \sqrt{\xi_m^2 + R^2 \sin^2\gamma + 2R\xi_m(\cos\alpha\cos A + \cos\beta\sin A)}}; \tag{11}$$

here α, β, γ are the angles formed by the vector R with the x, y, z axes of Cartesian coördinates; the z axis is assumed to coincide with the path of the perturbing star, and the meteor is placed in the xy plane; A is the azimuthal angle of the line of closest approach to meteor, recorded from the x axis in the xy plane.[1] The independent

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[1] The derivation of (11) is simple in Cartesian coördinates, where

$$\Delta u_x = \frac{2x_m}{\xi_m^2} - \frac{2x_s}{\xi_s^2};\ \Delta u_y = \frac{2y_m}{\xi_m^2} - \frac{2y_s}{\xi_s^2};\ \Delta u_z = 0;$$

and

$$\Delta u = \sqrt{\Delta u_x^2 + \Delta u_y^2 + \Delta u_z^2}.$$

variables are ξ_m, R, and the direction of R defined by α, β, and γ under the restriction $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$. From (11) it is possible to find the frequency of different values of Δu. The exact procedure, however, is too complicated; in this preliminary discussion we will substitute in (11), for the trigonometric functions, their average values calculated on the assumption of a random orientation of the vector R. In this way we obtain a certain approximate expression for the average value $\overline{\Delta u}$ of the vector Δu. For the function (11) this procedure means underestimating $\overline{\Delta u}$. Further, let Δv_a be the component of Δu along v_a, so that

$$\Delta v_a = \Delta u \cos\varphi, \tag{12}$$

where φ is the angle between Δu and v_a. Assuming a random frequency for this angle,[1] we have $|\overline{\cos\varphi}| = \frac{1}{2}$. Also, in (11), $|\overline{\sin\gamma}| = \frac{\pi}{4}$; $\overline{\sin^2\gamma} = \frac{2}{3}$; $\overline{\cos\alpha} = \overline{\cos\beta} = 0$. This gives for the order of magnitude of the average perturbation in v_a the expression

$$\overline{\Delta v_a} = \pm\frac{\pi}{4}\,\frac{R}{\xi_m\sqrt{\xi_m^2 + \frac{2}{3}R^2}}. \tag{13}$$

From (10) we find the expression for the changed perihelion distance

$$q_1 = \frac{R^2\,(v_a + \Delta v_a)^2}{2} = \frac{R^2 v_a^2}{2} + \frac{R^2}{2}\,(\Delta v_a)^2 + R^2 v_a \Delta v_a$$

and with the aid of (10) again

$$q_1 - q = \frac{R^2}{2}\,(\Delta v_a)^2 + R\sqrt{2q}\,\Delta v_a. \tag{14}$$

Substituting for Δv_a its average value from (13), retaining the ± sign, and putting $\varepsilon = \xi_m/R$, we get the average perturbation of q as follows:

$$\bar{q}_1 - q = \frac{\pi^2}{32}\cdot\frac{1}{\varepsilon^2(\varepsilon^2 + \frac{2}{3})} \pm \frac{\pi\sqrt{2q}}{4}\cdot\frac{1}{\varepsilon\sqrt{\varepsilon^2 + \frac{2}{3}}}. \tag{15}$$

The first member is always positive and gives the *average systematical increase* of q as the result of stellar perturbations; the second member is symmetrically negative and positive and cancels out in the mean; this member, being proportional to Δr_a, which was assumed to be the projection of a random vector Δu on r_a, may be used to estimate the dispersion of $\Delta q = q_1 - q$ around its average value given by the first member of the formula. Neglecting the dispersion in Δu, we get a minimum spread of Δq, according to the frequency function

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[1] This assumption is not exactly fulfilled, because φ depends upon α, β, γ: the vector v_a is at right angles to the vector R.

$$\sin\varphi\, d\varphi = dt, \tag{16}$$

where

$$t = \frac{\Delta q}{2\Delta q_0} = \cos\varphi$$

and Δq_0 is the absolute value of the second member in (15).

Now, the main spread in Δu, according to (11), depends upon the factor $\sin\gamma$, for given ξ and R, or given $\varepsilon = \frac{\xi}{R}$, except when $\varepsilon = 1$ which is a peculiar case. To allow for this spread, we have to put, instead of t,

$$t_1 = \frac{t}{\sin\gamma} \times \text{constant};$$

the frequency function of γ is $\sin\gamma$. The frequency-function influenced by the spread in Δu will be

$$\int_{\gamma=0}^{\gamma=\pi/2} dt_1 \times \text{frequency-function of } \gamma$$

$$= \int_0^{\pi/2} \frac{dt}{\sin\gamma}\cdot\sin\gamma\, d\gamma \times \text{constant} = dt \times \text{constant}.$$

Thus we see, that the peculiarity of the spread in Δu is such that it does not much influence the spread in t which is determined by (16). According to this formula, all values of Δq from $-2\Delta q_0$ to $+2\Delta q_0$ are equally probable.

Equation (3′), upon which the validity of our formulae depends, may be represented as

$$\frac{P}{t_0} = 1,11\frac{\sqrt{R}}{\varepsilon}.$$

For practically important values of ε it may be shown that $\frac{P}{t_0}$ is a great number for R exceeding 1000 a. u. The extreme limit of a rough applicability of our formulae may be estimated at $R > 300$ a. u.

The target area for an approach at a distance $\xi = \varepsilon R$ may be put equal to

$$\sigma = F \cdot \pi\xi^2 = F\pi\varepsilon^2 R^2. \tag{17}$$

The factor F is equal to 2 for $\varepsilon \leq 0.5$ and decreases slowly for greater values of ε toward 1; this factor depends upon the size of the overlapping fraction of the areas of two circles, each of radius ξ, centered at sun and meteor, the distance between the centers being thus R. The formula for F is

$$F = 2 - \frac{2\alpha}{\pi} + \frac{\sin 2\alpha}{\pi},$$

where

$$\cos \alpha = \frac{1}{2\varepsilon}.$$

The probable number of passages closer than $\xi = \varepsilon R$ within the time t years is given in the present case by

$$N_\epsilon = 3.10^{-18} \cdot F\pi\varepsilon^2 R^2 \cdot 2\pi t = 0,60.10^{-16} F\varepsilon^2 R^2 t. \tag{8'}$$

Table 1 gives the results of a sample computation for $q = 1$.

In this table $\varepsilon = \frac{\xi}{R}$, where ξ is the distance of closest approach either to meteor, or to sun, and R is the aphelion distance; q_1 is the changed perihelion distance; the average increase, $q_1 - q$, does not depend upon q, the initial perihelion distance, unless q is not small as compared with R. The effective limits of q may be somewhat surpassed in the actual case, but for practical purposes it is safe to assume that the maximum and minimum values of q_1, given in the table, symmetrical with respect to "q_1 average," represent the extreme limits of variation of q, and that the frequency function of q_1 is constant with respect to q_1, that is, that equidistant intervals of q_1 are equally probable, to their extreme limits. The negative values of "q_1 minimum" are introduced as a matter of convention, to indicate that the possible values include $q_1 = 0$. Thus, from the table we infer that for $\varepsilon = 0.125$ and less, the changed perihelion distance is always greater than $q = 1$. For $\varepsilon = 0.16$, only the fraction $\frac{1 - 0.9}{36.1 - 0.9}$ or 0.003 will have $|q_1|$ less than 1; for $\varepsilon = 0.25$ the same fraction is $\frac{2.0}{20.8}$ or ten per cent, and the

TABLE 1.

PERTURBATIONS OF THE PERIHELION DISTANCE.

$q = 1$

ε	0.1	0.125	0.16	0.20	0.25	0.32	0.40	0.50	0.64
F_ε^2	0.0200	0.0312	0.0512	0.0800	0.125	0.205	0.320	0.500	0.770
q_1 effective limit max.	73.6	51.4	36.1	25.1	18.2	12.8	9.44	6.97	5.06
q_1 effective limit min.	19.6	8.6	0.9	−1.3	−2.6	−3.0	−2.76	−2.27	−1.66
q_1, average	46.6	30.0	18.5	11.9	7.8	4.92	3.34	2.35	1.70
R, for $N_\varepsilon = 1$ and $t = 3.10^9$ years	17000	13500	10500	8300	6700	5200	4100	3300	2700
N_ε for $R = 2000$ and $t = 3.10^9$ y.	0.014	0.022	0.036	0.058	0.09	0.15	0.24	0.37	0.55
ε	0.80	1.0	1.28	1.6	2.0	2.56	3.2	4.0	5.12
F_ε^2	1.11	1.61	2.43	3.56	5.28	8.06	12.3	18.8	29.3
q_1 effective limit max.	3.81	2.91	2.22	1.82	1.52	1.33	1.21	1.13	1.09
q_1 effective limit min.	−1.07	−0.53	−0.06	0.26	0.52	0.69	0.79	0.87	0.91
q_1, average	1.37	1.19	1.081	1.038	1.016	1.0065	1.0028	1.0011	1.0005
R, for $N_\varepsilon = 1$ and $t = 3.10^9$ years	2240	1860	1510	1250	1020	830	670	540	430
N_ε for $R = 2000$ and $t = 3.10^9$ y.	0.80	1.15	1.76	2.56	3.84	5.86	8.9	13.8	21.7

fraction of perturbed orbits which will approach the sun to a distance closer than $|q_1| = 0.25$ a. u. is $\frac{0.5}{20.8} = 2.5$ per cent. From ε somewhat less than 0.20 to about 1.3, a certain fraction of the perturbed objects may hit the surface of the sun, when $|q_1| < 0.0045$; for $\varepsilon = 1.28$ this fraction is $\frac{0.0045 \times 2}{2.28}$ or one in 250. The 6th and 7th lines of the table refer to data connected with N_ε as defined by (8′); the probable fraction of objects which in the indicated interval of time have not undergone an approach, defined by the given value of ε, or closer, is

$$p_\varepsilon = e^{-N_\varepsilon}. \tag{9'}$$

Thus, from the table we infer that at $R = 10500$ the fraction $1/e$ of all objects has not suffered from approaches closer than $\varepsilon = 0.16$ or 1600 a. u. within 3000 million years; or that at $R = 2000$ a. u. the fraction $e^{-2,66}$ or about 8 per cent has not suffered changes from approaches closer than $\varepsilon = 1.6$ or 3200 a. u. The table, though constructed for $q = 1$, may be easily adapted to any other value of q, when q remains small compared with R; in agreement with formula (15), we must only add $q - 1$ to "q_1 average," and change the range of the effective limits of q_1 in the ratio $\sqrt{q}$. All the other data of the table remain unchanged. Any changes in N_ε, R or t in the last two lines of the table are to be made in agreement with formula (8′).

The most remarkable fact revealed by Table I is the tendency of the perihelion distance to increase continually. After a sufficient interval of time the perihelia will shift out of the sphere with radius $q = 1$, and even out of the known planetary limits of the solar system. The systematic increase in q, depending upon the probability of an approach, increases with the aphelion distance. Thus, for R sufficiently great, we may expect that most of the meteors, or comets, have already gone out of the region of observability. This limit of R may be estimated from the table at 10000 to 20000 a.u., assuming an age of 3.10^9 years, and would correspond to periods of revolution from 400,000 to one million years. It is not very probable that many of the observable meteors have periods of revolution greater than these figures, unless the total number of such meteors in the solar system is very great. In the latter case, because the spread in q_1 increases as $\sqrt{q}$, a small fraction of meteors possessing a large q may be changed by perturbations into those with a small value of q_1 and such objects may thus enter observable regions of the solar system. A state of statistical equilibrium will be installed when the number with small q changed into big q_1 will be balanced by the number with big q changed into small q_1. Because of the said systematical trend of the perturbations in q, the equilibrium will take place when the number with great q will be greater than the number with small q. It is easy to see that this means also a certain increase of concentration of meteoric matter in the nonobservable regions of the solar system, although the concentration is only a relative one, being counter-balanced by dilution in a greater volume of space. If, however, among our observable objects there is a sensible proportion of meteors with R exceeding 10000 a. u., it may be only the result of a very peculiar distribution of meteoric matter at the outskirts of the solar system—a kind of a meteoric cloud, or shell, of a density much higher than may be anticipated from the observed frequency of solar meteors caught on the surface of the terrestrial atmosphere. Unless this is the case, we may expect that the majority of solar meteors we observe comes from distances less than 10000 a. u.

This supposition means that an average solar meteor must have completed at least many thousand revolutions during its life time, being thus exposed for a total interval of a thousand years or more to comparatively high temperatures during perihelion passages. This deduction may have some bearing on the determination of the age of meteorites from their helium content, and upon the gradual loss of the more volatile constituents of the meteors. The heating effect of the sun is increased through the character both of the systematic and accidental variation of q. A meteor met by the earth must have q less than 1, and because of the systematic increase of q with time the probability is that in the past history of the meteor the average q was still smaller. Furthermore the variation of q is such that small changes of q occur much more often, and are more symmetrically distributed around the original value, than big perturbations. The result is a continuous unsteadiness of the orbit, a certain property of "mixing," so that the probability is considerable that the meteor, before being caught by the earth, or before "jumping out" as the result of a big perturbation, will undergo some very close approaches to the sun.

The stability of the orbit increases as the aphelion distance decreases. At $R = 2000$ a. u., for a period of revolution of 30000 years, the perturbations during $t = 3.10^9$ years are sensible, but not so revolutionary. Below 2000 a. u., the majority of the orbits may be regarded as statistically stable during the adopted interval of time.

The perturbations of the inclination have much in common with the perturbations in perihelion distance except for the systematical character. When the perturbations are small, the *average change* of inclination is given by $\overline{tg\Delta i} = \frac{q_1 - q}{2q}$; for stronger perturbations the conditions are more complicated. In the following table is given the average expected change in inclination for $t = 3.10^9$ years.

R	100	250	500	1000	1500	2000
$\overline{\Delta i}$	± 5.°5	± 14°	± 28°	± 55°	± 82°	± 110°

For other intervals of time $\overline{\Delta i}$ is proportional to $\sqrt{t}$. An average change $\overline{\Delta i} = \pm 90°$ already means a close approach to uniform distribution. According to statistics by Jantzen, 395 comets (1909) showed a more or less random distribution of the inclinations, with a slight preponderance of direct motions over retrograde ones; with an age of from 10^9 to 3.10^9 years, this would correspond to an average aphelion distance of 1500–2000 a. u., or a period of revolution of 20000–30000 years. For greater aphelion distances the distribution of inclinations should be practically uniform, being smoothed out by perturbations. Meteors seem to show a much stronger preference for direct motion,[1] which could be preserved in the course of time only at a smaller effective aphelion distance, about 1000 a. u., corresponding to a period of about 10000 years.

CONCLUSIONS.

1. The possible limits of the solar system, as set by stellar perturbations during the time elapsed since the origin of the system, may be estimated at 10^6 astronomical units; whether the actual limits reach so far, depends upon the presence of bodies at these distances.

2. Meteors, or comets of an aphelion distance exceeding 10000 astronomical units, are not very likely to occur among the observable objects, because of the rapid increase of the average perihelion distance due to stellar perturbations.

3. From the scarce data relating to the observed nonuniformity of the orbital inclinations, the effective aphelion distance of observable comets is estimated at 1500–2000 astronomical units, and of observable sporadic meteors at 1000 astronomical units.

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[1] E. Öpik, H. C. 355, p. 9, 1930.

4. At the aphelion distance of 1000 a. u., the orbit of a meteor during 3.10^9 years is comparatively stable; the average heating effect of the sun on an observable meteor corresponds in this case to from 100000 to 300000 approaches at the actual perihelion distance, or to about 30000 years of continuous heating to a temperature exceeding 0°C in vacuo, which may be sufficient for a stony meteorite to lose all its helium content. It is therefore probable that stony meteorites of solar origin yield very low ages according to the helium-radium method.

5. As the result of stellar perturbations, the number of meteors and comets of a given constant aphelion distance of 2000 a. u. or greater must increase with increasing perihelion distance, to a certain limit; the rate of relative increase in the number must be greater for greater aphelion distances. For objects with an aphelion distance below 1000 a. u., stellar perturbations could not change appreciably the original distribution of the orbital elements within 3.10^9 years.

The present paper gives only an approximate sketch of the situation. The approximate results are, however, quite sufficient to serve as bases for certain numerical estimates which appear to be useful and even necessary in the course of the systematic meteor research undertaken at Harvard Observatory under the initiative and organization of Dr. Shapley.

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FURTHER STUDIES ON THE FAUNA OF NORTH AMERICAN HOT SPRINGS.

By Charles T. Brues.

FURTHER STUDIES ON THE FAUNA OF NORTH AMERICAN HOT SPRINGS.[1]

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[1] From the Entomological Laboratory, Harvard University.

INTRODUCTION.

The material contained in the present paper represents the third contribution of the writer dealing with the fauna of thermal springs in the Western United States. His first paper, published in 1924 was based upon observations and collections made in the Yellowstone National Park during the summer of 1923. In 1927 he was able to spend several months in the adjoining states of Utah, New Mexico, Nevada and California making a field study of a number of scattered hot springs, and obtaining collections of the insects and other types of animal life that occur in these springs. The results of these investigations were published in 1928.

Again during the summer of 1930 the same general region was revisited, including Yellowstone Park and parts of Idaho as well as Nevada and California. Nearly four months were spent in the field where observations were made and collections obtained in considerably more than one hundred separate thermal springs. The writer is deeply indebted to Harvard University for a grant from the Milton Fund for Research which was given him to aid in the continuance of these investigations. They involved a very considerable amount of travel on account of the scattered locations of the springs and frequently isolated positions that they occupy with reference to ordinary lines of travel. The writer traveled in his own car with a trailer (a distance of something over 14,000 miles) carrying necessary camping equipment and supplies. In this way it was usually possible although not always easy to get at least within walking distance of most of the springs, which numbered nearly one hundred and fifty. In most of these material was collected, although some proved unsuitable, usually on account of having been too much "improved" and thus rendered unfit for biological work. A few that we had hoped to visit proved to be too far from passable roads and others in the Death Valley region in California were omitted as the summer season is not suitable for travel there on account of the excessive heat.

As may be seen by reference to the accompanying map (Fig. 1) an extensive area has now been covered in this way and together with the thermal springs visited previously, we now have material collected at 154 localities. Each of these is indicated on the map, with a number; these numbers were given in sequence as we visited the springs and are the ones referred to later in the text. Numbers 35 to 154 are dealt with in the present paper.

As before, determinations of the hydrogen ion concentration of the waters in which material was collected were made by the colorimetric method. A very compact kit made by the British Drug Houses, Ltd. of London was used. This contains a series of indicators, a supply of capillary tubes, pipette bulbs and small glass dishes, together with a series of standardized indicators in similar capillary tubes enclosed in light-tight envelopes to prevent fading. This, known as the B. D. H. Capillator proved very handy for use and highly satisfactory. The following indicators were used, with their ranges expressed in pH: bromphenol blue, 3.6–5.2; bromcresol purple, 5.2–6.8; phenol red, 6.0–8.4; thymol blue, 8.0–9.6. It is believed that the error cannot amount to more than 0.1 over the range between 7.2 and 9.6 which includes the great majority of the springs. Those above 9.6 and below 3.6 have not been differentiated, and quite a number fall into these classes.

The specific gravity of the water in every spring was also taken as a measure of its salinity. This was determined by a hydrometer reading to the fourth decimal place. By keeping the hydrometers scrupulously clean and taking the readings with careful reference to temperature it is believed that the specific gravity determinations are accurate to within 0.0004. All the determinations have been corrected to a uniform temperature of 15° Centigrade and thus give a general idea of the range of salinities to be found in the whole series of springs examined. On account of the great differences in salts present in the several springs the salinities are not biologically comparable as they are in the case of brackish waters which represent mainly dilutions of sea-water. As will be mentioned later the salinities fall mainly within a rather narrow range.

For recording the temperatures of water where collections were made thermometers reading to tenths of a degree Centigrade were used, previously calibrated to insure their accuracy.

The writer is indebted to several members of his family for assistance in collecting material and recording data. My wife spent much time in planning and arranging our itinerary and also made extensive collections of grasses associated with the hot springs. Determinations of the temperatures, specific gravity and hydrogen ion concentration of the waters were made in great part by my daughter, Alice M. Brues who also accompanied me during the entire trip. During a part of the time while my son Dr. Austin M. Brues and his wife were with us these data were taken and recorded by them. Without the interest and coöperation of these companions it would have been quite impossible to carry out such an extensive program in a region which presents so many difficulties to travel.

Much of the insect material obtained has been studied and identified by the writer but he is indebted to several other persons for assistance as mentioned in the more detailed account that follows. All of the beetles were determined by Mr. C. A. Frost who secured help from Mr. H. C. Fall and Mr. F. E. Winters. Mr. W. J. Clench and Mr. Arthur Loveridge, both of the Museum of Comparative Zoology, kindly determined the molluscs and amphibians respectively, while the identifications of the tadpoles were made by Dr. G. K. Noble of the American Museum of Natural History. The ostracod crustaceans were examined by C. H. Blake of the Massachusetts Institute of Technology and the fishes by Dr. Carl L. Hubbs of the University of Michigan. The series of Acarina were identified by Dr. Ruth Marshall of Rockford College. On account of the miscellaneous nature of the collections their identification has not been easy and the writer appreciates very keenly the inroads he has made upon the time of those who have aided him in this way.

Descriptive List of Hot Springs Studied.

These are listed and numbered serially in the order in which we visited them. In order to avoid any duplication of numbers or confusion in connection with my previous account (Brues '28), the present numbers begin with No. 35 as there are 34 springs described in the first paper. (See Fig. 1.)

No. 35. Thermopolis, Wyoming. Along the eastern bank of the Big Horn River close to the town of Thermopolis is a group of hot springs which have built up an extensive deposit of calcareous tufa that extends for a distance of about half a mile with a width of from 100 to 300 feet, overlooking the river. One very large spring at the southern end of the group furnishes water for a bath and sanitarium, but there is an extensive overflow from it and several smaller springs further down the river where our collecting was done. The springs are in a State Park as they were deeded many years ago to the State of Wyoming by the far-sighted old Indian Chief Washakie, with the provision that one-quarter of their flow should be set aside for the use of the public without cost. This has curbed to a great extent the exploitation of the springs by promoters of hot-water cures and sanatoria. The specific gravity of the water in the largest spring is 1.0056 and pH 7.9. As may be seen from the following analysis, there is a considerable amount of calcium carbonate and much less silica.

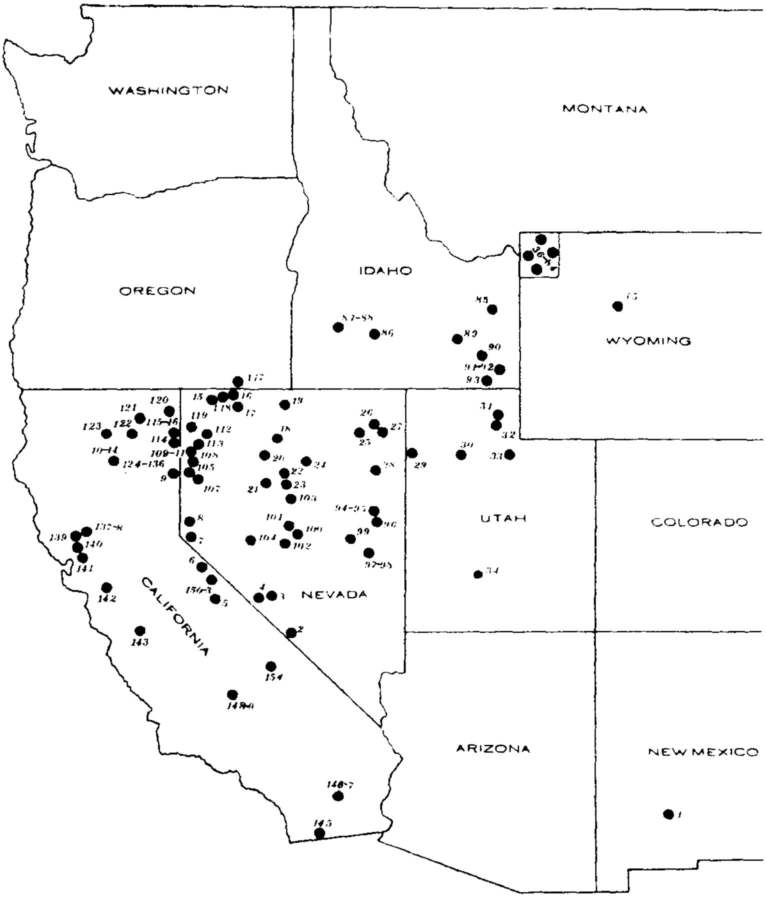
Potassium chloride	175.60
Sodium chloride	449.20
Sodium sulphate	449.20
Magnesium sulphate	259.10
Calcium sulphate	225.60
Calcium carbonate	693.70
Ferric oxide & alumina	3.90
Silica (SiO_2)	85.50
Total parts per million	2,226.00

No. 36. Washakie, Wyoming. This is located in the valley of the Little Wind River, about one and one-half miles west of Washakie. The basin of the spring is several hundred feet across, filled with water holding much fine material in suspension that causes a very noticeable bluish or yellowish opalescence when viewed from the shore. The pool is practically at the level of the surrounding semi-desert country and is surrounded by very little vegetation. It is used for a swimming pool by the Indians of the reservation in which it lies and is provided with a large bath house, but the shores have not otherwise been disturbed. The specific gravity of the water is 1.0052 and pH 7.8.

Nos. 37–42. West Thumb, Yellowstone National Park. These six springs are all close together, near the Ranger Station, but as they differ considerably both in the specific gravity and hydrogen ion concentration of the water, have been listed separately. No. 37 is just southwest of the pink mud geysers or "Paint Pots"; No. 38, just southeast of the Paint Pots, and the three following are successively to the northward of No. 38. The first three hold a good deal of clay in suspension, with little or no overflow, while the others are clear and flow eastward over and through the porous calcareous tufa which they are depositing. Following is the specific gravity and hydrogen ion concentration of the six springs.

	Specific Gravity	pH
No. 37	1.0056	8.9
No. 38	1.0025	6.8
No. 39	1.0039	6.3
No. 40	1.0058	7.3
No. 41	1.0045	8.4
No. 42	1.0046	7.1

No. 43. West Thumb, Yellowstone National Park. This is also near the shore of Yellowstone Lake, about three-quarters of a mile north of the previous group, its overflow running down a gentle slope to the edge of the lake. The specific gravity of the water is 1.0023 and its reaction quite strongly alkaline (pH 9.0).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IDAHO
OREGON
WYOMING
NEVADA
UTAH
COLORADO
CALIFORNIA
ARIZONA
NEW MEXICO
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109-11
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FIGURE 1. Outline map, showing geographical location of the hot springs in which material was collected. Each spring, or group of springs is indicated by a black spot.

No. 44. Lewis Lake, Yellowstone National Park. About twelve miles south of the West Thumb Station the road passes close to the edge of Lewis Lake and from this point, one half mile southwestward along the lake shore is an extensive flat swampy area fed by a hot spring, lying barely below the outlet to the lake. Here there are a number of pools and warm overflows with much vegetation in which we obtained a good deal of material. The specific gravity of the water is 1.0023 and its reaction only moderately alkaline (pH 7.8). There is little mineral deposit about this spring except a fine clayey, pale-colored mud.

No. 45. Norris Geyser Basin, Yellowstone National Park. This spring lies between the road and the Gibbon River opposite Roaring Mountain and not far north of the Norris Station. The water arises in a meadow area at a number of places and furnished good collecting at varying temperatures. The specific gravity of the water is 1.0040 and pH 7.9.

No. 46*a* and 46*b*. Bijah Spring, Yellowstone National Park. This locality is a short distance (1.6 miles) south of the previous one, where several springs emerge in a meadow within a distance of about 50 feet from one another. In spite of the proximity there is a difference in the specific gravity of two of these springs in which we obtained material so they have been listed separately (46a, 1.0039 and 46b, 1.0036) although the pH, 6.5, is the same in both. The temperature of these springs varies from 34.3° to 43.5°, and to judge from the absence of odor in the water the rather acid reaction is probably due to carbon dioxide.

No. 47. Indian Pond, Yellowstone National Park. Although this pond is evidently supplied in part by thermal water, its temperature at the time of our visit (July 27) was only 17.1° C. (62.8° Fahr.) along the shore and the specific gravity only 1.0001. It is really therefore practically an ordinary fresh-water pond. Some material was obtained here for comparison with that collected elsewhere in this vicinity.

Nos. 48 and 49. Beach Springs, Yellowstone National Park. The group of springs bearing this name is practically at the extreme northeastern corner of Yellowstone Lake, situated close to the lake shore. They are separated into two groups by a slight rise, the northernmost one (No. 48) containing water of a specific gravity of 1.0028 and pH of 6.5. The water is slightly muddy with a grayish clay border and a maximum temperature of about 34°. The water in the southern group of three more or less connected pools is less acid, with a pH of 7.2 and with a higher specific gravity (1.0051). The water is slightly warmer, ranging from 38.5° to 40°.

No. 50. Mary Bay, Yellowstone National Park. This spring is not far south of the preceding ones, also near the shore, but it has a large flow which finds its way to the lake as a small stream. The water is very hot at its source, with a specific gravity of 1.0022 and pH of 7.3, and as it cools contains much green vegetation.

No. 51. Mary Bay, Yellowstone National Park. These springs are near the lake shore just north of the base of Steamboat Point which rises sharply above them. They lie in a grassy slope well above the lake and are surrounded by soft, muddy, and crusty seepage areas which disappear at the gravelly beach next to the margin of the lake. The water is moderately saline, with a specific gravity of 1.0038.

Nos. 52–56. Turbid Lake, Yellowstone National Park. These springs are near the eastern shore of Turbid Lake, No. 52 between the East Entrance Road and a small stream that enters the lake not far north of the road; Nos. 53, 54 and 55 follow one another as one passes northward along the eastern shore, and No. 56 is near the northern corner of the lake. They differ among themselves greatly in salinity and hydrogen ion concentration in spite of their similar positions with reference to the lake shore. Nos. 52 and 53 are highly acid (pH 3.6) with specific gravities of 1.0025 and 1.0034 respectively while the others are much more alkaline (No. 54, pH 8.6; sp. gr. 1.0025; No. 55, pH 7.3; sp. gr. 1.0066; No. 56, pH 7.2; sp. gr. 1.0040).

No. 57. Tower Junction, Yellowstone National Park. Between Tower Falls and the junction with the road from Mammoth is a large spring to the west of the road which arises from the slope of a hill and supplies sufficient water to form several small streamlets. This water is conveyed for a short distance through a pipe, but both above and below this there is both running water and several small marshy areas which furnished good collecting. The temperature of the water is only about 45° at the hottest part, but on account of the good flow it cools rather slowly as it runs off. There is a smell of hydrogen sulphide and the pH is rather low (7.0), with a specific gravity of 1.0057.

No. 58. Whirligig Geyser, Yellowstone National Park. The water in the run-off from this geyser is extremely acid in reaction, pH below 3.6, and is moderately saline with a specific gravity of 1.0051. As with all geyser overflows the amount of water varies greatly and its temperature fluctuates in conformity with the activity of the geyser. Such locations are therefore not generally favorable for the occurrence of animal life of any kind.

No. 59. Near Minute Man, Yellowstone National Park. This is a warm flow at a culvert along the road in the Norris Geyser Basin about 100 yards south of the Minute Man Geyser. The water here is also highly acid with a pH of less than 3.6 which was the lower limit of the indicators we had with us. The specific gravity is 1.0026 and the temperature quite low, barely 25° in the warmest parts.

No. 60. Angel Terrace, Yellowstone National Park. This and the two following locations are near Mammoth Hot Springs. The first includes a spring (60a) just north of the once beautiful and well-watered Angel Terrace which was practically extinct at the time of our visit, and another (60b) known as the New Highland Terrace which lies directly southwest of 60a. The water here has a pH of 8.3 and specific gravity of 1.0040. We were disappointed to find that Bath Lake, a pond in which we had obtained much interesting material on the occasion of a previous visit in 1923 remains only as a dry depression, completely devoid of water.

No. 61. Soda Spring, Yellowstone National Park. Two springs within a few feet of each other are of quite different temperatures, one at 29.0°, known as Soda Spring and the other containing water as high as 39.2°. The composition of this spring is shown by the following analysis which is posted there by the Park authorities. The water is quite acid (pH 6.9) and moderately saline (specific gravity 1.0048).

Potassium chloride	93
Sodium chloride	182
Sodium sulphate	220
Magnesium sulphate	328
Calcium sulphate	392
Calcium carbonate (acid)	1,113
Ferric carbonate (acid)	34
Silica (SiO_2)	78
Total parts per million	2,440

No. 62. White Elephant, Yellowstone National Park. This is a hot pool at the western edge of the "Elephant," a mound of travertine still in slow process of deposition near the southern edge of the

active springs at Mammoth Hot Springs. The water is hot, over 50° at the center, but cools about the edge. The pH is 6.9 and the specific gravity 1.0058.

Nos. 63 and 64. Amphitheatre Springs, Yellowstone National Park. This is a group of pools in a depression about a mile to the east of the main road about five miles north of Norris Junction and a little more than a mile south of the Obsidian Cliff. There are a number of rather large pools which lie between two wooded hills and their overflows combine to form a small stream. The temperature of the water is nowhere very high, ranging between 30° and 40°. The water is highly acid (pH 3.6 or less) and the specific gravity 1.0043 and 1.0034 respectively.

Nos. 65–67. Firehole River, Yellowstone National Park. These springs are along the Firehole River near Old Faithful, opposite the lodge cottages. No. 65 is a small spring containing very little water of pH 8.3, of low temperature (32°–34°) but it yielded a good deal of material. No. 66 is a very hot pool, 100 feet east of No. 65, with the overflow filled with yellow and olive brown algæ. The pH of the water is 8.9 and its salinity rather low (1.0023). No. 67 is a fast flowing streamlet again 100 feet east of the last one, pH 8.2 and specific gravity somewhat higher (1.0044). It contained numerous insects and crustaceans.

No. 68. This and the six following springs are in the Shoshone Geyser Basin which lies about seven and one-half miles in a straight line south of Old Faithful at the extreme western extremity of Shoshone Lake. This area is accessible only by a trail about twelve miles in length and is little visited as there are no accommodations there for tourists. However, through the kindness of the park authorities at Old Faithful we were allowed the use of a ranger cabin that is maintained near the shore of the lake and were able to secure saddle and pack horses for the trip. The cabin is just a few yards west of the southernmost springs of the Yellow Crater group. This is a particularly good region for zoological work as it affords a great variety of springs in their original condition, all grouped on the slopes that rise to the west from the bank of Shoshone Creek at this place. Many of these are boiling or extremely hot, but the overflows and other cooler pools proved to be good for collecting. No. 68 is the run-off from a hot pool about 100 yards northeast of the cabin. The pool lies on a slight rise due to the deposits it has laid down and the overflow is in part rather shallow. This is a strongly alkaline spring with pH of

9.4 and a specific gravity of 1.0040. Collecting was done here in water as high as 41.6°–41.8°. This spring is evidently No. 11 of the Yellow Crater group described in Hayden's report for 1878 (vol. 2, p. 288).

No. 69. Shoshone Geyser Basin, Yellowstone National Park. This pool is about fifty yards east of the previous one and lies at a lower level, with a soft clay margin. It is quite shallow, with little vegetation and the temperature of the water does not rise above 32° at any place. It was inhabited by beetles, tadpoles and leeches at the time of our visit, but collecting was difficult on account of the very soft muddy bottom and surroundings. The water is less saline and less alkaline than in the previous spring, with a specific gravity of 1.0036 and pH of 8.2.

No. 70. This is a large pool less than 100 yards south and slightly west of the cabin on somewhat higher ground. It has a temperature of about 31.5° and is apparently devoid of algæ. It is moderately saline, specific gravity 1.0042 and rather strongly alkaline, pH 9.0.

No. 71. Near the west shore of Shoshone Creek about a quarter of a mile south of the previously mentioned springs is a large actively flowing spring which runs down into the creek. The overflow widens out considerably in a very irregular way due to the deposit which it lays down along the lines of flow. It thus spreads out more or less fan-shaped to the edge of the creek where it empties almost opposite to a small very active geyser known as the Minute Man that plays from an elevation near the east bank of the creek. The overflows contain a good growth of algæ in water of a specific gravity of 1.0046 and pH 8.8.

No. 72. This location is next below No. 71 also on the west side of the stream a distance of about 100 yards. The spring lies near the edge of a wooded area which approaches Shoshone Creek at this point. There is no direct overflow and the specific gravity of the water is rather high (1.0062) and the reaction moderately acid (pH 7.7).

No. 73. This is east and slightly north of the cabin at a distance of about 60 yards, and consists of a small spring and extensive seepage overflow that drains northeastward to Shoshone Creek. The water in this spring is very different from the others in this group, having an extremely acid reaction (pH 3.6 —) and a moderate salinity of 1.0058.

No. 74. This location includes several small springs near together on the east shore of Shoshone Creek almost directly east of the cabin.

They drain into the creek by a small streamlet with grassy sides and soft, pale clayey bottom. The water is quite acid (pH 6.4) and its specific gravity is 1.0047.

There are numerous small geysers in the Shoshone Geyser Basin and many highly varied and beautiful boiling or extremely hot springs, some of them of great size and depth. The majority are of course not suitable for the occurrence of animal life on account of their high temperature, intermittent overflow or rapidly changing run-off channels.

No. 75. Three Sisters, Yellowstone National Park. This is an overflow about 100 yards northwest of the Three Sisters Springs which are about half a mile west of Old Faithful Geyser. This overflow is quite shallow, extending for a considerable distance as a number of streamlets over the sloping formation close to the road leading to the Black Sand Basin. The water is highly alkaline, with a pH of 9.1 and specific gravity of 1.0046.

No. 76. Sunset Lake, Yellowstone National Park. This is the overflow from Sunset Lake which finds its way into Iron Creek near the center of the Black Sand Basin about one and one-half miles west of Old Faithful Geyser. The water is apparently quite similar to that of the previous spring; pH 9.0, specific gravity 1.0043.

No. 77. Lower Geyser Basin, Yellowstone National Park. This spring is another three-quarters of a mile further along the Black Sand road as it turns northeastward beyond Sunset Lake, and lies next to this road at the point where a branch road leads to the White Pyramid Geyser Cone. It forms a brownish pool surrounded by low vegetation, the water highly alkaline (pH 9.5) and of moderate salinity (1.0056).

No. 78. Biscuit Basin, Yellowstone National Park. This is an overflow which finds its way into the Firehole River just north of the foot-bridge which leads to Biscuit Basin from the automobile road about two miles north of Old Faithful. Like the previous springs the water is highly alkaline (pH 9.6 +) with a specific gravity of 1.0040.

No. 79. Sprite Pool, Yellowstone National Park. This is a very large overflow of varying temperature which extends over a considerable area. It has laid down a large amount of deposit over which the hot waters pour in great volume. They contain quantities of yellowish and greenish algæ and deposition is evidently proceeding at a rapid rate. From its size, extensive flow and large amount of deposited material, this spring gives evidence of considerable age and uniform flow. The water has a specific gravity of 1.0039 and pH of 8.9. Here we found numerous specimens of *Thermacarus nevadensis*, the first taken in 1930, and were able to establish its occurrence in the Park.

No. 80. Firehole Spring, Yellowstone National Park. This is an extremely hot pool which lies at the end of a branch road that leads off eastward from the southern edge of the Lower Geyser basin toward Firehole Lake. In the overflow some collections were made in water of pH 8.2, specific gravity 1.0046.

No. 81. Tangle Creek, Yellowstone National Park. This is the overflow from Hot Lake near the eastern margin of the Lower Geyser Basin. The water runs rapidly along a well-defined stream-bed, east of and more or less parallel with the road that leads from the Great Fountain Geyser toward Hot Lake. The temperature drops gradually as one passes down stream and was near 33° along the stretch where we made our collections. The water is heavily charged with silica which is deposited along the course of the stream as a smooth black coating, which gives a peculiar appearance to the surface of immersed objects, as though they had been dipped in a black varnish. Stones and other loose objects are cemented to the bed of this stream by the deposit. We had visited the same place in 1923 and found the same conditions prevailing in 1930, seven years later. The deposition must be very rapid, as on the former occasion we found a piece of burlap in the water which had been completely covered with the black silica as though coated with tar. The characteristic insect of this location is *Ambrysus heidemanni*, found abundantly in both years. The water showed a specific gravity of 1.0037 and pH of 8.4.

Nos. 82 and 83. Near Ink Well, Yellowstone National Park. These are both near the Ink Well which lies close to a foot-bridge across the Firehole River about a mile north of Old Faithful. No. 82 is on the west side of the river in a rather low swampy place; the water is highly alkaline (pH 9.6 +) with a specific gravity of 1.0038. No. 83 is a very extensive flow arising south of the foot-bridge on a sinter elevation whence it flows northward, expanding irregularly to form numerous depressions more or less clogged with algæ and depositing material. It yielded a considerable amount of material. The water is rather strongly alkaline (pH 9.1); specific gravity 1.0039.

No. 84. Near Spasmodic Geyser, Yellowstone National Park. This spring is quite similar to the last with a large overflow that runs off in a southerly direction from its point of origin south and slightly west of the Spasmodic Geyser. It is well supplied with algæ and supports a quite considerable fauna in strongly alkaline water (pH 9.3) with a specific gravity of 1.0032.

No. 85. Heise, Idaho. These springs arise on the northeastern bank of the Snake River at the Heise Hotel and Bath Houses, at a distance of twenty miles almost directly northeast of Idaho Falls, although the distance by road is between 45 and 50 miles from that city. The springs arise on a promontory overlooking the river and about forty feet above it. There was formerly a series of hot baths next to the springs, but these have been abandoned and the water is now piped for a distance of several hundred feet to the hotel. Our collecting was done in the seepages directly along the side of the elevation on which the springs emerge and in small pools derived from the leaky pipe which carries the water. The water is heavily charged with salts, showing a specific gravity of 1.0087, but unfortunately our record of the pH has been mislaid. The proprietor of the hotel informed us that the corrosive action of the water on iron or galvanized piping is so great that it requires very frequent replacement. The swampy slope watered by the springs is muddy, without much evidence of mineral deposits and supports a good deal of vegetation, mainly the alkali-grass, Distichlis.

No. 86. White Arrow Sanitorium, Blanche, Idaho. This spring arises in Gooding County, about nine miles north of Bliss, a station on the Oregon Short line and also on the transcontinental highway "U. S. 30," about 100 miles east of Boise. From Bliss it may be reached over a good secondary road, although the approach from the east is barely more than possible for an automobile. The spring has a large flow which forms a stream that flows for a distance of several hundred yards to a flat area where it disappears and at several places along its course part of the water is led off into ditches for irrigation. Collecting at a number of places along the stream yielded a fairly varied series of aquatic insects at temperatures between 30° and 41°. The water is rather saline, with a specific gravity of 1.0060, and only moderately alkaline (pH 8.6).

Nos. 87 and 88. Mountain Home, Idaho. Like the preceding locality, Mountain Home is also on the Oregon Short Line and "U. S. 30", about 50 miles west of Bliss and an equal distance east of Boise on the highway. The springs are about eight miles northeast of the town and are easily reached over a good secondary road. They arise at an altitude of 3700 feet through lava about a mile south of the rhyolite of Mount Bennett on the sloping edge of a large flat area close to high lava cliffs that rise up about 1,000 feet to the northeast. The five springs form two groups each giving rise to a small stream. After about 50 yards the two unite and pursue a winding course through the sage-brush flat finally finding their way into Bennett Creek and thence to the Snake River. The springs vary in temperature from 40° to 75° and although some collecting could be done close to the springs, the temperature of the stream, whose flow is said to be about 120 cubic feet per minute, is not cool enough for animal life until it has flowed out for a considerable distance. The slope about the springs is grassy and more or less marshy, although the stream below cuts its way quite sharply with steep sides and little vegetation appears aside from the desert vegetation which, as is generally the case next to water of any kind, grows with noticeable luxuriance. The combined flow of two of the springs, which are of lower temperature than the others we have listed as No. 87, with highly alkaline (pH 9.6) water of but little salinity (1.0029). The flow of all combined after the flows from both groups of springs have united forms No. 88, having a higher salinity (1.0036), but the same hydrogen ion concentration. No doubt the greater salinity of the stream is due to the greater solvent power of the hotter water in the second group of three springs.

No. 89. Indian Springs, Idaho. These large springs are reached by a short side road which leaves the main highway (U. S. 30) just southwest of American Falls, or about 17 miles west of Pocatello, Idaho. They have been transformed into a resort with large bathing pool, but are nevertheless still highly interesting. The springs arise at the edge of a hill that borders the northern edge of an extensive flat area. One of these has been led from its origin in a small gully to the west of the buildings into a warm pool in which the proprietor maintains a colony of goldfish. This pool and the water between it and the origin of the spring furnished a good series of specimens. Further eastward, perhaps 100 yards distant we made a second series of collections in the rocky bed of the stream which is derived partly from the spring that supplies the bath house and from others that occur along the hill slope at that point. These latter springs also supplied material. The temperature of the water is moderate, between 32° and 33°, but it is well mineralized with a specific gravity of 1.0031. It is but slightly alkaline; pH 8.4.

No. 90. Lava Hot Springs, Idaho. This little town is on the

Partneuf River twelve miles east of McCammon which is on the Oregon Short Line 23 miles south of Pocatello. It may be reached by a road ten miles long that leaves the highway (U. S. 91) at a point two miles south of McCammon. Here a number of hot springs issue from the bank of the river. Some of these supply a large public natatorium, but there are others along the river just below a large park, accessible by a steep and rather precarious path that descends about 100 feet to the level of the river. Hot water issues at several places along the bank of the river and in the bed of the stream at the shore. The hot water thus mingles with the cold stream water so that it is difficult to determine temperatures, except where pools of constant temperature are formed. As happens under such conditions the hot water tends to flow over the surface and crayfish that were living on the bottom were killed by heat in removing them through the supernatant hot water for preservation. The water is quite saline, with a specific gravity of 1.0053.

No. 91. Cleveland, Idaho. This locality is southeast of the last one and northeast of Preston, Idaho from which it is accessible by road, a distance of twenty-five miles. There are about six springs in a group in a pasture just south of the road and next to the village store. The ground slopes at first slightly and then steeply to the south where it borders on a broad flat area along the edge of Cottonwood Creek which flows eastward and almost immediately enters Bear River which flows south at this point. The springs form large shallow pools with grassy borders, separated by marshy land near the road and by more dense, brushy vegetation below on the steep lower slope. The water is used for irrigation and for watering stock although it is rather saline (pH 7.5). These pools support a varied fauna and we secured much material here at temperatures ranging from 28° to 42°.

No. 92. Cleveland, Idaho. This location is very near to the previous one, on the other side of the Bear River near the summit of a steep slope that rises above the river. The spring arises near the road that extends along the eastern side of the river and fully 100 feet above the stream, so that looking across one may see the other springs and below them the clover fields that they irrigate. This spring has a quite extensive flow and is rapidly depositing a calcareous material on the side of the hill on its descent to the river. Undoubtedly this spring has built up the elevation from which it now emerges as the surrounding soil has the characteristic appearance of hot springs deposits. The occurrence of the thermophilous mite, Thermacarus, here, also suggests that it is a permanent spring of considerable age.

No. 93. Preston, Idaho. This spring is about four miles northwest of the town, on the western side of the Bear River, only a few hundred feet from the shore. It has a rather large flow, most of which is led to a bath-house, but the remainder flows out into a clayey, muddy flat covered with sparse, low vegetation, consisting mainly of alkali grass (Distichlis). The water issues from near the base of a slope that rises just to the west of the road and after passing under the road spreads out, flowing northward into the marsh just described where it is gradually absorbed without flowing into the nearby river. The water emits a vile, more or less sulphurous odor which permeates also the surrounding whitish mud and soil. The water which is very highly saline, with a specific gravity of 1.0150 is slightly alkaline (pH 8.0), and was found to contain numerous larvæ of Ephydra. In relation to the springs at Cleveland, also on the Bear River, the present location is about 18 miles southwest in a straight line.

Nos. 94 and 95. Cherry Creek, Nevada. This may be reached from the new transcontinental highway U. S. 50. At the town of Schelbourne about 100 miles south of Wendover, Utah, a secondary road leads northwestward 16 miles to the town of Cherry Creek and the springs are a couple of miles south of the town. They may be approached also from the south from Ely over a secondary road, a distance of about 50 miles. These springs are on a ranch which sets on the slope of hills that rise to the west, surrounded by sage-brush vegetation. The water of the main spring (No. 95) issues in large quantity from a crater-like depression in a bare mound of deposit and is used for irrigation and domestic purposes. Several hundred feet further south is a smaller flow (No. 94) surrounded by sparse vegetation which disappears after flowing down a gully for a short distance. The water of both springs is very hot (above 55°) where it emerges, but cools as it runs off down the slope. Both springs have a pH of 8.4, but the specific gravity of No. 94 is less (1.0038) than that of No. 95 which is 1.0043.

No. 96. Monte Neva, Nevada. This large spring is about 15 miles north of Ely, Nevada on the secondary road to Cherry Creek which lies west of the main highway (U. S. 50) that leads from Ely to Wendover, Utah. There is a copious flow of very clear, hot water that has been partially impounded into a large pool from which it is led off through a ditch to be cooled and used in a well-appointed nearby bath-house, swimming pool and hotel. The water is depositing large quantities of mineral matter as it cools and there is considerable seepage from the pool which is elevated above the surrounding land, undoubtedly on material deposited by the spring. Along the ditch much of the deposit is forming at the surface of the water along the edges, in liver-like outgrowths with lobate or scalloped edges, showing noticeable bands of ferruginous red color. The water is not highly saline, showing a specific gravity of 1.0038 and is slightly alkaline with a pH of 8.4.

Nos. 97 and 98. Sunnyside, Nevada. This locality is on a secondary road that leaves the main Ely-Tonopah highway 28 miles from Ely and leads southward to the Sunnyside Ranch some 45 miles further. The springs are several miles beyond the ranch, and on the way the road crosses a warm stream formed by the overflow from the spring. No. 97 is the stream near the bridge at the crossing. Here the temperature of the water is 30.8° and the pH 9.3. The spring itself is about a mile further south and may be easily located by two isolated buttes of Silurian age that rise near it. It flows a very large volume of water at a temperature near 34°, which forms a large tule-bordered pool with a considerable extension to the westward. The outlet is to the east from whence the previously mentioned spring flows north as a good sized stream. The water at its source (No. 9) is more saline, with a specific gravity of 1.0042 and less strongly alkaline pH 8.3. This is undoubtedly a very old spring, for although it is not now depositing mineral matter in any quantity, the surrounding sage-brush plainly shows evidence of extensive hot-springs formations. In places it is cut by gullies or shows mounds that rise above the level of the plain over an area of several square miles, often to a height of forty feet.

No. 99. Duckwater, Nevada. This location may be reached in two ways from the transcontinental highway "U. S. 50" by traveling south about 40 miles. One road leads off about 20 miles west of Ely going through Hamilton an old mining town now nearly deserted; the two meet at Duckwater which is just south of the spring. Both roads are very poor and rough and the one through Hamilton has a number of steep grades. Probably the other and shorter road over which we returned is the better. This spring seems to issue from a fissure and has a rather large flow of clear water, having a specific gravity of 1.0032 and pH of 8.0, and a temperature of about 32° at its source.

It does not appear to be actively depositing mineral matter, but is raised above the surrounding land on deposits evidently laid down in the past. The water flows out as a small winding stream that soon disappears in a marshy pasture. The stream contains a good growth of algæ, a number of insects and here we found the remarkable new fish, *Crenichthys nevadæ*. To judge from the large flow, evidence of extensive formations and presence of this peculiar fish, this spring is undoubtedly a very ancient one.

No. 100. Antelope Valley, Nevada. This spring is reached by a road that leaves the main Eureka-Austin highway about 25 miles west of Eureka. This little used road leads south and the spring is reached after traveling eleven miles. This location is in a sloping grassy valley with a seep of cold, potable water and a short distance further down is a large hot spring emitting a copious flow of water at about 60°. It is led off through a ditch and fallen-down building which was probably at one time a bath-house. Near here are some sulphur fumaroles. The timbers of the house were blackened by the sulphur fumes. As the water flows down along a winding course through the sage brush and widens out into a grassy marshy area we made extensive collections both in the marsh and in the eddies of the swift stream. Near its source, the water is depositing mineral matter in considerable amounts, but further down only a whitish clayey mud. The water has a specific gravity of 1.0032 and strongly alkaline reaction of pH 9.1.

No. 101. Birch Creek Ranch. This is on a secondary road that leaves the main Eureka-Austin highway twelve miles east of Austin, Nevada. The Birch Creek Ranch is only two or three miles south on this road and from the ranch a trip of two or three miles to the east across a wide valley, over an almost impassible road leads to the spring. This arises on the lowest slope of the mountains that lie to the east and flows for a short distance to two deserted and dilapidated "bath-houses" after which it spreads out into a marshy area. There is some mineral deposit about and below the spring, but little more than fine clay seems to be forming at the present time. The water has a specific gravity of 1.0032 and pH of 7.3.

No. 102. Darroughs Hot Springs, Nevada. This locality is about fifty miles south of Austin, Nevada from which it can be reached by secondary roads. One of these extends directly south from Austin, and the other and better one is the road already mentioned (Spring No. 101) that leads south from the highway 12 miles east of Austin.

The springs lie in the Big Smoky Valley at the foot of the Toyabe Range which rises directly to the west. There are several springs in this group, the largest actively boiling and another emitting a quantity of steam which is used for steaming hams by the ranch owners. The springs cover an area of several acres; one to the northwest and at a higher level than the hottest ones furnished good collecting. There is some calacareous deposit in the vicinity of the present outlets and generally within a radius of several hundred feet. The specific gravity of the water is 1.0038 and the pH 7.3 in the spring where the collecting was done. An analysis of the water from the boiling spring according to Meinzer ('17), p. 89 and p. 154, is given below:

CONSTITUENTS IN PARTS PER MILLION

Silica (SiO_2)	88
Calcium	13
Magnesium	3
Sodium and Potassium	80
Carbonates (Co_3)	31
Bicarbonates (HCO_3)	102
Sulphates (SO_4)	60
Chlorine	15
Total dissolved solids	392

No. 103. Cortez, Nevada. This spring is forty-five miles northeast of Austin, Nevada and may be reached over a secondary road that extends northward from the main highway about three miles east of Austin. This road leaves the highway in the mountains just east of Austin and descends to Grass Valley, proceeding northward in the floor of the valley past the ranch where the springs are located. Here an actively flowing spring issues from a calcareous mound whence it is led through a swimming pool and later used for irrigation. The spring is on the east side of Grass Valley and the overflow forms a large green area below it, traversed by tule-bordered ditches. The water issues at a temperature of over 70° and it is still very warm when it reaches the meadow. The water is depositing large quantities of calcareous material near the spring, but itself is not highly mineralized, showing a specific gravity of 1.0041 and mildly alkaline reaction (pH 8.5). We secured interesting and extensive collections here.

No. 104. Monte Christo, Nevada. This locality is rather difficult of access over a road which extends south of the transcontinental highway 83 miles west of Austin and 97 miles east of Reno, Nevada.

This side road is passable, but difficult, winding through mountains for about 30 miles to the spring which is at the eastern edge of a large alkali flat. It is possible to drive westward across this flat and join a road at Deadhorse Well which leads through Rawhide and thence north to the highway, which we did on returning. This is a large spring with extensive flow, the overflow forming several large pools and a marshy area. It arises near the southern tip of the Sand Springs Range, presumably through lava rock as the road is through lava for some miles to the north. There is little mineral deposit from the spring which issues at a level barely above the alkali flat. The water has a specific gravity of 1.0052 and pH of 8.7.

Nos. 105, 106 and 107. Pyramid Lake, Nevada. Pyramid Lake lies in western Nevada, north and slightly east of Reno, Nevada. About fifty miles over a very good secondary road one can drive to the Post Office of Pyramid on the western shore of the lake. The lake receives its name from a large hot-spring tufa cone that rises high above its waters near the eastern shore. At the present time the flow of the water from the "pyramid" is much reduced but there are a few tiny geyser-like outlets not far above the surface of the lake and others at or just below the surface. As the sides are very steep the overflows pass quickly into the lake and we found no opportunity to do any collecting here. However, at the northern extremity of the lake there are a number of small springs issuing close to the shore, near a series of tufa-covered rocky prominences, The Pinnacles, that line the shore. These springs are all of small size and very limited flow, but in several of these we were able to collect a considerable amount of material. They vary but little in salinity, and not greatly in pH, all being moderately alkaline.

	Specific Gravity	pH
No. 105	1.0064	8.0
No. 106a	—	8.4
No. 106b	—	8.4
No. 107	1.0074	8.4

The water of Pyramid Lake had a specific gravity of 1.0062 at the time we were there, but in 1877 was stated by Hague and Emmons ('77) to be much lower (1.0027). At the present time the lake is receiving less water than normally due to an artificial diversion of the Truckee River, its level has dropped noticeably, and no doubt, the future will see it unfit to support the abundant fish now found there as its salinity is increased through evaporation.

No. 108. Gerlach Hot Springs, Gerlach, Nevada. Gerlach lies north and slightly east of the northern end of Pyramid Lake about 30 miles in a straight line. It may be reached by a road said to be reasonably good that extends northward from Pyramid passing west of the Lake Range, thence across the southwest arm of the Black Rock Desert and then north along the edge of the desert. We ourselves followed a road that crosses the Lake Range at the northeast corner of Pyramid Lake and thence northward along the southeast arm of the desert. This proved to be only a sheep trail for some 20 miles after crossing the mountains, but was finally negotiated after a very difficult and arduous day's travel. The springs form a group of three pools about half a mile west of the town of Gerlach, one of which is some twenty feet in diameter filled with water near the boiling point. The overflow waters a green meadow of considerable size and in the partially cooled run-off considerable material was collected. The water is quite strongly saline with a specific gravity of 1.0061 and rather acid reaction (pH 7.5). There is practically no hard mineral deposit about these springs although they are slightly above the level of the surrounding country due to clayey material which is also noticeable along the course of the overflows.

Nos. 109, 110, and 111. Sixteen miles northwest of Gerlach, Nevada. Proceeding northward from Gerlach which lies at the extreme southern end of the Granite Range along the edge of the desert and thence west and north is a group of several large pools producing a large flow, sufficient to irrigate a good-sized ranch. They reach the surface in a low meadow bordering an alkali flat and about a mile east of the mountains of the Granite Range that rise rather steeply to the west. Of the three pools which we examined, one small one is extremely hot (about 75°) while the other two, one of which is perhaps 75 feet in diameter, are only warm (32.5° and 33.7°). The large pools are all surrounded by a luxurious fringe of tule rushes, next to the shore which rises sharply several feet above the water. This is evidently an ancient group of springs, possibly not all from the same source considering the great difference in temperature, as it seems probable that the supply of water in the large cooler pools must be in part derived from the adjacent mountains, especially as there are other cool or cold springs (*e. g.*, No. 113) further north along the eastern edge of the Granite Range, in essentially similar positions. The specific gravity and pH of these springs are as follows: No. 109, sp. gr. 1.0029, pH 8.5; No. 110, sp. gr. 1.0024, pH 8.3; No. 111, sp. gr. 1.0042, pH 8.7.

No. 112. Black Rock Desert, Nevada. Although we were unable to do any collecting in this spring it is so remarkable in appearance and location that it seems worth while to describe it briefly. It lies near the eastern edge of the completely barren Black Rock Desert about 40 miles northeast of Gerlach from which it may be reached by driving over the absolutely flat clayey floor of the desert to a point near the conspicuous, dark, rocky hill from which the desert derives its name. On account of the mirages it is difficult to estimate distance, or to be sure of the location of distant objects. The spring is a large pool of very hot water set near the base of some hills, whence it flows over a gentle slope to the plain that sets just above the level of the desert, to disappear in this area which is covered with a sparse growth of desert plants. The pool is about 50 feet in diameter, surrounded by a large area of highly luxuriant tule rushes which also surround the overflow and make the locality a most conspicuous and pleasing, verdant landmark. The pool is accessible where the very hot water issues at the upper, eastern edge, but the rushes and swampy soil make access to the cooler parts very difficult. So quickly is the water absorbed that no actual overflows could be found of sufficient size for collecting. The composition of the water is apparently quite similar to the springs just described northwest of Gerlach on the other side of the desert, with a specific gravity of 1.0046 and pH of 8.4.

No. 113. Soldier Meadows, Nevada. These springs are about 35 miles almost directly north of Gerlach, Nevada, very close to the road that leads to Soldier Meadows along the western side of the Black Rock Desert. The road here passes through a sparse growth of desert shrubs very little above the floor of the desert but at the eastern base of mountains that rise more than two thousand feet above it. Although of undoubted thermal nature the water is cool. It waters an area of perhaps an acre of alkali grass (Distichlis) with a mixture of some other low herbs and a portion that is covered with water is well populated by frogs and toads. The water has a specific gravity of 1.0007 and pH of 8.8.

No. 114. Hausen, California. These springs are at the south end of Surprise Valley at the southern end of Lower Lake, just over the California border and about twelve miles south of Eagleville. There is a large flow of warm water which supplies a small bath-house and serves to irrigate a large field. The springs are only moderately warm and our collecting was done at 36.2° in an extensive overflow that forms a gulley near the several houses that compose the town. The water has a specific gravity of 1.0033 and pH of 7.5.

Nos. 115 and 116. Surprise Valley, California. Between Hausen and Eagleville there are a number of hot springs that issue from the gravel or boulders of the hillside between the road and the lake bed to the east. These form a series of irregular small rocky streams that serve to water small areas. Some are merely seepages, but others have considerable flows that run for distances of 100 yards or more. In most cases the water is only slightly warm but two were of sufficient size and hot enough to be of considerable interest and yielded an extensive series of specimens. The water is of moderate salinity, 1.0027 and 1.0035 and the pH 8.3 and 7.6 respectively for the two springs listed.

No. 117. Denio, Oregon. This is a group of several springs six and one-half miles south of Denio, Oregon which lies just over the state line in Nevada so that the springs are in the latter state. We visited these springs in 1927 (No. 16 at that time) and were surprised to find after a lapse of three years that they had greatly decreased in flow, especially the isolated spring about a quarter of a mile east of the others which was nearly dry although previously flowing actively. This may be temporary, as a lake just to the north previously filled with water was entirely dry. However, as the flow from these springs is very hot this does not seem entirely likely as they do not appear to have their origin in meteoric water. The water was still very hot (about 75°), but its specific gravity had increased from 1.0015 to 1.0024 and its pH had changed from 8.5 to 7.5. Possibly these springs are destined soon to become extinct and join some of their neighbors whose former presence is now indicated only by their fast weathering deposits dotted over this vast desert region.

No. 118. South of Denio, Oregon. This is a very large and extremely interesting hot spring which lies west of No. 117 about four miles. It may be reached by traveling south five and one-half miles from No. 117 and over a right hand road that leads northwest another four miles to the spring. We had almost reached this spring in 1927, but were forced to turn back to repair our engine. An enormous volume of water at about 52° issues here in a large pool surrounded by a grassy meadow from which a rapidly flowing stream some eight feet wide extends south and later eastward, first through the desert and later into land which it irrigates. The stream is filled with fleshy green algæ and we obtained an extensive collection. This is a very active spring and although it has built up only a comparatively small amount of mineral deposit is undoubtedly of great age and permanence. The water has a salinity of 1.0030 and pH of 8.4. It arises well removed from any mountains, just west of the northern tip of the Pine Forest Mountains in an extensive flat area sparsely clothed with desert plants at an altitude of about 4,000 feet.

No. 119. Fish Springs, Nevada. These springs form one of the very few watering places along the distance of more than one hundred miles between Cedarville, California and Denio, Oregon. Although perhaps of thermal origin the water is cold and potable. The flow is small in amount and disappears in the arid soil within a quarter of a mile from its source. The spring and overflow support numerous aquatic insects and we secured a collection for comparison with others made in the hot springs of the same region. Fish Springs arise from the western slope of a low range of rocky hills. The water leaves no white deposit, but flows over black mud containing innumerable minute particles of opal evidently derived from opalized wood exposed on the slopes above.

No. 120a. Cedarville, California. Just east of Cedarville a distance of five miles on the "dry weather" road that crosses the middle alkali lake, a short side road to the south leads to this spring which is only about half a mile from the main road. Like many such springs the water contains easily detectable traces of radium and this spring is being exploited as a cancer sanitarium. It issues as a large steaming pool on a slight elevation above the desert which borders the alkali flat that lies between it and Cedarville. The water flows into a large tule-bordered pool and thence off into the desert. The water is very hot, but cools in the second pool where some collections were made as well as in the overflow ditch. The water is depositing small amounts of clay only. It is moderately mineralized with a specific gravity of 1.0037 and slightly alkaline (pH 8.2). No. 120b. This is really some of the overflow from the same spring which reaches the side of the main road just north of No. 120a. Here the density of the water is slightly reduced (1.0034) but the pH of 8.2 is unchanged.

No. 121. Kelly's Hot Spring, Canby, California. This is about four miles west of the town of Canby on the highway that leads to Alturas southwestward toward Redding, California, at a point 25 miles from Alturas. Here water issues at the boiling point from a large depression with considerable movement and the evolution of large bubbles of gas. The discharge, which is said to average 325 gallons a minute is pumped in part to a swimming pool and flows in part into a meadow just below the spring. The spring does not seem to be depositing much mineral matter at present but there is much white deposit in the neighborhood, indicating that it is the remnant of a more extensive spring or group of springs previously active, and cinnabar is found in small quantities in the deposits. The overflow extends for a long distance and afforded good collecting. The water is only moderately saline (specific gravity 1.0032) with a pH of 8.4.

No. 122. Bassett Hot Spring, Bieber, California. This is about two and one-half miles northeast of Bieber which is 59 miles from Alturas on the same highway as No. 121. The water issues at the level of the surrounding ground with a flow of about 175 gallons a minute whence it is led into a large cement swimming tank. The water is moderately mineralized with a specific gravity of 1.0037 and pH of 8.4. Its composition is given as follows, in parts per million:

Sodium chloride	166.6
Calcium sulphate	66.6
Calcium acid carbonate	14.1
Sodium sulphate	488.0
Calcium carbonate	26.5
Total parts per million	761.8

The water is thus seen to be of the sulphate type, lacking in silica and magnesium.

No. 123. Big Bend Hot Springs, Henderson, California. Together with the two springs just mentioned as Nos. 121 and 122, this group issues along the course of the Pit River near the town of Henderson. Henderson may be reached from the same highway as the previous springs by going north a distance of about 18 miles on a secondary road that leaves the highway at Montgomery Creek, 40 miles northeast of Redding. The springs issue along the steep bank of the river through boulders and gravel, one well above the river and some others near its edge; one higher up supplies a bath-house (No. 123a). Our collecting was done mainly about a quarter of a mile down stream from the bath-house in the overflow from a spring which issues on the slope and flows into the river, possibly augmented by others that add to its volume, further down the slope and into the edge of the river. The specific gravity of the water varies from 1.0040 (No. 123a at bath-house) to 1.0043 (No. 123b, down stream) but the pH is 7.4 at both places.

No. 124. Mount Lassen National Park, California. This and the following thirteen locations (Nos. 124–136) are in various parts of Lassen Park and are in general characterized by their variation of hydrogen ion concentration, several showing a pH of less than 3.6 and none more than pH 7.4. No. 124 is just above the new graded road that leads from Mineral, Cal. into the park to Lake Helen, about one and one-half miles below the lake. Here the water issues in a steeply sloping meadow filled with grass and much vegetation, including many conspicuous plants of Veratrum. Through the middle of this slope there runs a streamlet of cold water, and the warm springs lie to the southern side of this with which their overflows joins. The water has a moderate specific gravity of 1.0037 and extremely low pH (3.6 or below).

No. 125. This spring lies only a few feet above the last, running off through a soft muddy gully lined with gray, clayey deposits. It is also extremely acid (pH 3.6 or less) and with a slightly higher salinity (1.0043).

No. 126. This is in the same group with the last two a few feet to the left of the others. The water has a strong odor of hydrogen sulphide and a higher pH (7.1). Its salinity is lower (1.0025).

No. 127 to 132. Bumpass Hell, Mount Lassen National Park. These are in the large depression known as Bumpass Hell which includes a great variety of steam-vents, mud-pots, boiling springs, warm springs and small seepages scattered mainly over an area of perhaps ten or fifteen acres, near the higher western portion. The activity of these springs undoubtedly varies greatly due to the supply of water from the slopes above and the quantity of melting snow present at different times. When we visited this locality previously in 1927 there was considerably more water present and the combined overflow which runs off to the south from the lower end of the basin formed a sizable warm stream of suitable size for bathing, while in 1930 with less snow on the upper slopes this stream was greatly reduced in volume. The several springs in this group are highly varied in appearance and temperature, salinity and reaction and undoubtedly also in the composition of their dissolved materials. There is much siliceous sinter about some of the hotter pools and in general the formations remind one of those in the Norris Basin in Yellowstone Park. As will be seen from the data below these springs vary considerably in salinity, but all are quite acid.

	Specific Gravity	pH
No. 127	1.0028	6.0
No. 128	1.0028	6.7
No. 129	1.0029	6.6
No. 130	1.0056	6.8
No. 132	1.0033	4.2

No. 133 to 136. Sulphur Works, Mount Lassen National Park. The formations to which this name has been given are associated with a stream that crosses the main park road at an elevation of about 6600 feet. They lie above and to the north of the road from which they are easily reached by a trail that follows the stream. There are many slopes and mounds of white and yellowish buff-colored material which has been deposited by hot springs in the past and much greenish, bluish or reddish clay of similar origin surrounds some of the springs. Nos. 133 and 134 are two pools with very acid water of pH 3.6 or less and moderate salinities of 1.0047 and 1.0043 respectively. No. 135 is a warm stream that flows over a rocky course. It contained numerous insects and tadpoles. The water is of much higher pH (7.4) and of lower salinity (1.0034). No. 136 is at the side of this same stream further down where the water is barely more acid (pH 7.3).

Nos. 137 and 138. Wilbur Hot Springs, California. These springs rise in the small canyon or valley of Sulphur Creek in Colusa County. They may be reached by road, 31 miles west of the town of Colusa, or by driving west over a very steep and winding road 21 miles from Williams which is on U. S. Highway 99. The waters from a number of springs, some of high temperature, join and form a stream that flows eastward down the valley. Our collecting (No. 137) was done at about 100 yards below the resort where the stream has a temperature of 32.8°, pH of 8.5 and very high specific gravity of 1.0217. Near the edge of the stream in a pool (No. 138) partly diluted with other, probably rain-water, the specific gravity is less, 1.0081 but the pH is the same. The water in these springs contains a large amount of sodium chloride, and gives off a very strong odor of hydrogen sulphide. It probably contains alkaline sulphides in solution as a silver coin placed in the main, most rapidly flowing spring, is completely blackened in a few seconds. It has a most unpleasant salty and sulphurous taste but is evidently quite harmless as it is drunk with impunity by the inmates of the sanitarium. There is very little mineral deposit about these springs.

No. 139. Clear Lake Park. Traveling west from Wilbur Hot Springs the road crosses the mountains after a stiff climb and again descends passing along a branch of Cache Creek at a distance of about 14 miles. This is evidently thermal water although not very warm (28.3°) with a specific gravity of 1.0053 and pH of 7.5. Some collecting was done here.

No. 140. Seigler Hot Springs, California. These springs are at a resort in Lake County, reached over a secondary road northwest from Middletown at a distance of fourteen miles. They form a group of about a dozen springs, mostly small, on the slope that borders the edge of a small creek, associated with lava rock. Our collecting was done in some pools and a sluice-way which leads the water to a bathing pool. The water is moderately saline (sp. grav. 1.0057) and nearly neutral in reaction (pH 7.3), containing much sodium and magnesium with chlorides and carbonates, but almost no sulphates.

No. 141. Calistoga, California. There are several hot springs in or near the town which lies near the head of the Napa Valley. The largest is augmented by a dug well, made into a large pleasure resort and swimming pool, and completely ruined for any biological work. Near the outskirts of the town is an intermittent spring which has been made into an artificial geyser by sinking a large iron pipe into the soil. From this the warm overflow runs along the road to form a tule-bordered stream that yielded a few specimens. The water is only slightly mineralized (sp. grav. 1.0035) with moderately alkaline reaction (pH 8.4).

No. 142. Gilroy Hot Springs, California. The springs are about twelve miles east of Gilroy over a characteristically dry and dusty secondary road which leaves the U. S. Highway 101 at Gilroy, south of San José. This place has been a resort for some sixty years, built about a moderate-sized warm spring that issues in a ravine on the slope of Coyote Creek, about 200 feet above the stream. The specific gravity of the water is 1.0049 and the pH 7.3.

No. 143. Paraiso Hot Springs, California. A number of small springs form this group which is pleasantly set near the upper end of a small valley that opens eastward into the larger Salinas Valley. It may be approached over a secondary road that leaves the U. S. Highway "101" several miles south of the town of Soledad and leads westward up the valley to the springs. The springs have been known for a long time, receiving their name from some early Catholic missionaries who selected the spot as a better place to live than the adjacent lowlands. It is now a health resort and bathing place. The flow of these springs is small, but in some seepages, beneath the shadows of a group of palms that still remain as evidence of the first white inhabitants, we collected a considerable amount of material. Although said to be a cure for persons suffering from malaria, we found in these springs the only larvæ of malarial mosquitoes collected during the summer. The water has a pH of 7.4 and specific gravity of 1.0042.

No. 144. Paso Robles Mud Baths. These are on the main north and south highway "U. S. 101" about two and one-half miles north of the town of Paso Robles, California. An imposing, but deserted building and swimming pool stand as mute evidence of an attempt to popularize these mud-baths as a sanitarium. East of the building in a swamp are some warm pools and seepages with noticeable odor of hydrogen sulphide, where our collecting was done. The water has a pH of 7.4 and specific gravity of 1.0051.

No. 145. Jacumba Hot Springs. These are in the small town of Jacumba in the southeastern part of San Diego County a few miles east of the divide that separates the coastal drainage from that of the Colorado Desert and less than a mile from the Mexican boundary. Jacumba is on the transcontinental U. S. Highway "80" about 80 miles east of San Diego. The water issues in a field and along the side of a small creek, whence it is led through a ditch into a small public park where it cools considerably below the temperature of 36° noted at the source. Collecting was done both in the field and in the park. The pH of the water is 7.4, but the specific gravity varies from 1.0041 in the field to 1.0021 in the mixed flows that pass through the park.

Nos. 146 and 147. Fish Springs, California. These are on the north and south "U. S. Highway 99" close to the northwestern shore of the Salton Sea, 49 miles northwest of Brawley, Cal. One may drive to the springs over a rough road, a distance of about a mile east of the highway. The spring has been enclosed in a concrete tank which sets in a grove of small trees, surrounded by rank growth of tule and coarse grass, the tule extending over a wide area toward the shore of the Salton Sea. The temperature of the water is 30° or well below the daily summer air temperature of about 41° (106° Fahr.) which prevails at this altitude of about 200 feet below sea-level in the Imperial Valley. Nearby is another small spring (No. 147) about the same temperature and specific gravity (1.0060), but of higher pH (7.7). We visited also the "Mud Volcanoes" which lie on the edge of the Salton Sea about 15 miles north of Brawley. These are boiling and steaming mud pots on low cones of grey clay. They were until recently below the level of the water in the sea but have now been exposed by the receding water. There is but little water and no sign of life associated with these strange formations.

Nos. 148 and 149. Democrat Springs, California. These are in the valley of the Kern River, northeast of Bakersfield, from which they may be reached by traveling about 35 miles up the river valley. There are several springs which arise near the eastern bank of the river, about 50 or 75 feet up the rather steep slope. The largest spring is used for a bath-house but the others have been only slightly disturbed. There is evidently a considerable difference in the composition of the water in the different springs as the two in which we obtained specimens differed greatly in salinity and reaction as follows: No. 148, sp. grav. 1.0017, pH 7.4; No. 149, sp. grav. 1.0047, pH 8.7.

Nos. 150–153. Mono Lake, California. This highly alkaline lake is partly fed by a number of hot or warm springs that issue at a number of places along its shore. The waters of the lake due to continual evaporation are highly concentrated showing a specific gravity of 1.0409 and an extremely high pH of more than 9.6. As can be seen from the following analysis it contains a very large amount of sodium bicarbonate and considerable borax.

CONSTITUENTS IN PARTS PER MILLION.

Potassium chloride	1,834.2
Sodium chloride	18,506.8
Sodium sulphate	9,869.0
Sodium bicarbonate	34,150.1
Magnesium bicarbonate	334.9
Calcium bicarbonate	81.0
Sodium borate	200.0
Aluminum oxide	3.0
Silica	70.0
Total solids	65,149.0

This analysis was made many years ago and probably the water is more concentrated at the present time.

In Mono Lake is Paoha Island near the western shore of which a large warm spring arises several hundred feet inland and well above the level of the lake. The overflow from this (No. 150) flows toward the lake as a small stream bordered by grass and tule. The specific gravity of the water is 1.0045 and its pH 8.3, much lower than that of the lake. Near the eastern side of Paoha Island and reached by a trail about a mile in length which passes over a ridge of hills is another spring (No. 151). This forms a large pool about 50 feet in diameter, set in a crater like depression about 75 feet deep. This pool is said to have appeared after an explosion of gas coincident with the drilling of an oil well. It is known as Red Lake on account of the color of the water which is like that of diluted catsup. The red coloring matter is in suspension and would appear to be due to chromogenic bacteria although we were unable to more than guess at this conclusion. The only macroscopic life appears to be a slimy blue-green alga growing in quantity on submerged objects. The water is more than four times as saline as that of the lake, with a specific gravity of 1.1881 and with a pH of more than 9.6.

Along the western shore of the lake are a few warm springs that enter near the water-level. One of these (No. 153) with a specific gravity of 1.095 and pH of 9.6 yielded some specimens.

No. 154. Coso Hot Springs, California. These are near the southwest corner of Inyo County about 40 miles west of the southern part of Death Valley. They may be reached by a rather poor secondary road which extends eastward a distance of about twenty miles from the main road between Mojave and Bishop at a point about 80 miles north of the former and 100 miles south of the latter. The group includes one large spring and a number of smaller ones that cover an area of about an acre. There is a considerable amount of whitish-muddy deposit, more or less covered with coarse short grasses, which borders the eastern edge of a sinter mound. In the gulch east of the springs are indications of former hot springs in the form of whitish deposits but the present activity seems to be confined to the group mentioned. The water is only slightly saline (sp. grav. 1.0036) with a pH of 7.3.

SYSTEMATIC ACCOUNT OF THE FAUNA.

The following pages contain a detailed consideration of the zoological material, mainly insects, which we collected, together with data taken from the literature relating to the fauna of thermal springs in various parts of the world, and to the relations of other animals to high temperatures. As I have cited many publications in my previous papers on this subject, only material contained in contributions since 1928 is considered, in addition to a small amount or relevant data which I had previously overlooked or failed to discuss.

PROTOZOA.

We ourselves have made no collections of Protozoa, but one paper relating to the occurrence of these animals in hot springs has recently appeared.

Ciofalo in an extensive communication to the Royal Academy of Sciences of Acireale ('27) has given an account of the fauna of a number of Sicilian hot springs. He confined his attention mainly to the Protozoa of which he lists 65 species in addition to a few mentioned only by generic names. The temperatures of these springs where his material was collected varied from below 30° to over 60° but no Protozoa were obtained in any case at temperatures of above 44°, although other springs at 46° and 50° were examined as well as some of still higher temperature. The major groups of Rhizopoda, Foraminifera, Helizoa, Mastigophora, Ciliata and Acineta are all represented and considering the completeness of Ciofalo's observations it is perfectly evident that none of these types of Protozoa are particularly resistant to high temperatures. Many of the species are apparently widespread, common forms and none, so far as indicated by Ciofalo are species restricted to thermal springs. Considering the great difficulty of making authoritative specific determinations of Protozoa from preserved specimens, the most important data in this contribution relate to the upper limits of temperature at which Protozoa were obtained.

NEMATODA.

We ourselves have made no attempt to collect nematodes in thermal waters, but since my first visit to Yellowstone Park, Hoeppli ('26) has published an account of some collections of free living nematodes made in various parts of Yellowstone Park. I have summarized his records below as they add greatly to the data that I was able previously to gather from other published sources. Most of the following species are recorded as in water of "atmospheric temperature" which probably means a summer temperature somewhere in the neighborhood of 15°–20° centigrade; some are however considerably higher.

Mononchus brachyuris Bütschli, var. *macrodenticulatus* Micoletzky. Jupiter Terrace, in water near atmospheric temperature. This is a cosmopolitan species and has been found in thermal water at Ragaz, Switzerland by Hofmanner and Menzel ('15) at a temperature of 25°.

Mononchus macrostoma Bastian. Soda Spring, at 30.5°. According to our own records Soda Spring (No. 61) is quite acid (pH 6.9) and moderately saline (sp. gr. 1.0048).

Plectus parvus Bastian. Overflow from the Castle Geyser at atmospheric temperature. Known also from soil and fresh water.

Plectus paracommunis Hoeppli. Angel Terrace at atmospheric temperature.

Trilobus gracilis Bastian, var. *allophysis* Steiner. Overflow from Castle Geyser at atmospheric temperature.

Aphelenchus parietinus Bastian. Overflow from Castle Geyser at atmospheric temperature.

Diplogaster macrura Hoeppli. Overflow from Castle Geyser at atmospheric temperature.

Macrolaimus (Macrolaimoides) setosus Hoeppli. Jupiter Terrace almost at atmospheric temperature. This group is mainly marine although a few are known to occur in brackish or fresh water and in soil.

Chromadora nana Hoeppli. Jupiter Terrace, at atmospheric temperature. A related form, *Chromadora salinarum* has been found (Linstow '01) in saline water at 32° in Europe.

Chronogaster gracilis Cobb. Overflow from mud volcano at 25°–30°.

Dorylaimus thermæ Hoeppli. In several geyser overflows in water of 38° or 40°. This would appear to be a truly thermal species, especially interesting in connection with Issel's ('06) discovery of *Dorylaimus atratus* inhabitating thermal water in Italy at 40°. This species was first discovered by Cobb, but not described by him. He reported it first from a "scum-covered, small, shallow warm spring of 53 degrees Centigrade." Cobb suspected that the nemas might be actually living in water slightly cooler, and this seems probable as Hoeppli's records are 38° in a hot pool near the Castle Geyser and 40° in a hot pool near the main road one mile from Old Faithful in the direction of the Lower Geyser Basin.

From the data so far accumulated relating to thermal nematodes, it is evident that members of this group are quite generally distributed in hot springs, and that the dearth of records is due only to a lack of interest on the part of helminthologists. No doubt some nematodes will be found to be partial to this environment, if not restricted to it.

CRUSTACEA.

Members of this group are frequently encountered in thermal waters, especially small amphipods which are often numerous in water of moderate temperature. Unfortunately I am not able to report upon the Amphipoda at present. There are also a number of species of Ostracoda, met with less frequently, but of very great interest as certain members of this group occur in water at unusually high temperatures. An account of the distribution of the Ostracoda which we collected both in 1927 and 1930 is included in the present paper.

On one occasion a species of crayfish was taken in the stream close by the outlets of hot springs at Lava Hot Springs, Idaho, but not actually in the warm water. At this particular spot, hot water was flowing out over the surface of the stream, so that when the crayfish were removed from the bottom through the upper stratum of water they succumbed to the heat of the surface layer. Living on the bottom they are of course in cool water. They cannot therefore be included as members of the thermal fauna.

Ostracoda.

Among the collections made in 1927 were several lots of ostracod crustaceans and in 1930 a number of additional series was obtained. I have been fortunate to be able to have these identified by Mr. Charles H. Blake of the Massachusetts Institute of Technology who is particularly interested in this group. The following notes include much information kindly furnished by Mr. Blake, following the painstaking care which he gave to the two collections.

Eight species are included, and of the six which have been identified without question, all represent new species or varieties, included in five genera. This fact is rather surprising, but it must be remembered that very little collecting of these small animals has been done in the region from whence the present specimens came. From this fact it follows that it is difficult to judge whether any of the species are restricted to water of high temperature. It seems highly probable, however, that at least one form, *Potamocypris brunnea* Blake is a truly thermal species as it was taken only in water of extremely high temperature, *i. e.* at 49°–50° C., and the same is probably true of *P. varicolor* Blake collected at 44.5° and 48.3° C. Also, as is well known, at least one Algerian ostracod, *Cypris balnearia* Moniez (cited in my previous paper, 1928, p. 216) occurs only in water of high temperature. Unfortunately neither this species nor *Cypris thermalis* Costa have ever been described with sufficient accuracy to place them with certainty in any modern genus and their relationships to the species here mentioned cannot be considered at the present time.

It is of interest also, to note that in three of the species, *Chlamydo-*

theca bruesi Blake, *Potamocypris perbrunnea* Blake and *P. varicolor* Blake, both sexes are present in numbers. The species of these genera possess males only under tropical conditions, which of course are reproduced in hot springs in spite of the very cold winters which prevail over much of the area included in the present investigations.

In the short summary that follows, I have included more specific information on the occurrence of the several species.

Family Cypridæ.

Subfamily Candoninæ.

Physocypria brunnea Blake.

This was obtained at only one locality, in Hot Spring No. 91, Cleveland, Idaho in one of the pools containing water at 28.2°, specific gravity 1.0062, and pH 7.5. Most of the species of Physocypria are tropical in distribution although there are a few in the north temperate region where it is replaced by numerous species of Cadona, which is not known to extend into the tropics or southern hemisphere.

Subfamily Cyprininæ.

Cyprinotus sp. nov. ?

This species, probably undescribed, related to *C. salinus* occurring in brackish water, was obtained at two quite widely separated localities.

Number	Locality	Temperature	Specific Gravity	pH
3	Goldfield, Nevada	37.3°	1.0020	8.0
89	Indian Springs, Idaho	32.5°	1.0031	8.4

Eucypris calida Blake.

In the overflow from Hot Spring No. 150 on Paoha Island in Mono Lake, California at a temperature of 32.0° in water of specific gravity 1.0045 and pH 8.3.

Eucypris ? sp.

A species, probably of this genus was taken at three localities, all in Nevada, as follows:

Number	Locality	Temperature	Specific Gravity	pH
94	Cherry Creek, Nevada	30.4°	1.0038	8.4
97	Sunnyside, Nevada	30.8°	1.0029	9.3
101	Austin, Nevada	36.2°	1.0038	7.3

Chlamydotheca bruesi Blake.

Collections of this species were made at four localities at temperatures ranging from 31.6° to 42.0°, indicating that it is able to live at high temperatures, although quite probably not restricted to hot springs. The genus is confined to the New World and only one other species, the tropical *C. barbadensis* is known to possess males. Males of *C. bruesi* were present in the material from Battle Mt., Nevada and Cleveland, Idaho.

Number	Locality	Temperature	Specific Gravity	pH
21	Sou Hot Springs, Nevada	31.6°	1.0020	8.1
23	Battle Mt., Nevada	33.4°–42.4°	1.0008	8.0
91	Cleveland, Idaho	42.0°	1.0062	7.5
109	Gerlach, Nevada	36.6°	1.0029	8.5

Herpetocypris chevreuxi, var. **americana** Blake.

This form is a variety of an African species, which Mr. Blake informs me is so near to the typical form from Algeria that he considers it to be only a well-marked variety. We collected it in two localities as follows:

Number	Locality	Temperature	Specific Gravity	pH
97	Sunnyside, Nevada	30.8°	1.0029	9.3
120b	Cedarville, California	28.4°	1.0034	8.2

It will be noted that neither of these temperature records is very high.

Potamocypris Brady.

Potamocypris varicolor Blake.

Two lots of this species were taken at very high temperatures in northern Nevada.

Number	Locality	Temperature	Specific Gravity	pH
103	Cortez, Nevada	44.5°	1.0041	8.5
118	Denio, Nevada	48.3°	1.0030	8.4

Potamocypris perbrunnea Blake.

Number	Locality	Temperature	Specific Gravity	pH
18	Golconda, Nevada	49°–50°	1.008	8.1
20	Winnemucca, Nevada	49°–50°	1.0014	8.6

This species was taken at extremely high temperatures in these two springs which are within a short distance of one another. They were present in immense numbers in the Golconda spring and repeated observations in the shallow water near the margin of the pool showed temperatures of 49°–50°, so that there can be no mistake in the thermometer readings. This is one of the hottest locations in which we have taken living animals and the individuals were all extremely active at the time of our visit which was shortly before noon. It is evident that the preceding species is also highly adapted to heat, although the readings in the case are noticeably lower.

FAMILY DARWINULIDÆ.

Darwinula Brady & Norman.

Darwinula sp.

An undetermined species of this peculiar genus was found at two localities. Aside from a few tropical species, one occurs in the north temperate zone. Our two lots were taken as indicated below.

Number	Locality	Temperature	Specific Gravity	pH
101	Austin, Nevada	36.2°	1.0038	7.3
150	Mono Lake, California	31.9°	1.0045	8.3

There are two references to Ostracoda in thermal springs that are not cited in my former papers. Issel ('06) refers to the occurrence of *Cytheridia torosa* in an Italian hot spring at a temperature of 35°–36° C., and Long early published a note on the presence of a "small bivalve testaceous animal" in a hot spring in Arkansas. The latter is undoubtedly an ostracod and the locality is quite probably the present city of Hot Springs, Ark.

THERMOSBÆNACEA.

THERMOSBÆNIDÆ.

In 1924 Monod ('24) described a most remarkable abberrant Crustacean, Thermosbæna, from a hot spring (El Hamma) in north Africa near Gabes in Tunis. This form he has more recently ('27) placed in a distinct order of Malacostraca. Thermosbæna occurs in water at 45° in the pool fed by this hot spring and Monod supposes that it is a subterranean form which reaches the pool from the depths of the spring which are somewhat hotter (48°). This supposition seems rather rash since no reference is made to the animals actually emerging in the flow of subterranean water, but there seems to be no question that these crustaceans are restricted to the very warm water like Thermacarus among the mites and certain ostracod crustaceans.

ISOPODA.

Aside from the few Isopoda reported in thermal springs mentioned in my former paper, there is another described in 1922 from Australia by Chilton ('22). This is a member of the family Phreatoicidæ, *Phreatoicus latipes*, discovered under peculiar circumstances in the interior desert region near Coward in central Australia. It was first found in great numbers in water from a hot water bore at a temperature not recorded, but evidently quite high as steam was rising from the surface. Later the species was found within a radius of some 30 miles in springs varying greatly in salinity and temperature from some of which it had evidently found its way to the new source although no springs occur within several miles of the bore. The species related to this form occur in surface and underground fresh-water streams.

ACARINA.

In my previous paper ('28), I have given an account of a most remarkable mite collected in several hot springs in Nevada and California during 1927. This species, *Thermacarus nevadensis* Marshall is the second species of a genus known to occur also near Lake Baikal in Siberia where it is represented by the type species, *T. thermobius* Sokolow. Thermacarus is the only known representative of the family Thermacaridæ and appears to be entirely restricted to hot springs of quite high temperature. During 1930 we secured much additional material of this species and collected also a number of other mites. Dr. Ruth Marshall has examined all of this material and has kindly identified the several forms so far as possible. Some species, represented only by immature specimens or by insufficient series are of necessity dealt with only in a preliminary way.

In addition to the species listed beyond, we encountered on several occasions extremely small, red, first stage mites swarming in dense clusters about grass stems or other small objects just above the surface of the water onto which they quickly pass when disturbed, spreading out as minute specks almost invisible to the unaided eye. I am at a loss to associate these with any of the fully developed forms that we collected. They appear to be restricted to water of rather high temperature and may perhaps represent the first stage of Thermacarus, although this is only a surmise.

FAMILY EYLAIDÆ.

One Japanese species of Eylais has been described by Uchida ('27) from a hot spring in Formosa where it was collected in water at 42°. We had consequently hoped to find other representatives of this genus in North American thermal waters. We were disappointed, however, for although we obtained a species of Eylais at three widely separated localities they were nowhere in really warm water (28.2° or lower). In two places (Cleveland, Idaho; hot spring No. 91 and Gerlach, Nevada, hot spring No. 113) the water though cool showed specific gravities of 1.0062 and 1.0007, respectively giving evidence of its thermal origin.

FAMILY PROTZIIDÆ.

Protzia sp. nov.

A species of this genus, according to Dr. Marshall undoubtedly undescribed, and probably one restricted to thermal waters was taken at a number of localities, as follows:

Number	Locality	Temperature	Specific Gravity	pH
57a	Tower Junction, Yellowstone Pk.	44.8°	1.0057	7.0
66	Firehole River, Yellowstone Pk. . .	38.4°	1.0024	8.9
67	Firehole River, Yellowstone Pk. . .	36.2°	1.0044	8.2
75	Three Sisters, Yellowstone Pk. . .	34.0°	1.0046	9.1
78	Biscuit Basin, Yellowstone Pk. . . .	34°–35°	1.0040	9.6+
114	Hausen, California	36.2°	1.0033	7.5
150	Mono Lake, California	32.8°	1.0045	8.3

The field data indicate that this mite withstands rather high temperatures and so far as they extend do not suggest that it inhabits generally cool springs as the lowest temperature recorded is 32.8°. All of the springs contain a considerable amount of dissolved salts as the specific gravity ranges from 1.0033 to 1.0057. The pH covers a wide range also, from 7.0 to more than 9.6. This small genus includes a few species, several of which are represented in Europe. A species, not named, is said by Wolcott to occur in Yellowstone Park. It may be identical with the one here discussed.

FAMILY SPERCHONIDÆ.

Two members of this family are included in our collections. One is the well-known European *Sperchon glandulosus* Koenike which has a very wide distribution, including the United States and Canada. It appears in our collections from two hot springs as follows:

Sperchon glandulosus Koenike.

Number	Locality	Temperature	Specific Gravity	pH
75	Three Sisters, Yellowstone Pk . . .	34.0°	1.0046	9.1
78	Biscuit Basin, Yellowstone Pk . . .	34°–35°	1.0040	9.6

Both springs appear to be closely similar on the basis of specific gravity and pH.

Sperchon, or allied genus.

The other form which belongs to Sperchon or an allied genus was collected at two widely separated localities, also in water of only moderately high temperature as follows:

Number	Locality	Temperature	Specific Gravity	pH
130	Bumpass Hell, Lassen Pk.	35.6°	1.0056	6.8
150	Mono Lake, California	32.8°	1.0045	8.3

FAMILY LIMNESIIDÆ.

Tyrrellia sp.

A species of Tyrrellia, probably undescribed was obtained on Paoha Island in Mono Lake, California at a temperature of 32.8°. This spring which yielded also a variety of other material has specific gravity of 1.0045 and pH of 8.3.

FAMILY THERMACARIDÆ.

Thermacarus nevadensis Marshall.

During 1927 this mite was obtained at four localities and in 1930 particular care was taken to search for it as it is one of the most interesting forms that we have discovered. Eleven additional localities have now been added, greatly extending the range of the species and adding other information. The table given on the opposite page includes also the previous records secured in 1927.

The greater number of localities are in Nevada, but it also extends generally into Yellowstone Park on the north, into Idaho and northwestward to near Denio which lies on the Nevada-Oregon state line. Whether it may occur also in hot springs in Alaska is a matter of surmise, but the genus may be expected in places intermediate between the western United States and Siberia where the only other known species has been found. There seems to be no question that this mite is restricted to thermal springs as none of our temperature records are below 34° C. and the majority are much higher. Also, all specimens seen were in actively flowing springs of large size and those

Number	Locality	Temperature	Specific Gravity	pH
6	Bridgeport, California	34.0°	1.0030	7.5
7	Minden, Nevada	45.0°	1.0000	9.6
25	Deeth, Nevada	43.0°	1.0008	7.4
28	Ruby Valley, Nevada	41.8°	1.0009	9.4
68	Shoshone Basin, Yellowstone Pk.	39.2°	1.0040	9.4
77	Lower Geyser Basin, Yellowst. Pk	39.1°	1.0056	9.5
79	Sprite Pool, Yellowstone Pk.	41.0°	1.0039	8.9
87	Mountain Home, Idaho	42.6°	1.0027	9.6+
92	Cleveland, Idaho	39.1°	1.0067	8.1
100	Antelope Valley, Nevada	40.4°	1.0032	9.1
103	Cortez, Nevada	44.5°	1.0041	8.5
104	Monte Cristo, Nevada	38.3°	1.0052	8.7
108	Gerlach, Nevada	36.3°	1.0061	7.5
118	Denio, Oregon	50.8°	1.0030	8.4
123a	Henderson, California	36.8°	1.0040	7.4

giving evidence of having been present for a long time with little change, as indicated by the deposits that they had produced, present flow, etc. This is what might be expected of a small flightless purely aquatic arthropod unable to persist in water other than that of hot springs as their chances of passing from one spring to another must be very slight. The highest temperature recorded (50.8°) was in the overflow from a very large spring a few miles south of Denio, Oregon, under such circumstances that unusually accurate observations could be made. The stream formed by this spring is in places fifteen feet in width, quite deep and a large volume of water flows rapidly through it. The mites were abundant on the surface of the algæ in the water, many of them more or less inside the thick, matted, infolded masses. Under such circumstances no extensive temperature changes are possible during short periods of time and no rapid movement of the mites to cooler water along the edge of the stream is possible. This temperature therefore, represents a normal environment, where the mites were rapidly swimming or crawling about, and it represents the highest temperature record which we have been able to secure for any of the forms of animal life that we have actually collected. It is certainly well above the lethal temperature for most members of the thermal fauna.

When present, Thermacarus is usually abundant and may be seen crawling about on algæ, in the mud or frequently swimming about in the water with rather slow movements. When undisturbed, it appears to move quite decidedly faster in those springs where the water is at a higher temperature but in the absence of actual measurements no definite statement can be made.

The tolerance of this species for varying salinity and hydrogen ion concentration seems to be very great as it occurs in water of specific gravity from 1.0000 to 1.0061 and pH of from 7.4 to above 9.6. Certainly in the case of Thermacarus, temperature alone is the main factor determining its distribution, as even in winter the size of the springs in which it occurs, precludes any considerable lowering of the temperature of the medium.

INSECTS.

As before, our last series of collections contains representatives of six orders of insects. Each of these is considered in detail on the following pages.

HEMIPTERA.

Members of this order are not very abundant in hot springs, especially in water of rather high temperature. However, certain forms have appeared quite consistently at a series of localities. Our collections contain representatives of five families, as follows: Gerridæ, Notonectidæ, Naucoridæ, Belostomatidæ and Corixidæ.

GERRIDÆ.

Two species of Gerris, *G. remigis* Say and *G. gillettei* Leth. and Serv. and a single series of nymphs which cannot be determined specifically. They come from several springs (Nos. 69, 100, 123a, 131) always at very moderate temperatures, ranging as high as 34.2 in only one case. These records agree well with our previous observations and show that although purely surface species they exhibit no tendency whatever to enter the thermal environment.

NOTONECTIDÆ.

Two species of Notonectidæ obtained by us previously in hot springs were obtained again at several stations, so that we now have quite an extensive series of records.

Notonecta unifasciata Guérin (*N. indica* Linn.).

Number	Locality	Temperature	Specific Gravity	pH
3	Goldfield, Nevada	35.4°	1.0020	8.0
20	Winnemucca, Nevada	36.3°	1.0014	8.6
21	Osobb Valley, Nevada	31.6°	1.0021	8.1
23	Battle Mt., Nevada	30.0°	1.0008	8.0
28	Ruby Valley, Nevada	33.0°	1.0009	9.4
86	White Arrow Sanatarium, Nev.	30.0°	1.0040	8.6
97	Sunnyside, Nevada	30.8°	1.0029	9.3
120b	Cedarville, Cal.	28.4°	1.0034	8.2
121	Canby, Cal.	38.5°	1.0032	8.4
122	Bieber, Cal.	39.5°	1.0037	8.4

The above table which summarizes all our collections of two seasons shows quite conclusively that this Notonecta and apparently *N. kirbyi* Hungerford also, which we collected for a second time at Sunnyside, Nevada (hot spring No. 97) does not find the thermal environment very congenial. Practically all the specimens were taken in only moderately warm water, and none were seen in water at 40° or above. It is evident also, as I had surmised on the basis of our earlier observations, that Notonecta is not tolerant of a high degree of salinity since we have found it restricted approximately to the lower half of the salinity-range.

NAUCORIDÆ.

A considerable series of specimens belonging to two species of Ambrysus were taken at several localities, serving to enlarge considerably records of the occurrence of these waterbugs in thermal springs.

Ambrysus heidemanni Montd.

Adults of *Ambrysus heidemanni* Montd. were found at three places, one of which was exactly the same station at which we first discovered it in Yellowstone Park seven years previously, indicating that it is certainly a regular inhabitant of this spring.

Number	Locality	Temperature	Specific Gravity	pH
81	Tangle Creek, Yellowstone Park	32.8°	1.0037	8.4
82	Ink Well, Yellowstone Park	30.9°	1.0038	9.6+
97	Sunnyside, Nevada	30.8°	1.0029	9.3
6	Bridgeport, Cal.	34.0°	1.0030	7.5
(1923)	Tangle Creek, Yellowstone Park	35.5°	—	—

Ambrysus pulchellus (?) Mont.

Taken at one locality, Sunnyside, Nevada in water at 33.6°, specific gravity 1.0043, pH 8.3.

Ambrysus sp.

Nymphs, most probably of *A. heidemanni* were taken in three other springs, at the following places:

Number	Locality	Temperature	Specific Gravity	pH
71	Shoshone Basin, Yellowstone Park	27.8°	1.0046	8.8
77	Lower Geyser Basin, Yel'stone Pk	38.6°	1.0056	9.5
99	Duckwater, Nev.	29.0°	1.0032	8.0
105	Pyramid Lake, Nevada	32.0°	1.0064	8.0

Arctocorixa omani Hungerford

Number	Locality	Temperature	Specific Gravity	pH
69	Shoshone Basin, Yellowstone Park	31.2°	1.0036	8.2

Trichocorixa sp.

Number	Locality	Temperature	Specific Gravity	pH
147	*Fish Springs, Cal.*	32.2°	1.0058	7.4

Corixid nymph

Number	Locality	Temperature	Specific Gravity	pH
150	Paoha Isl., Mono Lake...........	31.9°	1.0045	8.3

ODONATA.

A very considerable number of dragon-fly and damsel-fly nymphs were obtained in 1930, representing a number of species that we had not taken previously as well as additional material of forms already contained in our 1927 series.

Nine genera and numerous species are included, but in most cases from a single or from only a few localities. A notable exception is the widespread *Mesothemis simplicicollis* now known from thermal water at ten places. None of the forms appear to be especially adapted to thermal life as the temperature records are usually rather low (rarely over 40°) and there is a wide range in the salinity of the several springs, never in excess of 1.0061 however. No nymphs were taken in any of the extremely alkaline nor except in a single instance in highly acid springs as the other pH records range from 7.4 to 9.3.

I have appended below the data upon which the foregoing remarks have been based, as well as some of the more interesting details relating to the various species.

FAMILY CŒNAGRIIDÆ.

Argia Rambur.

There are ten series of nymphs belonging to this genus, probably representing more than one species, but I have not been able to place them with certainty as belonging to any species of which the nymphs have been described. They are included as a single group on page 232.

An unidentified species of Argia has been reported from Las Vegas, New Mexico by Needham and Cockerell ('03) in warm streams running off from the hot springs at that place, and we have one species, probably an Argia from a highly acid spring, No. 59 in Yellowstone Park (specific gravity 1.0026, pH 3.6 –).

Number	Locality	Temperature	Specific Gravity	pH
51	Mary Bay, Yellowstone Park	38.5°	1.0038	—
89	Indian Springs, Idaho	32.5°	1.0031	8.4
97	Sunnyside, Nev.	30.8°	1.0029	9.3
102	Darrough's Hot Springs, Cal.	36.5°	1.0025	9.0
103	Cortez, Nevada	40.8°	1.0041	8.5
109	Gerlach, Nevada	35.0°	1.0029	8.5
115	Surprise Valley, Nevada	33.9°	1.0027	8.3
123a	Henderson, California	37.6°	1.0040	7.4
143	Paraiso Hot Springs, California	33.4°	1.0042	7.4
149	Democrat Springs, California	31.4°	1.0047	8.7

Chromagrion conditum Hagen.

A single nymph which agrees well with Garman's description of this species ('27) was taken at hot spring No. 77 in the Lower Geyser Basin in Yellowstone Park at a temperature of 38.6°. This is a strongly alkaline spring (pH 9.5) having a specific gravity of 1.0056. The species does not appear to have been recorded from so far west hitherto.

Ischnura Charpentier.

One species of Ischnura is represented by nymphs in our collections.

Ischnura perparva ? Selys.

Nymphs which appear to belong to this widespread Western species were collected at four localities. Needham and Cockerell ('03) have reported the species as breeding in "alkali" water.

Number	Locality	Temperature	Specific Gravity	pH
—	Squaw Lake, Yellowstone Park	Cold		
104	Monte Cristo, Nevada	38.3°	1.0052	8.7
108	Gerlach, Nevada	36.3°	1.0061	7.5
109	Gerlach, Nevada	35.0°	1.0029	8.5

FAMILY GOMPHIDÆ.

Ophiogomphus or **Herpetogomphus.**

One large nymph belonging to one or the other of these genera was obtained at Lava Hot Springs, Idaho (hot spring No. 90) along the edge of the stream into which the springs enter, at a temperature between 33° and 40°. This is the first record known to me of any species of Gomphidæ in a hot spring.

FAMILY LIBELLULIDÆ.

Libellula Linné.

Nymphs of Libellula were obtained at only four localities. One lot contains only a single small individual, but the others are full grown and I have been able to identify them without much question as two species. The first, taken at Cortez, Nevada in hot spring No. 103 together with nymphs of *Mesothemis simplicicollis*, is *Libellula forensis* Hagen at a temperature of 40.8°. The second, *Libellula saturata* Hagen, is more interesting as it has been previously found breeding in hot springs. I have already mentioned (Brues '24) its occurrence in Yellowstone Park on the authority of Needham ('04). The latter also records this species from White's Warm Springs, Idaho and we have now collected it in hot springs in both Nevada and California. Our data are as follows:

Number	Locality	Temperature	Specific Gravity	pH
97	Sunnyside, Nevada...............	30.8°	1.0029	9.3
143	Paraiso Hot Springs, Cal.........	33.4°	1.0042	7.4

The temperatures are not high in any case and this species appears to be one that has extended its range into cooler regions by breeding in thermal water.

Plathemis subornata Hagen.

Nymphs which seem undoubtedly to be this species were taken at Henderson, California (hot spring No. 123a) at 37.6°, the water having a density of 1.0040 and pH of 7.4. The nymph of this species was first described as "Dythemis sp?" by Needham and Cockerell from Dimmit Lake, an alkaline pond near Roswell, New Mexico.

Orthemis ferruginea Fabr. ?

A series of nymphs from Antelope Valley, Nevada (hot spring No. 100) in water of 35.8°, specific gravity 1.0032 and pH 9.1. These agree well with Needham's description ('04), except that there is only one mental seta on each side, whereas he figures a group of four and several adjacent minute ones. My identification is therefore quite possibly incorrect.

Mesothemis simplicicollis Say.

This is the most abundant dragon-fly which we have found in the thermal springs visited. In 1927 it was obtained at three places and we are now able to add seven more localities. These are as follows:

Number	Locality	Tempera-ture	Specific Gravity	pH
1	Hot Springs, New Mexico	39.5°	1.0039	8.1
5	Convict Lake, California	43.0°	1.0012	8.5
15	Denio, Oregon	35.2°	1.0000	8.4
44	Lewis Lake, Yellowstone Park	30.8°	1.0023	7.8
100	Antelope Valley, Nevada	35.8°	1.0032	9.1
103	Cortez, Nevada	40.8°	1.0041	8.5
104	Monte Cristo, Nevada	38.3°	1.0052	8.7
109	Gerlach, Nevada	35.0°	1.0029	8.5
114	Hausen, California	36.2°	1.0033	7.5
116	Surprise Valley, Nevada	40.2°	1.0035	7.6

Pantala flavescens Fabr.

A nymph of this species was taken at Hot Springs, New Mexico in 1927, but I failed through inadvertence to include the record in my previous account, and am now able to add another locality.

Number	Locality	Tempera-ture	Specific Gravity	pH
1	Hot Springs, New Mexico	39.5°	1.0039	8.1
69	Shoshone Basin, Yellowstone Park	31.7°	1.0036	8.2

PLECTOPTERA.

A comparatively small amount of material representing this order is contained in the collections obtained during 1930. It shows quite conclusively that mayflies cannot be considered as forming a part of the true thermal fauna since the temperatures recorded are consistently low, although there is a sufficient number of records for one to expect higher records if these animals actually do commonly invade water of rather high temperature.

Our collections are listed in detail below simply to demonstrate this fact. In 1927 we obtained mayfly nymphs in two springs in Nevada, also at low temperatures (31.6°–36.0°).

Callibaëtis sp.

Nymphs of a species of this genus were obtained at several localities, but always at comparatively low temperatures so that they give no indication of any tendency for these insects to adapt themselves to a really thermal environment. In addition to those listed below two collections were made in cool water, one in Squaw Lake, Yellowstone Park and in a cool pool, probably of thermal origin (sp. gr. 1.0007 and pH 8.8) near Gerlach, Nevada.

Number	Locality	Temperature	Specific Gravity	pH
97	Sunnyside, Nevada	30.8°	1.0029	9.3
123a	Henderson, California	35.0°	1.0040	7.4
140	Paraiso Hot Springs, California	33.4°	1.0042	7.4

Baëtis.

Two series of nymphs of this genus were obtained, one in cool water in Indian Pond, Yellowstone Park in water practically without salinity (sp. gr. 1.0001) and a second lot at Indian Springs, Idaho in water at 32.8°. The latter is a true thermal spring with water of a specific gravity of 1.0031 and pH of 8.4, but the temperature is by no means high.

Leptophlebia sp.

A single series of small nymphs of a species of Leptophlebia were obtained in hot spring No. 90 at Lava Hot Springs, Idaho at a temperature of about 33° C. where the small springs flow directly into the creek. As there is rapid mixture of the water from the springs and that of the river the density and temperature of the spots where they were living cannot be accurately determined.

An entirely different type of mayfly nymph was taken together with the Leptophlebia mentioned above at Lava Hot Springs, Idaho (hot spring No. 90). I have been unable to identify this form, which has lateral, plate-like gills, each plate furnished below with tufts of short filamentous projections.

TRICHOPTERA.

In spite of the general abundance of caddis worms in the several types of fresh water, few records exist of their occurrence in thermal waters. We had hoped on our last trip to secure at least a few additional records, but met with very poor success. Limnephilid larvæ were found on only three occasions, first in spring No. 47b (Indian Pond, Yellowstone Park) in water of low temperature that is practically non-saline; second in spring No. 65 (Firehole River, Yellowstone Park) in water at 32.4° and pH 8.3; third in spring No. 98 (Sunnyside, Nevada) in water of 1.0042, pH 8.3 at 33.6°. There appear to be three species represented.

I have also a few Limnephilids of still another species given me by Mr. S. W. Denton which he collected in a hot spring west of Reno, Nevada in water of unrecorded temperature.

There can be no question that Trichoptera form a very small and unimportant part of the thermal fauna.

Although numerous Trichoptera have been collected in brackish water, with the exception of some that I have cited in my previous account, most of these occur in water of moderate salinity. Thus in his extensive lists of Finnish caddis worms from brackish water, Siltala ('06) finds many species in water with about one-half percent salt. This is approximately the salinity of many of the hot springs which we have investigated. On this basis we might expect caddis flies to be more abundant than they actually are and we must believe that temperature is the limiting factor in so greatly restricting the trichopterous fauna of thermal waters.

DIPTERA.

This order is abundantly represented in the thermal fauna by aquatic larvæ belonging to several families, more especially the Chironomidæ, Culicidæ, Stratiomyiidæ, Tabanidæ and Ephydridæ. Of these certain chironomids and stratiomyiids frequently occur in water of especially high temperature, while certain ephydrids, as is well known, are able to develop in water of very great salinity. With the exception of a very few forms that could not be placed with any certainty, all of our collections are included in the detailed account that follows.

CHIRONOMIDÆ.

A very considerable series of larvæ belonging to this family were obtained during the course of the summer, represented by nearly sixty lots. As before, we found them to be one of the very characteristic groups of the thermal fauna, represented by eight species belonging to four genera. These, together with our previous records include twelve species belonging to five genera.

They are as follows:

Tanypus monilis Linn?
Tanypus sp.
Chironomus tentans Fabr.
Chironomus sp. No. 1.
Chironomus sp. No. 2.
Chironomus sp. No. 3.
Chironomus sp. No. 4.
Orthocladius sp.
Orthocladius sp. No. 2.
Cricotopus fasciatus Panzer.
Cricotopus sp.
Tanytarsus exiguus Johannsen.

Fully one-third of the springs where we were able to make collections yielded specimens, usually in considerable abundance, although since most of the forms live in mud they are to be found frequently only on rather close examination. As will be seen from the data on the following pages, several species, notably *Chironomus tentans*, *Chironomus* sp. No. 2 and *Chironomus* sp. No. 4 occur commonly at temperatures of 40° or above, although so far as can be stated at present none appear to be restricted to thermal waters.

Of two of these forms, *Chironomus tentans* and *Chironomus* sp. No. 4 we were fortunate to secure a long series of records and these show clearly that in the case of these species at least, the hydrogen ion concentration of the water seems to play an important rôle in determining their occurrence in hot springs. Thus the first is typically an inhabitant of acid waters and the second of alkaline ones. None of the records include any springs of very high salinity, *i e.* above 1.0062, in spite of the fact that some other members of this family occur in strongly brackish water or in the sea.

One striking feature of the present list is the recurrence of species which we took previously at other localities, a fact which indicates a greatly limited thermal fauna, notwithstanding the large extent of the family Chironomidæ.

Chironomus tentans Fabr.

This species which has been provisionally identified and referred to in my two previous publications ('24; '28) was taken again at eleven localities, all of them in Yellowstone Park. In each case the larvæ were taken in acid water (pH 3.6 – to 7.9) and we have a previous record of pH 8.8. Thus, although there is considerable range, it is very clear that the species occurs generally in acid waters, whereas a species such as *Chironomus* sp. No. 4 prefers alkaline water ranging from pH 7.1 to 9.6. These two species show without question that the reaction of the water is an important factor in determining their distribution.

Number	Locality	Temperature	Specific Gravity	pH
38	West Thumb, Yellowstone Park ..	34.2°	1.0025	6.8
39	West Thumb, Yellowstone Park ..	36.8°	1.0039	6.3
40	West Thumb, Yellowstone Park ..	37.0°	1.0058	7.3
45	Norris Geyser Basin, Yell. Pk....	44.8°	1.0040	7.9
49	Beach Springs, Yellowstone Park .	39.5°	1.0051	7.2
58	Whirligig Geyser, Yellowstone Pk.	38.2°	1.0051	3.6–
59	Near Minute Man, Yell. Pk......	25.4°	1.0026	3.6
63	Amphitheatre Springs, Yell. Pk. ..	30.6°	1.0043	3.6–
72	Shoshone Geyser Basin, Yell. Pk..	35.8°	1.0062	7.7
73	Shoshone Geyser Basin, Yell. Pk..	40.1°	1.0058	3.6
74	Shoshone Geyser Basin, Yell. Pk..	35.3°	1.0047	6.4

Chironomus sp. No. 2.

This species which we took previously ('28) in Lassen National Park in hot spring No. 12 reappeared in our last series of collections from five localities in Yellowstone Park and one in Mt. Lassen Park.

Most of the larvæ were living in water of slightly or highly alkaline reaction, as can be seen from the following list of stations.

Number	Locality	Temperature	Specific Gravity	pH
38	West Thumb, Yellowstone Park ..	34.2°	1.0025	6.8
81	Tangle Creek, Yellowstone Park..	30.0°	1.0037	8.4
82	Ink Well, Yellowstone Park......	30.0°	1.0038	9.6+
83	Ink Well, Yellowstone Park......	36.6°	1.0039	9.1
84	Near Spasmodic Geyser, Yell. Pk.	41.0°	1.0032	9.3
126	Mount Lassen Park..............	22.2°	1.0025	7.1

Chironomus sp. No. 4.

This species which we took at a series of localities in 1927 was again well represented in our collections made in 1930. The very characteristic shape and dentition of the labium make it easily recognizable and there can be no doubt that all the larvæ belong to a single species, although there is some slight variation in the form of the pale colored median teeth; these are sometimes less acute than in the specimen figured (Brues '28; fig. 4, no. 12).

The following list includes twelve additional records for this species:

Number	Locality	Temperature	Specific Gravity	pH
77	Lower Geyser Basin, Yell. Park...	38.6°	1.0056	9.5
78	Biscuit Basin, Yellowstone Park..	35.6°	1.0040	9.6+
95	Cherry Creek, Nevada...........	30.4°	1.0043	8.4
100	Antelope Valley, Nevada.........	40.4°	1.0032	9.1
102	Darroughs Hot Springs, Nevada..	40.8°	1.0025	9.0
103	Cortez, Nevada..................	40.8°	1.0041	8.5
104	Monte Cristo, Nevada...........	38.3°	1.0052	8.7
114	Hausen, California..............	36.2°	1.0033	7.5
116	Surprise Valley, Nevada.........	40.2°	1.0035	7.6
123a	Henderson, California...........	35.0°	1.0040	7.4
147	Fish Springs, California..........	32.2°	1.0060	7.7
150	Paoha Island, Mono Lake, Cal....	38.3°	1.0045	8.3

Together with eight localities listed in my previous paper it is clear that the larvæ of this form occur very generally in water of high temperature as there are eight records from water above 40°, ranging as high as 43.0°. The range with reference to hydrogen ion concentration is mainly in the alkaline side, from 7.1 to 9.6, but only two records are below 7.5.

Two lots of larvæ from hot spring No. 87 at Mountain Home, Idaho (pH 9.6 +; sp. gr. 1.0027) and No. 114 at Hausen, California (pH 7.5; sp. gr. 1.0033) are similar to the above species, but differ in the form of the labium which has the teeth more widely spaced in the central group of four and the small lateral tooth more clearly separated. They may represent another species.

Tanytarsus exiguus Johannsen.

One series of larvæ seem to belong to this species which is already known from the eastern United States and from Idaho. They were collected in hot spring No. 97 at Sunnyside, Nevada, in water of 1.0029, pH 9.3 at a temperature of 30.8°.

Cricotopus trifasciatus Panzer.

Numerous larvæ which agree with the description and figure of this species were taken in hot spring No. 66 near the bank of the Firehole River in Yellowstone Park. This is a widespread holarctic form whose larva has been described by Malloch ('15). The labium has a rounded median tooth, a bifid lateral tooth and a gradually decreasing series beyond. The specimens were in water at 38.4°, pH 8.9; specific gravity 1.0023.

Cricotopus sp.

This is the same species that we collected in 1927 at Beowawe, Nevada in highly alkaline water having a pH of 9.6 or over. It appeared at four other localities in Yellowstone Park in water of moderately or strongly alkaline reaction, but always at low or moderately low temperatures.

Number	Locality	Temperature	Specific Gravity	pH
43	West Thumb, Yellowstone Park	23.2°	1.0023	9.0
72	Shoshone Geyser Basin, Yell. Pk.	35.4°	1.0062	7.7
75	Three Sisters, Yellowstone Park	37.2°	1.0046	9.1
83	Ink Well, Yellowstone Park	36.6°	1.0039	9.1

Orthocladius sp.

This form was found in one of our previous collections from Steamboat Springs, Nevada (hot spring No. 8) and we collected it again in 1930 at two localities as follows:

Number	Locality	Temperature	Specific Gravity	pH
89	Indian Springs, Idaho	32.5°	1.0031	8.4
121	Kelly Hot Springs, Canby, Calif.	38.5°	1.0032	8.4

Orthocladius sp. No. 2.

A series of larvæ taken in hot spring No. 80, Firehole Pool, Yellowstone Park represent a species quite similar to one figured by Malloch ('15, plate 29, fig. 17) as Orthocladius, sp. E. The labium is more pointed medially so that the minute lateral teeth set further back on the sides and it may represent another species; it is quite clearly different from the Orthocladius recorded above and in my previous paper ('28, p. 189). The larvæ were at a temperature of 34°, sp. gr. 1.0046, pH 8.2.

DIXIDÆ.

The larva of a species of Dixa was obtained in Indian Pond, Yellowstone Park in cool water. However, this is not truly thermal water as the specific gravity is very low, only 1.0001 in spite of numerous small thermal springs that surround its shores. We found it in none of the truly thermal, warm, saline waters and may probably regard the group as not adapted to such an environment.

CULICIDÆ.

In the present collections four species are represented by fifteen lots of specimens collected in twelve different springs. None of these were found at very high temperatures, and with the data that we have secured in previous years in this region, the statement may be made that mosquitoes do not readily adapt themselves to life in hot springs.

Culex tarsalis Coquillett.

This widespread species seems to be the commonest mosquito found breeding in thermal waters in the region investigated, as we obtained larvæ on nine occasions. Seven other records are contained in my previous paper ('28) so that we now have a fairly accurate notion concerning the requirements of this species. As the series is so complete, I have combined the present data with those relating to the other springs in the same region.

Number	Locality	Temperature	Specific Gravity	pH
1	Hot Springs, New Mexico	39.0°	1.0039	8.1
5	Convict Lake, Calif.	35.2°	1.0012	8.8
7	Minden, Nevada	30.0°	1.0000	8.8
23	Battle Mountain, Nevada	36.0°	1.0008	8.0
23a	Battle Mountain, Nevada	36.0°	1.0021	8.5
26	Wells, Nevada	35.2°	1.0018	7.9
28	Ruby Valley, Nevada	35.6°	1.0009	9.4
37	West Thumb, Yellowstone Park	Cool	1.0056	8.9
38	West Thumb, Yellowstone Park	34.2°	1.0025	6.8
61	Soda Spring, Yellowstone Park	29.0°	1.0048	6.9
72	Shoshone Basin, Yellowstone Park	35.4°	1.0062	7.7
113	Soldier Meadow, Nevada	Cold	1.0007	8.8
121	Canby, California	38.5°	1.0032	8.4
138	Wilbur Hot Springs, California	28.0°	1.0081	8.5
143	Paraiso Hot Springs, California	33.4°	1.0042	7.4
145	Jacumba Hot Springs, California	36.0°	1.0041	7.4

Although usually in quite alkaline water, the larvæ occur at a lower pH than my previous records showed as one pool at West Thumb in Yellowstone Park contained larvæ thriving at pH 6.8. They show also great tolerance with respect to dissolved salts as the specific gravity of the water in the several springs ranges from 1.0000 to 1.0081 as determined in the field.

Aëdes pullatus Coq.

Larvæ of this species were collected in highly acid water with a pH of 3.6 or less near the eastern shore of Turbid Lake in Yellowstone Park, in company with larvæ of Ephydra. This is ordinarily a snow-water form, but in the present instance although the water was cold at the place the larvæ were taken its specific gravity was 1.0025.

Culicella inornata Williston.

The stout-bodied larvæ of this species were encountered at three stations, all in California.

Number	Locality	Temperature	Specific Gravity	pH
129	Bumpass Hell, Lassen, N. P.	27.6°	1.0029	6.6
138	Wilbur Hot Springs, California	28.4°	1.0081	8.5
143	Paraiso Hot Springs, Cal.	33.4°	1.0042	7.4

As there seems to be some variation in the insertion of the tufts of the anal brush there may be a possible error in the determination of this species, since some of the specimens in two of the lots will run to *C. maccrackenæ* in Dyar's table on the basis of this character. I am reasonably certain, however, that the larvæ are all *C. inornata*. The occurrence of this species in the overflow from the Wilbur Hot Springs shows that it will endure a considerable concentration of salts, which agrees well with the data cited in my previous paper ('28, p. 185) concerning the related *C. incidens* Thomson.

Anopheles pseudopunctipennis Theobald.

This malarial mosquito was collected at two places:

Number	Locality	Temperature	Specific Gravity	pH
143	Paraiso Hot Springs, Cal.	33.4°	1.0041	7.4
147	Fish Springs, Cal.	33.2°	1.0060	7.7

These records are of considerable interest as this species of Anopheles seems to be the only one that has been found in thermal waters. As cited in my earlier paper ('24, p. 394) it has been twice taken in truly thermal water, once in Peru and again in the Las Vegas hot springs in New Mexico. As this species is a common carrier of malaria its presence in the Paraiso Hot Springs might easily interfere with the advertised efficacy of the water as a cure for malaria.

SIMULIIDÆ.

We have collected only a single lot of larvæ belonging to this family during the course of our work and I have found no published records of them from hot springs. However, since my previous paper was published, my friend Mr. S. W. Denton visited a number of hot springs in the west and brought back some insects. Among these is a single larva of Simulium from Lawton hot spring, near Ashland, Oregon.

We found larvæ of a species *Simulium* in very large numbers attached to stones in the overflow from hot spring No. 135 near the "Sulphur Works" in Lassen National Park at a temperature of 30.6° C. This water has a specific gravity of 1.0034 and pH of 7.4. The larva collected by Mr. Denton belongs to the same species as nearly as I can tell, but I have been unable to identify them. The larvæ agree well with Malloch's description (Malloch '14) of *S. vittatum* Zett. and I had provisionally regarded them as this species, but the discovery of a single pupa in the lot makes this determination doubtful unless the pupa belongs to a different species, which does not seem probable. The pupa has only 10 branches to the respiratory filaments, whereas *vittatum* has sixteen. *S. jenningsi* Malloch has ten, but my larvæ do not agree with this species. It seems probable therefore that they belong to some species of which the preparatory stages have not yet been described.

STRATIOMYIIDÆ.

Forty-three lots of larvæ of Stratiomyiidæ are contained in the present collections, mainly species of Stratiomyia and Odontomyia. Members of this family develop under the most diverse situations almost always in rather acid waters of pH 8.5 or below, although the range for the individual species is usually rather wide. In one very striking case however, larvæ of a peculiar species of Stratiomyia with unusually long anal segments were taken only in extremely acid springs. This species (*Stratiomyia* No. 6) was collected in four springs, all having a pH of less than 3.6.

Without question the preference for generally acid springs is related to the need for calcium carbonate which is regularly deposited in the integument of the larvæ. Johannsen has pointed out that these larvæ always bear considerable amounts of this substance in the integument regardless of whence they may have come, and notes that it may be removed by immersing the larvæ in dilute hydrochloric acid for a few moments. Following this procedure in the preparation of specimens for study, I have noted no cases among the hot springs material in which the effervescence failed to occur.

Much of the present material represents species found in our collections of previous years, but there are several additional species and a much greater series of some of the forms, as will be evident from the following more detailed statement.

Stratiomyia sp. No. 1.

This seems to be the most abundant species in the hot springs we have examined as there are eight localities represented in the 1930 collections. These together with three previous records show the following distribution.

Number	Locality	Temperature	Specific Gravity	pH
5	Convict Lake, California	35°	1.0012	8.5
11	Bumpass Hell, Lassen	30°–40°	1.0016	5.7–6.5
13	Drakesbad, Lassen	28°	1.0021	6.7
46b	Bijah Spring, Yellowstone Park	34.3°	1.0036	6.5
57a	Tower Junction, Yellowstone Park	36.5°	1.0057	7.0
91	Cleveland, Idaho	39.1°	1.0062	7.5
103	Cortez, Nevada	40.8°	1.0041	8.5
—	Cortez, Nevada	Cold	—	—
113	Gerlach, Nevada	Cold	1.0007	8.8
116	Surprise Valley, Nevada	40.2°	1.0035	7.6
123a	Henderson, California	36.0°	1.0040	7.4
135	Sulphur Works, Lassen	32.6°	1.0034	7.4

There is some variation in these larvæ, but I cannot separate them into groups and think all must represent one species. It will be noted that there is a well-marked tendency for this form to occur in more acid springs than most other Stratiomyiidæ. Also it was not taken at very high temperatures and was found in two cold springs as well.

Stratiomyia sp. No. 3. (Fig. 2, A).

A rather large, heavy-bodied larva with the body segments very strongly constricted off from one another, especially on the posterior half of the body. Body above not strongly pigmented, except the puparia, which are almost black. All except the first and last body segments with five narrow longitudinal dorsal blackish stripes. These are most conspicuous on the anterior part of the first three segments and on the posterior part of the last three (exclusive of terminal segments) segments since at these places the space between the stripes is strongly decolored. Body hairs weak and pale-colored. Sides of body segments strongly bulging between the sutures which thus appear to be much more constricted than usual even in small specimens. Terminal segment (Fig. A) twice as long as wide at base, the anal slit placed near the base; ventral surface with a curved row of small setæ near base and four arranged in a quadrilateral beyond the middle.

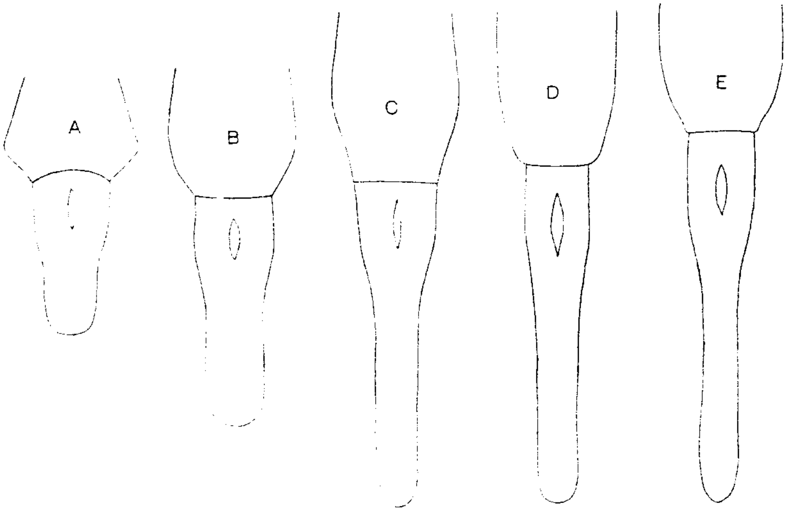

FIGURE 2. Outlines of apices of abdomen in ventral view of larvæ of several species of Stratiomyia. A, Stratiomyia, sp. No. 3; B, Stratiomyia sp. no. 4; C, Stratiomyia, sp. No. 5; D and E, Stratiomyia sp. No. 6.

This species was not obtained in 1927, but appears in the material from four widely separated localities in the later collections. These are as follows.

Number	Locality	Temperature	Specific Gravity	pH
91	Cleveland, Idaho	38.0°	1.0062	7.5
128	Bumpass Hell, Lassen	34.2°	1.0028	6.7
140	Siegler Hot Spring, Calif.	42.8°	1.0057	7.3
143	Paraiso Hot Spring, Calif.	33.4°	1.0042	7.4

The specimens vary considerably in size, the puparia measuring from 25–30 mm. in length.

Stratiomyia sp. No. 4. (Fig. 2, B).

A rather long slender species with elongate terminal segment. Body segments separated by moderately deep constrictions that are more conspicuous in fully matured specimens, especially behind. Body very dark above, the five dorsal longitudinal dark bands practically continuous along the entire length of the body. Surface quite strongly pale hairy, each segment at the lateral edge bearing two stouter setæ. Terminal segment (Fig. 2, B) slightly less than three times as long as wide at base; with a curved row of four setæ near base, one at each side of the slit near middle, one at middle of lateral margin, four minute ones in a quadrilateral just behind the middle. Mature length about 25–30 mm.

This species is similar to the one designated as No. 2 and may be the same, but the body is very conspicuously striped in all of five specimens. They come from hot spring No. 128 in Bumpass Hell, Mt. Lassen National Park in water of a density of 1.0028 and pH of 6.7, at a temperature of 34.2°.

Stratiomyia sp. No. 5. (Fig. 2, C).

A large, pale species with long, tapering terminal segment. Entire length 35 mm.; terminal segment 6 mm. Constrictions between body segments weak anteriorly; more noticeable, but not deep behind. Pigmentation weak, the dorsal longitudinal bands more conspicuous on the segments behind where the body color between them is almost white; almost obliterated on the anterior part of the segments where the body color is darker, the median band, however, practically continuous. Body above rather sparsely hairy, the hairs on the lateral margin unusually dense; no larger setæ present however on the lateral margins. Terminal segment (Fig. C) long and slender, slightly more than four times as long as wide at base, its bristles very weak.

Two specimens from hot spring No. 128 in Bumpass Hell, Mt. Lassen National Park, California taken in company with the previous species in water of a density of 1.0028, pH 6.7 and temperature of 34.2°. It is at once distinguished from all the larvæ of Stratiomyia we have collected, except the one following (No. 6) by its very long terminal segment.

Stratiomyia sp. No. 6. (Fig. 2, D, E).

A large species, the mature larvæ from 32–40 mm. in length with extremely long slender terminal segment. Color usually very dark above, the median dorsal stripe quite distinct, but the lateral ones clearly indicated only near the sutures by paler spots which lie between the stripes. In specimens that appear to be entirely mature they have the color more uniform with the stripes practically absent. Dorsal surface more or less white pilose, the lateral margins without any conspicuous enlarged setæ. Terminal segment extremely long, measuring from 10 to 12 mm. in length and about 5 or 6 times as long as wide (Fig. D, E).

This species obtained only in Yellowstone Park, in four extremely acid hot springs where the pH is below 3.6, the lower limit at which we could make colorimetric determinations. It is no doubt restricted to such highly acid water. I have also several larvæ which seem to be the same, taken in a hot spring at Banff, Canada by Mr. D. J. Zinn.

The localities are as follows:

Number	Locality	Temperature	Specific Gravity	pH
53	Turbid Lake, Yellowstone Park ..	37.8°	1.0034	3.6–
58	Whirligig Geyser, Yellowstone....	38.2°	1.0051	3.6–
59	Near Minute Man, Yellowstone ..	25.4°	1.0026	3.6–
63	Amphitheatre Springs, Yellowstone	39.4°	1.0043	3.6

Odontomyia sp. No. 1.

This species was originally taken by us at Hot Springs, New Mexico, but there is in the present collection another specimen apparently the same from Soda Spring (hot spring No. 61) in Yellowstone Park. Considering the wide difference in location however, I hesitate to consider them as identical. A third locality is also listed below.

Number	Locality	Temperature	Specific Gravity	pH
1	Hot Spring, New Mexico.........	38.7°	1.0039	8.1
61	Soda Spring, Yellowstone........	29.0°	1.0048	6.9
101	Austin, Nevada................	36.2°	1.0038	7.3

Odontomyia sp. No. 2.

Larvæ of this undetermined species were taken at three localities during 1927 and five additional lots were secured in 1930. The species is not at all restricted in occurrences either in regard to temperature or specific gravity.

Number	Locality	Temperature	Specific Gravity	pH
4	Silver Peak, Nevada	39.6°–46.7°	1.0212	8.3
10	Brokeoff, Mt. Lassen	43°–47°	1.0014	7.3
22	Battle Mt., Nevada	38°	1.0014	7.8
101	Austin, Nevada	36.2°	1.0038	7.3
103	Cortez, Nevada	44.5°	1.0041	8.5
113	Gerlach, Nevada	Cold	1.0007	8.8
123a	Henderson, California	37.6°	1.0043	7.4
135	Sulphur Works, Lassen	32.6°	1.0034	7.4

All the records are from springs with moderately high hydrogen ion concentration.

Odontomyia sp. No. 3.

We encountered this species only once, and then not alive in a hot spring near Golconda, Nevada in 1927, but last year it appeared in our collection from three other localities.

Number	Locality	Temperature	Specific Gravity	pH
18	Golconda, Nevada	Dead specimens	1.0008	8.1
57a	Tower Junction, Yellowstone	36.5°	1.0057	7.0
91	Cleveland, Idaho	39.1°	1.0062	7.5
94	Cherry Creek, Nevada	30.4°	1.0038	8.4
103	Darrough's Hot Spring, Nevada	36.5°	1.0025	9.0

Odontomyia sp. No. 4.

This form we took previously only twice in our 1927 collections.

Number	Locality	Temperature	Specific Gravity	pH
57a	Tower Junction, Yellowstone	36.5°	1.0057	7.0
61	Soda Spring, Yellowstone	39.2°	1.0048	6.9
103	Cortez, Nevada	44.5°	1.0041	8.5
123a	Henderson, California	37.6°	1.0043	7.4
147	Fish Springs, California	32.2°	1.0058	7.7
	Unrecorded locality	40.8°	—	—

The three records of unusually high temperatures indicate a great tolerance to heat.

Odontomyia sp. No. 5.

Two lots of larvæ are apparently this species, although most of the specimens are considerably larger than the original ones from Mount Lassen Park. All are from water of quite or very high temperature.

Number	Locality	Temperature	Specific Gravity	pH
10	Brokeoff Mountain, Mt. Lassen	47°	1.0014	7.3
61	Soda Spring, Yellowstone	29.0°	1.0048	6.9
91	Cleveland, Idaho	39.1°	1.0062	7.5
101	Austin, Nevada	36.2°	1.0038	7.3
109	Gerlach, Nevada	36.6°	1.0029	8.5

Oxycera (?) sp.

The same species which I referred to this genus with some doubt, in 1928 (p. 197) was taken again at several widely separated localities in Yellowstone Park, Idaho and California in water of rather uniformly high temperature. As indicated by the data given below there is a rather wide variation in the specific gravity of the water in the several springs (1.0014 to 1.0067) but the hydrogen ion concentration ranges only from 7.0 to 8.3. I have also a larva collected in a hot spring at Banff, Canada by Mr. D. J. Zinn. It is quite evident that this species occurs very generally in thermal springs.

Number	Locality	Temperature	Specific Gravity	pH
10	Brokeoff, Mt. Lassen	43°–47°	1.0014	7.3
57	Tower Junction, Yellowstone	36.5°	1.0057	7.0
60a	Angel Terrace, Yellowstone	38.0°	1.0040	8.3
91	Cleveland, Idaho	39.1°	1.0062	7.5
92	Cleveland, Idaho	36.7°	1.0067	8.1
101	Austin, Nevada	36.2°	1.0038	7.3
135	Sulphur Works, Lassen	32.6°	1.0034	7.4
140	Siegler Hot Spring, California	42.8°	1.0057	7.3

In addition to the material listed above, there is a single peculiar larva of small size (about 5 mm. in length) from hot spring No. 57, at Tower Junction in Yellowstone Park. This bears a transverse series of large, spatulate, fimbriately margined bristles on the body segments. Each series includes four dorsal bristles and a marginal one at each side. It may possibly belong to Nemotelus or some related genus.

TABANIDÆ.

Fifteen lots of larvæ of this family were obtained in 1930, all similar and in all probability referable to *Tabanus punctifer* O. S. Previously we obtained larvæ of this species at several localities and adults are commonly seen in the neighborhood of hot and alkaline bodies of water. There is a great difference in the size of the individual larvæ, some being quite small and others evidently nearly mature. The species always develops in alkaline water; our records indicate a pH of 7.4 to 9.6, and it is peculiarly tolerant of highly saline water as we have found it in water ranging as high as 1.0146 at Hot Springs, Utah and 1.0217 at Wilbur Hot Springs, California, the latter water being the vilest we have ever attempted to taste. On six occasions the larvæ were at a temperature of over 40°, showing also a high tolerance for heat.

Below are the combined records of our two years' collections:

Number	Locality	Temperature	Specific Gravity	pH
5	Convict Lake, California	43.0°	1.0012	8.5
6	Bridgeport, California	38.5°	1.0030	7.5
20	Winnemucca, Nevada	38.0°	1.0014	8.6
24	Beowawe, Nevada	39.6°	1.0020	9.6
31	Hot Springs, Utah	35.3°	1.0146	8.2
32	Ogden Canyon, Utah	—	1.0063	8.3
51	Mary Bay, Yellowstone Park	38.5°	1.0038	—
71	Shoshone Basin, Yellowstone Park	40.8°	1.0046	8.8
77	Lower Geyser Basin, Yell. Park	38.6°	1.0056	9.5
94	Cherry Creek, Nevada	36.3°	1.0038	8.4
95	Cherry Creek, Nevada	36.3°	1.0043	8.4
100	Antelope Valley, Nevada	40.4°	1.0032	9.1
102	Darrough's Hot Springs, Nevada	38.2°	1.0025	9.0
103	Cortez, Nevada	40.8°	1.0041	8.5
104	Monte Cristo, Nevada	38.3°	1.0052	8.7
106b	Pinnacles, Pyramid Lake, Cal.	33.6°	—	8.4
116	Surprise Valley, Nevada	40.2°	1.0035	7.6
118	Denio, Oregon	42.2°	1.0030	8.4
123a	Big Bend Hot Spring, Cal.	37.6°	1.0040	7.4
137	Wilbur Hot Springs, Cal	32.8°	1.0217	8.5
143	Paraiso Hot Spring, Cal.	33.4°	1.0042	7.4

EPHYDRIDÆ.

Our previous collections made in 1927 contained five series of larvæ belonging to the genus Ephydra and in 1930 we were able to add 24 additional records. More than one species may be present, but I have been unable to distinguish them satisfactorily. Only two lots come from highly saline water, one from Wilbur Hot Springs (No. 137) and one from warm seepage along the shore of Mono Lake. The latter are probably *Ephydra hians* Say which occurs abundantly in the still more highly saline waters of the lake itself as well as in various other saline and alkaline bodies of water in the Western United States. Possibly the others are also the same species as I cannot distinguish them after a rather careful examination and they agree with the description given by Aldrich ('12) of *E. hians* which he obtained in several widely separated localities. Unfortunately there is much yet to be learned concerning the actual distribution of the several species of Ephydra that occur in the far western states and I have already recorded the occurrence of *E. pectinulata* Cresson in a spring in the Norris Geyser Basin in Yellowstone Park ('24). These larvæ are also indistinguishable so far as I can see from those in my present collections, so that these may represent this species, at least in part, especially since most of the series are from locations in Yellowstone Park.

By comparison with the series of determinations which we made of the hydrogen ion concentration of the water at each location, the collections of Ephydra larvæ at once group themselves into two series. One contains 18 lots taken in springs of fairly or very high pH, ranging from 6.5 to 9.6 + and the other includes six lots all from very strongly acid water of pH 3.6 or less. This may not be important since there were very few springs in which the water did not fall into one or the other of these groups. However, I have listed the two series separately below.

Ephydra sp. probably[1] *E. pectinulata* Cress.

Number	Locality	Temperature	Specific Gravity	pH
43	West Thumb, Yellowstone Park	23.2°	1.0023	9.0
48	Indian Pond, Yellowstone Park	28.4°	1.0028	6.5
49	Beach Springs	35.0°	1.0051	7.2
56	Turbid Lake, Yellowstone Park	40.6°	1.0040	7.2
57b	Tower Junction, Yellowstone Park	40.2°	1.0057	7.0
66	Firehole River, Yellowstone Park	38.4°	1.0024	8.9
68	Shoshone Basin, Yellowstone Park	39.2°	1.0040	9.4
71	Shoshone Basin, Yellowstone Park	40.8°	1.0046	8.8
75	Three Sisters, Yellowstone Park	36.0°	1.0046	9.1
76	Sunset Lake, Yellowstone Park	32.2°	1.0043	9.0
77	Lower Geyser Basin, Yell. Park	33.9°	1.0056	9.5
79	Sprite Pool, Yellowstone Park	36.3°	1.0039	8.9
80	Firehole Pool, Yellowstone Park	36.0°	1.0046	8.2
83	Near Ink Well, Yellowstone Park	39.3°	1.0038	9.1
84	Near Spasmodic Geyser, Yell. Pk	32.3°	1.0032	9.3
93	Preston, Idaho	36.0°	1.0150	8.0
137	Wilbur Hot Springs, Calif.	32.8°	1.0217	8.5
149	Democrat Springs, Calif.	24.8°	1.0047	8.7
153	Shore of Mono Lake, Calif.	18.8°	1.0195	9.6+

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[1] Except Numbers 137 and 153.

Ephydra sp., perhaps *E. pectinulata* Cress.

Number	Locality	Temperature	Specific Gravity	pH
52	Turbid Lake, Yellowstone Park . .	19.3°	1.0025	3.6
53	Turbid Lake, Yellowstone Park . .	37.8°	1.0034	3.6
58	Whirligig Geyser, Yellowstone Pk.	38.2°	1.0051	3.6−
59	Near Minute Man, Yell. Park	24.6°	1.0026	3.6−
63	Amphitheatre Springs, Yell. Pk. . .	27.4°	1.0043	3.6
73	Shoshone Basin, Yellowstone Park	40.1°	1.0058	3.6

In addition to the above, there are three lots of larvæ which seem referable to this family. They are small (8–10 mm. in length) with cylindrical body beset with minute black bristly hairs arranged much as in Ephydra, but without the stout segmentally disposed rows of hooks characteristic of that genus. The respiratory apparatus is a forked tube, each branch of which is tipped with a heavily chitinized dark brown cylinder as in Ephydra.

The localities are as follows:

Number	Locality	Temperature	Specific Gravity	pH
57a	Tower Junction, Yellowstone Park	39.2°	1.0057	7.0
61	Soda Spring, Yellowstone Park . . .	29.0°	1.0048	6.9
95	Cherry Creek, Nevada.	30.4°	1.0057	8.4

COLEOPTERA.

The beetle fauna of thermal springs is very abundant and varied, including members of most of the purely aquatic families. The present collections contain representatives of Haliplidæ, Dytiscidæ, Hydrophilidæ, Helmidæ and Heteroceridæ, but no Gyrinidæ, although a Gyrinus was taken by us in 1927, and if notice had been taken of riparian, mud-inhabiting forms several other groups would have been added.

Altogether many hundreds of specimens were taken and had it not been for the kind interest of Mr. C. A. Frost, who has undertaken to identify the material I should have been unable to give any account of them. Mr. H. C. Fall has also examined some of the material which was sent to him by Mr. Frost and the series of Laccobius and Helophorus have been in the hands of Mr. F. E. Winters. To these gentlemen I am also greatly indebted.

The following detailed account includes all of the species collected, with the exception of the Helmidæ and a very few others that could not be definitely determined.

HALIPLIDÆ.

Peltodytes callosus Lec.

This species was taken at two places in California in addition to a spring in Mt. Lassen Park listed in my previous paper ('28).

Number	Locality	Temperature	Specific Gravity	pH
120b	Cedarville, California	36.2°	1.0037	8.2
122	Henderson, California	39.5°	1.0040	7.4

DYTISCIDÆ.

Deronectes (*Hydroporus*) **griseostriatus** Deg.

Number	Locality	Temperature	Specific Gravity	pH
96	Monte Neva, Nevada	36.3°	1.0038	8.4
119	Fish Springs, Nevada	Cold	1.0002	—

Deronectes striatellus Lec.

This species was taken in six hot springs of very different character over a wide geographical range, but not at very high temperatures.

Number	Locality	Temperature	Specific Gravity	pH
64	Amphitheatre Springs, Yell. Pk.	34.2°	1.0043	3.6−
75	Three Sisters, Yellowstone Park	37.2°	1.0046	9.1
78	Biscuit Basin, Yellowstone Park	35.0°	1.0040	9.6+
119	Fish Springs, Nevada	Cold	1.0002	—
130	Bumpass Hell, Mt. Lassen Park	26.3°	1.0056	6.8
135	Sulphur Works, Mt. Lassen Park	32.6°	1.0034	7.4

Laccophilus mexicanus Aubé.

Number	Locality	Temperature	Specific Gravity	pH
102	Darrough's Hot Springs, Nevada	40.2°	1.0025	9.0
104	Monte Cristo, Nevada	38.3°	1.0052	8.7
113	Soldier Meadows, Nevada	19.2°	1.0007	8.8
118	Denio, Oregon	39.0°	1.0030	8.4
120b	Cedarville, California	36.2°	1.0034	8.2

This species would appear to extend its range northward in thermal waters through Nevada as the last locality is in the extreme eastern part of California. Two of the records indicate rather high temperatures.

Laccophilus decipiens Lec.

Six records of this species are all from springs of moderate temperature and alkalinity, quite similar to one noted previously ('28, Brues).

Number	Locality	Temperature	Specific Gravity	pH
96	Monte Neva, Nevada	36.3°	1.0038	8.4
97	Sunnyside, Nevada	30.8°	1.0029	9.3
104	Monte Christo, Nevada	38.3°	1.0037	8.4
108	Gerlach, Nevada	36.3°	1.0061	7.5
121	Kelly Hot Springs, California	38.5°	1.0032	8.4
122	Bassett's Hot Springs, California	39.5°	1.0037	8.4

Hydrovatus brevipes Sharp.

A single record is from Cedarville, California in hot spring No. 120b in water of 36.2°, specific gravity, 1.0034, pH 8.2.

Bidessus affinis Say.

In addition to four previous records (Brues '28) twelve may now be added for this widespread form. The temperatures range up to 41°, but nearly all fall below 39°, in alkaline water from pH 7.7 to 9.6 +. The salinities range up to 1.0150.

Number	Locality	Tempera-ture	Specific Gravity	pH
66	Firehole River, Yellowstone Park	38.4°	1.0024	8.9
69	Shoshone Basin, Yellowstone Park	31.2°	1.0036	8.2
72	Shoshone Basin, Yellowstone Park	34.5°	1.0062	7.7
78	Biscuit Basin, Yellowstone Park	35.0°	1.0040	9.6+
80	Firehole Pool, Yellowstone Park	36.0°	1.0046	8.2
93	Preston, Idaho	36.0°	1.0150	8.0
94	Cherry Creek, Nevada	—	1.0038	8.4
95	Cherry Creek, Nevada	36.3°	1.0043	8.4
113	Soldier Meadows, Nevada	19.2°	1.0007	8.8
114	Hausen, California	36.2°	1.0033	7.5
120b	Cedarville, California	36.2°	1.0034	8.2
121	Kelly Hot Springs, California	38.5°	1.0032	8.4

Cœlambus lutescens Lec.

Six springs yielded specimens of this species in addition to an atypical example from hot spring No. 77 which is probably the same. They all come from noticeably alkaline water (pH 8.3–9.5), only one record indicating a temperature of more than 40°. One previous record obtained in 1927 is from Denio, Oregon in hot spring No. 16 at 39°.

Number	Locality	Tempera-ture	Specific Gravity	pH
77	Lower Geyser Basin, Yell. Park	33.9°	1.0056	9.5
96	Monte Neva, Nevada	36.3°	1.0038	8.4
97	Sunnyside, Nevada	30.8°	1.0029	9.3
100	Antelope Valley, Nevada	35.8°	1.0038	9.1
103	Cortez, Nevada	40.8°	1.0041	8.5
118	Denio, Oregon	39.0°	1.0030	8.4
150	Paoha Isl., Mono Lake, Calif.	32.0°	1.0045	8.3

Cœlambus nigrescens Fall.

This species was taken only once, in company with the preceding one at Denio, Oregon at 39.0°.

Cœlambus impressopunctatus Sch.

Collected only at Hausen, California in hot spring No. 114; temperature 36.2°, specific gravity 1.0033, pH 7.5.

Cœlambus sp. (near *artus* Fall).

A single hot spring locality is in spring No. 69 in the Shoshone Geyser Basin, Yellowstone Park in water at 31.2°, specific gravity, 1.0036, pH 8.2.

Cœlambus collatus Fall.

This form was taken at two places in northwestern Nevada.

Number	Locality	Temperature	Specific Gravity	pH
108	Gerlach, Nevada................	36.3°	1.0061	7.5
113	Soldier Meadows, Nevada........	19.2°	1.0007	8.8

Cœlambus fastidiosus Fall.

One series from the Pinnacles, Pyramid Lake, Nevada in hot spring No. 106a at 39.0°, specific gravity, 1.0064, pH 8.4.

Cœlambus compar Fall (?).

One series from the White Arrow Sanitarium Springs near Blanche, Idaho, in hot spring No. 86 in water at 39.0°, sp. gr. 1.0040, pH 8.6.

Cœlambus sp.

Females of an undetermined species were taken at two localities, one at the very high temperature of 44.5°.

Number	Locality	Temperature	Specific Gravity	pH
103	Cortez, Nevada..................	44.5°	1.0041	8.5
120b	Cedarville, California...........	28.4°	1.0034	8.2

Cœlambus sp.

Another undetermined species is represented in two collections.

Number	Locality	Temperature	Specific Gravity	pH
113	Soldier Meadows, Nevada	19.2°	1.0007	8.8
121	Kelly Hot Springs, California	39.5°	1.0032	8.4

Cœlambus sellatus Lec.

A single series of this species was taken in hot spring No. 77 in the Lower Geyser Basin, Yellowstone in water at 33.9°, sp. gr. 1.0056, pH 9.5.

Cœlambus thermarum Darlington.

This species, which was described from material collected by us in two Nevada hot springs in 1927 appears in the collections from three more springs as follows:

Number	Locality	Temperature	Specific Gravity	pH
20	Winnemucca, Nevada	38.0°	1.0014	8.6
23a	Watt's Ranch, Nevada	30.0°	1.0021	8.6
103	Cortez, Nevada	44.5°	1.0041	8.5
113	Soldier Meadows, Nevada	19.2°	1.0007	8.8
120b	Cedarville, California	28.4°	1.0034	8.2

It will be noted that the temperature record from one spring is unusually high and that the pH range, at least as here indicated, is not wide.

Agabus griseipennis Lec.

Two records for this species were obtained, in addition to two others previously noted ('28). These indicate the occurrence of this form quite generally in very warm water.

Number	Locality	Temperature	Specific Gravity	pH
16	Denio, Oregon	39.0°	1.0015	8.5
23a	Watt's Ranch, Nevada	36.0°	1.0021	8.5
87	Mountain Home, Idaho	42.4°	1.0027	9.6+
96	Monte Neva, Idaho	36.3°	1.0038	8.4

Agabus lutosus Lec.

A single specimen was taken in hot spring No. 130 at Bumpass Hell, Mt. Lassen Park in water at 26.3°, sp. gr. 1.0056, pH 6.8.

Agabus obliteratus Lec.

One series from the Shoshone Geyser Basin, Yellowstone Park, hot spring No. 69. They were taken at 31.7° in water with a specific gravity of 1.0036 and pH of 8.2.

Agabus seriatus Say.

Two hot springs of widely different reaction yielded specimens of this species.

Number	Locality	Temperature	Specific Gravity	pH
86	White Arrow Springs, Idaho	41.0°	1.0040	8.6
124	Mt. Lassen Park	30.6°	1.0037	3.6−

Rhantus binotatus Harris.

This widely distributed species appears in two series from hot springs, at moderate temperatures.

Number	Locality	Temperature	Specific Gravity	pH
95	Cherry Creek, Nevada	30.4°	1.0043	8.4
96	Monte Neva, Nevada	36.3°	1.0038	8.4

HYDROPHILIDÆ.

Ochthebius bruesi Darlington.

This species was described from material collected by us in 1927, and appears in series from four more hot springs in the present collections. All are from moderately or highly alkaline waters in several cases at quite high temperatures.

Number	Locality	Temperature	Specific Gravity	pH
24	Beowawe, Nevada	38.8°	1.0020	9.6
103	Cortez, Nevada	40.8°	1.0041	8.5
113	Soldier Meadows, Nevada	19.2°	1.0007	8.8
120a	Cedarville, California	43.0°	1.0037	8.2
120b	Cedarville, California	36.2°	1.0034	8.2

Ochthebius rectus Lec.

Two localities are represented.

Number	Locality	Temperature	Specific Gravity	pH
120b	Cedarville, California	36.2°	1.0034	8.2
121	Kelly Hot Springs, California	39.5°	1.0032	8.4

Ochthebius lineatus Lec.

One specimen of this species was taken in Fish Springs, California (hot spring No. 147) at a temperature of 32.2°, sp. gr. 1.0060, pH 7.7.

Ochthebius interruptus Lec.

To a previous record from Wells, Nevada may be added Cortez, Nevada (hot spring No. 103 at 44.5°, sp. gr. 1.0041, pH 8.5).

Helophorus oblongus Lec.

Number	Locality	Temperature	Specific Gravity	pH
47b	Indian Pond, Yellowstone Park	Cold	1.0001	—

Helophorus lecontei Knisch.

Number	Locality	Temperature	Specific Gravity	pH
96	Monte Neva, Nevada	36.3°	1.0038	8.4
120b	Cedarville, California	36.2°	1.0034	8.2

Helophorus obscurus Lec.

Number	Locality	Temperature	Specific Gravity	pH
96	Monte Neva, Nevada	36.3°	1.0038	8.4
102	Darrough's Hot Springs, Nevada	36.8°	1.0025	9.0
120b	Cedarville, California	36.2°	1.0034	8.2

Helophorus lineatus Say.

Number	Locality	Temperature	Specific Gravity	pH
89	Indian Springs, Idaho	33.2°	1.0031	8.4
91	Cleveland, Idaho	28.2°	1.0062	7.5
93	Preston, Idaho	36.0°	1.0150	8.0
94	Cherry Creek, Nevada	—	1.0038	8.4
95	Cherry Creek, Nevada	36.3°	1.0043	8.4
96	Monte Neva, Nevada	36.3°	1.0038	8.4
101	Birch Creek Ranch, Nevada	36.2°	1.0038	7.3
102	Darrough's Hot Springs, Nev.	36.8°	1.0025	9.0
103	Cortez, Nevada	40.8°	1.0041	8.5
104	Monte Christo, Nevada	38.3°	1.0052	8.7

Tropisternus dorsalis Brulle.

With four previous records (Brues '28) and five in the present collections, this species has now been taken in nine hot springs at temperatures up to 42.2° and between pH limits of 7.5–8.6 and always in water of low salinity (1.0012 to 1.0041).

Number	Locality	Temperature	Specific Gravity	pH
96	Monte Neva, Nevada	36.3°	1.0038	8.4
103	Cortez, Nevada	40.8°	1.0041	8.5
118	Denio, Oregon	42.2°	1.0030	8.4
121	Kelly Hot Springs, Calif.	39.3°	1.0032	8.4
122	Bassett's Hot Springs, Calif	39.5°	1.0037	8.4

Tropisternus ellipticus Lec.

Two more records (four in all) show this species to have a distribution similar to that of *T. dorsalis*.

Number	Locality	Temperature	Specific Gravity	pH
113	Soldier Meadows, Nevada	19.2°	1.0007	8.8
143	Paraiso Hot Springs, Calif.	33.4°	1.0042	7.4

Tropisternus californicus Lec.

This species appeared at seven localities although we had taken it only twice before, in the Yellowstone and near Battle Mountain, Nevada. It was taken in more acid and more alkaline water than the two preceding forms and in more saline water, ranging from pH 6.5 to 9.1 and sp. gr. 1.0007 to 1.0062.

Number	Locality	Temperature	Specific Gravity	pH
46b	Bijah Spring, Yellowstone Park	34.3°	1.0036	6.3
86	White Arrow Springs, Idaho	41.0°	1.0040	8.6
91	Cleveland, Idaho	28.2°	1.0062	7.5
96	Monte Neva, Nevada	36.3°	1.0038	8.4
100	Antelope Valley, Nevada	35.8°	1.0032	9.1
103	Cortez, Nevada	40.8°	1.0041	8.5
113	Soldier Meadows, Nevada	19.2°	1.0007	8.8

Tropisternus sublævis Lec.

We had not taken this species in hot springs before, but the present series includes eleven different localities scattered over a wide area. Several records are from water at more than 40°; the pH range is wide extending from 6.8 to 9.6 +, and in only one case does the salinity exceed 1.0048.

Number	Locality	Temperature	Specific Gravity	pH
38	West Thumb, Yellowstone Park..	36.2°	1.0025	6.8
61	Soda Spring, Yellowstone Park...	29.0°	1.0048	6.9
69	Shoshone Basin, Yellowstone Park	31.2°	1.0036	8.2
86	White Arrow Springs, Idaho.....	41.0°	1.0040	8.6
87	Mountain Home, Idaho.........	42.4°	1.0027	9.6+
91	Cleveland, Idaho...............	28.2°	1.0062	7.5
96	Monte Neva, Nevada..........	36.3°	1.0038	8.4
102	Darrough's Hot Springs, Nevada..	36.8°	1.0025	9.0
103	Cortez, Nevada.................	40.8°	1.0041	8.5
113	Soldier Meadows, Nevada........	19.2°	1.0007	8.8
120b	Cedarville, California...........	36.2°	1.0034	8.2

Hydrous triangularis Say.

This familiar large species was only taken once alive, although we have seen it on several occasions floating dead on the surface of very hot water which it had attempted to enter. The specimen came from water at 36.8° in hot spring No. 118 near Denio, Oregon.

Paracymus subcupreus Say.

To four previous records we are now able to add twenty-four additional ones. They indicate their occurrence in salinities varying from 1.0025 to 1.0047 and at hydrogen ion concentrations from pH 6.4 to 9.6 +. Eight records of over 40° indicate that it may inhabit extremely warm water.

Number	Locality	Temperature	Specific Gravity	pH
45	Norris Geyser Basin, Yell. Pk.	44.8°	1.0040	7.9
65	Firehole River, Yellowstone Park	33.2°	—	—
67	Firehole River, Yellowstone Park	36.2°	1.0044	8.2
71	Shoshone Basin, Yellowstone Park	40.8°	1.0046	8.8
72	Shoshone Basin, Yellowstone Park	35.4°	1.0062	7.7
74	Shoshone Basin, Yellowstone Park	35.3°	1.0047	6.4
86	White Arrow Springs, Idaho	39.0°	1.0040	8.6
87	Mountain Home, Idaho	35.5°	1.0027	9.6+
91	Cleveland, Idaho	28.2°	1.0062	7.5
97	Sunnyside, Nevada	30.8°	1.0029	9.3
98	Sunnyside, Nevada	33.6°	1.0042	8.3
99	Duckwater, Nevada	29.0°	1.0032	8.0
100	Antelope Valley, Nevada	35.8°	1.0032	9.1
102	Darrough's Hot Springs, Nevada	36.8°	1.0025	9.0
103	Cortez, Nevada	40.8°	1.0041	8.5
104	Monte Cristo, Nevada	38.3°	1.0052	8.7
114	Hausen, Calif.	36.2°	1.0033	7.5
115	Surprise Valley, Calif.	34.2°	1.0027	8.3
116	Surprise Valley, Calif.	40.2°	1.0035	7.6
118	Denio, Oregon	39.0°	1.0030	8.4
120a	Cedarville, Calif.	43.0°	1.0037	8.2
120b	Cedarville, Calif.	36.2°	1.0034	8.2
121	Kelly Hot Springs, Calif.	38.5°	1.0032	8.4
141	Calistoga, Calif.	42.8°	1.0035	8.4

Paracymus elegans Fall.

This small species was taken once in a hot spring in the Kern River valley in California at Democrat Springs in water of rather low temperature (30.2°), sp. gr. 1.0017, pH 7.4.

Chasmogenus normatus Lec.

Four localities yielded specimens of this species, only one in water above 40° and all in water of nearly neutral reaction (7.3–7.4). All are in California.

Number	Locality	Temperature	Specific Gravity	pH
123a	Henderson, Calif.	37.6°	1.0040	7.4
140	Seigler Hot Springs, Calif.	42.8°	1.0057	7.3
143	Paraiso Hot Springs, Calif.	33.4°	1.0042	7.4
148	Democrat Hot Springs, Calif.	24.8°	1.0017	7.4

Enochrus (*Philhydrus*) **carinatus** Lec.

All but one of five localities are in Yellowstone Park. The species occurs in quite saline water and only, so far as our records extend in rather strongly alkaline water.

Number	Locality	Temperature	Specific Gravity	pH
60a	Angel Terrace, Yellowstone Park	37.6°	1.0040	8.3
84	Near Spasmodic Geyser, Yell. Pk.	32.0°	1.0032	9.3
102	Darrough's Hot Springs, Nev.	36.8°	1.0025	9.0
106a	Pinnacles, Pyramid Lake, Calif.	29.0°	1.0064	8.4
107	Pinnacles, Pyramid Lake, Calif.	36.5°	1.0074	8.4

Enochrus conjunctus Fall.

Nineteen records were obtained for this species indicating a wide range of tolerance to varying salinity, hydrogen ion concentration and temperature, although only two series came from water above 40° and none above 41°.

Number	Locality	Temperature	Specific Gravity	pH
46a	Bijah Spring, Yellowstone Park	39.3°	1.0039	6.5
50	Mary Bay, Yellowstone Park	40.1°	1.0022	7.3
53	Turbid Lake, Yellowstone Park	34.0°	1.0034	3.6
67	Firehole River, Yellowstone Park	36.2°	1.0044	8.2
69	Shoshone Basin, Yellowstone Park	31.2°	1.0036	8.2
84	Near Spasmodic Geyser, Yell. Pk.	32.0°	1.0032	9.3
91	Cleveland, Idaho	28.2°	1.0062	7.5
102	Darrough's Hot Springs, Nev.	36.8°	1.0025	9.0
103	Cortez, Nev.	40.8°	1.0041	8.5
108	Gerlach, Nev.	32.8°	1.0061	7.5
120a	Cedarville, California	36.2°	1.0037	8.2
121	Kelly's Hot Springs, Calif.	38.5°	1.0032	8.4
124	Mt. Lassen Park, Calif.	30.6°	1.0037	3.6–
126	Mt. Lassen Park, Calif.	31.2°	1.0025	7.1
127	Bumpass Hell, Mt. Lassen Park	33.5°	1.0028	6.0
128	Bumpass Hell, Mt. Lassen Park	34.2°	1.0028	6.7
130	Bumpass Hell, Mt. Lassen Park	35.6°	1.0056	6.8
131	Bumpass Hell, Mt. Lassen Park	34.2°	—	—
150	Paoha Isl., Mono Lake, Calif.	32.8°	1.0045	8.3

Enochrus pectoralis Lec.

Number	Locality	Temperature	Specific Gravity	pH
143	Paraiso Hot Springs, Calif.	33.4°	1.0042	7.4

Enochrus nebulosus Say.

Nine series of this species were obtained in addition to eleven previously noted (Brues '28). As will be seen from the data given below this form seems to be restricted to water of alkaline reaction (pH 7.4–9.6 +) and ranges into water slightly above 40°.

Number	Locality	Temperature	Specific Gravity	pH
67	Firehole River, Yellowstone Park.	35.5°	1.0027	9.6+
77	Lower Geyser Basin, Yell. Pk.....	33.9°	1.0056	9.5
87	Mountain Home, Idaho.........	35.5°	1.0027	9.6+
91	Cleveland, Idaho...............	28.2°	1.0062	7.5
99	Duckwater, Nevada.............	29.0°	1.0032	8.0
100	Antelope Valley, Nevada........	39.2°	1.0032	9.1
103	Cortez, Nevada	40.8°	1.0041	8.5
121	Kelly Hot Springs, Calif.........	38.5°	1.0032	8.4
143	Paraiso Hot Springs............	33.4°	1.0042	7.4

Enochrus nigrellus Sharp.

Number	Locality	Temperature	Specific Gravity	pH
143	Paraiso Hot Springs, Calif........	33.4°	1.0042	7.4

Cimbiodyta dorsalis Mots.

This southern Californian species was taken twice, one record indicating a quite considerable northern extension of its range.

Number	Locality	Temperature	Specific Gravity	pH
113	Soldier Meadows, Nevada.......	19.2°	1.0007	8.8
143	Paraiso Hot Springs, Calif.......	33.4°	1.0042	7.4

Laccobius agilis Rand.

Number	Locality	Temperature	Specific Gravity	pH
40	West Thumb, Yellowstone Park	37.0°	1.0058	7.3
49	Squaw Lake, Yellowstone Park	38.5°	1.0051	7.2
50	Mary Bay, Yellowstone Park	38.3°	1.0022	7.3
54	Turbid Lake, Yellowstone Park	23.4°	1.0021	8.6
55	Turbid Lake, Yellowstone Park	36.8°	1.0066	7.3
56	Turbid Lake, Yellowstone Park	40.6°	1.0040	7.2
60a	Angel Terrace, Yellowstone Park	38.0°	1.0040	8.3
60b	Highland Terrace, Yell. Park	37.6°	—	—
61	Soda Spring, Yellowstone Park	29.0°	1.0048	6.9
68	Shoshone Basin, Yellowstone Park	41.8°	1.0040	9.4
70	Shoshone Basin, Yellowstone Park	31.0°	1.0042	9.0
72	Shoshone Basin, Yellowstone Park	35.4°	1.0062	7.7
75	Three Sisters, Yellowstone Park	37.2°	1.0046	9.1
78	Biscuit Basin, Yellowstone Park	35.0°	1.0040	9.6+
79	Sprite Pool, Yellowstone Park	36.3°	1.0039	8.9
80	Firehole Pool, Yellowstone Park	36.0°	1.0046	8.2
84	Spasmodic Geyser, Yell. Pk.	32.0°	1.0032	9.3
92	Cleveland, Idaho	39.1°	1.0067	8.1
96	Monte Neva, Nevada	36.3°	1.0038	8.4
98	Sunnyside, Nevada	33.6°	1.0042	8.3
101	Birch Creek Ranch, Nevada	36.2°	1.0038	7.3
103	Cortez, Nevada	40.8°	1.0041	8.5
113	Soldier Meadows, Nevada	19.2°	1.0007	8.8
150	Paoha Isl., Mono Lake, Calif.	32.8°	1.0045	8.3

Laccobius piceus, var.

Number	Locality	Temperature	Specific Gravity	pH
37	West Thumb, Yellowstone Park	Cool	1.0056	8.9
57a	Tower Junction, Yellowstone Park	40.2°	1.0057	7.0
58	Whirligig Geyser, Yell. Park	38.2°	1.0051	3.6
77	Lower Geyser Basin, Yell. Park	33.9°	1.0056	9.5
80	Firehole Pool, Yellowstone Park	36.0°	1.0046	8.2
84	Spasmodic Geyser, Yell. Park	32.0°	1.0032	9.3
150	Paoha Isl., Mono Lake, Calif.	32.8°	1.0045	8.3

Laccobius bruesi Winters

Number	Locality	Temperature	Specific Gravity	pH
121	Kelly Springs, Canby, Calif.	39.5°	1.0032	8.4
137	Wilbur Hot Springs, Calif.	32.8°	1.0150	8.5

Laccobius ellipticus Lec.

Number	Locality	Temperature	Specific Gravity	pH
86	Blanche, Idaho	39.0°	1.0040	8.6
105	Pyramid Lake, Nevada	29.8°	1.0064	8.0
106a	Pyramid Lake, Nevada	29.0°	—	8.4
150	Paoha Isl., Mono Lake, Calif.	32.8°	1.0036	7.3

Cercyon sp.

Number	Locality	Temperature	Specific Gravity	pH
55	Turbid Lake, Yellowstone Park . .	36.0°	1.0066	7.3

HETEROCERIDÆ.

Heterocerus sp.

Number	Locality	Temperature	Specific Gravity	pH
91	Cleveland, Idaho	41.2°	1.0062	7.5
93	Preston, Idaho	36.0°	1.0150	8.0
102	Darrough's Hot Springs, Nevada	36.8°	1.0025	9.0
150	Paoha Isl., Mono Lake, Calif.	40.8°	1.0045	8.3

Heterocerus collaris Kies.

Number	Locality	Temperature	Specific Gravity	pH
99	Duckwater, Nevada.............	29.0°	1.0032	8.0

Heterocerus brunneus Mels.

Number	Locality	Temperature	Specific Gravity	pH
99	Duckwater, Nevada.............	29.0°	1.0032	8.0
103	Cortez, Nevada.................	40.8°	1.0041	8.5

COLEOPTEROUS LARVÆ.

CICINDELIDAE.

Certain hot springs, especially those where there is much soft clayey deposit about their borders, harbor in this warm soil the larvæ of cicindelid beetles which seem to thrive in the burrows which they excavate often very close to the water. I have previously reported the occurrence of these larvæ in such situations and we secured additional material in about five hot springs during 1930. It is impossible to identify the larvæ specifically, but adults taken flying in these localities indicate that they are the same species previously noted (Brues '28, p. 180) and also in addition *Cicindela latesignata* Lec., var. *obliviosa* Casey, previously known from Southern California, but taken by us much farther north about hot spring No. 120b at Cedarville, Calif. and no doubt representing a northward extension of this form in connection with hot springs. Other collections of Cicindelid larvæ are from the warm soil about the following springs: Nos. 40, 67, 126, 131, and 150.

DYTISCIDAE.

Our collections of dytiscid larvæ are much less numerous than might be expected from the large series of adult beetles obtained. They appear to represent four genera as nearly as I have been able to place them with the use of Wilson's account ('23) and the recent keys of Needham ('27).

Dytiscus sp.

One large larva from hot spring No. 47b (Indian Pond, Yellowstone Park) is without question a larva of some species of Dytiscus, perhaps *D. marginicollis* Lec., a species that I obtained previously in hot spring No. 23 near Battle Mountain, Nevada. In both cases, the water was very slightly saline (1.0001 and 1.0008).

Cybister sp.

In hot spring No. 113 near Gerlach, Nevada we collected a larva of Cybister agreeing closely with Wilson's description and figures of *C. fimbriolatus* Say. We had previously taken an adult of this genus in hot spring No. 21 at Sou Hot Springs, Nevada, but found no adults on the last trip.

Laccophilus sp.

Four springs yielded specimens of larvæ belonging to this genus. The records are as follows:

Number	Locality	Temperature	Specific Gravity	pH
91	Cleveland, Idaho	38.0°	1.0062	7.5
100	Antelope Valley, Nev.	39.2°	1.0032	9.1
101	Birch Creek, Nev.	36.2°	1.0038	7.3
121	Canby, California	38.5°	1.0032	8.4

It is probable that the last, from Kelly Hot Springs, Canby, California is *L. decipiens*, as we obtained adults of that species there.

Bidessus or **Cœlambus**.

Four series of larvæ were obtained that run to Hydroporus in the keys, but as the most closely related adults found were members of Cœlambus and Bidessus it is probable that they belong to one of these genera, probably the latter as one of the springs containing these larvæ (No. 66) yielded specimens of *Bidessus affinis*.

Number	Locality	Temperature	Specific Gravity	pH
63	Amphitheatre Springs, Yell. Pk. . .	27.4°	1.0043	3.6
66	Firehole River, Yell. Pk.	38.4°	1.0024	8.9
67	Firehole River, Yell. Pk.	36.2°	1.0044	8.2
75	Three Sisters, Yell. Pk.	34.0°	1.0046	9.1

HYDROPHILIDAE.

Tropisternus.

Larvæ of this genus were collected in thirteen of the hot springs visited during 1930, as follows:

Number	Locality	Temperature	Specific Gravity	pH
38	West Thumb, Yellowstone Park . .	34.2°	1.0025	6.8
44	Lewis Lake, Yellowstone Park . . .	30.8°	1.0023	7.8
47a	Indian Pond, Yellowstone Park . .	Cool	1.0001	7.1
65	Firehole River, Yellowstone Park .	33.2°	—	8.3
66	Firehole River, Yellowstone Park .	38.4°	1.0024	8.9
67	Firehole River, Yellowstone Park .	36.2°	1.0044	8.2
68	Shoshone Basin, Yellowstone Park	39.2°	1.0040	9.4
69	Shoshone Basin, Yellowstone Park	31.2°	1.0036	8.2
77	Lower Geyser Basin, Yell. Pk. . . .	38.6°	1.0056	9.5
91	Cleveland, Idaho	41.2°	1.0062	7.5
102	Darrough's Hot Spring, Nev.	36.5°	1.0025	9.0
113	Gerlach, Nev.	19.2°	1.0007	8.8
149	Democrat Springs, California	31.4°	1.0047	8.7

Enochrus.

Three localities yielded larvæ of this genus as follows:

Number	Locality	Temperature	Specific Gravity	pH
50	Mary Bay, Yellowstone Park	38.3°	1.0022	7.3
67	Firehole River, Yellowstone Park .	36.2°	1.0044	8.2
126	Mt. Lassen National Park	31.2°	1.0025	7.1

Helmis.

Larvæ, presumably *H. similis* or allied were numerous in hot spring No. 89 at Indian Springs, Idaho, where numerous adult beetles of this species were collected. Others were taken in No. 57a at Tower Junction, Yellowstone Park.

MOLLUSCA.

A considerable series of molluscs is contained in the material collected. Through the great kindness of my good friend, W. J. Clench, of the Museum of Comparative Zoölogy, I am enabled to report on the several species, for which he has given me authoritative determinations.

In all, members of nine families are represented, all of them gastropods, except a single lot of Sphærium. In a few cases it has not been possible to give specific names at present, especially for certain lots of Physa, as well as for an Amnicola and a Lymnæa from hot spring No. 99, at Duckwater, Nevada, which are probably undescribed.

Succineidæ.

Succinea rusticana Gould.

Number	Locality	Temperature	Specific Gravity	pH
103	Cortez, Nevada	40.8°	1.0041	8.5
114	Hausen, California	36.2°	1.0033	7.5
115	Surprise Valley, Nevada	34.2°	1.0027	8.3
116	Surprise Valley, Nevada	40.2°	1.0035	7.6

This species appears to frequent only water of slight alkalinity, but extends into water of quite high temperature.

Lymnæidæ.

Lymnæa palustris haydeni Lea.

Number	Locality	Temperature	Specific Gravity	pH
51	Mary Bay, Yellowstone Park	38.5°	1.0022	7.3
54	Turbid Lake, Yellowstone Park	23.4°	1.0021	8.6

This is the same species taken by us previously near the first of the localities cited above. It does not appear to occur at high temperatures nor in highly alkaline nor in acid waters.

Lymnæa sp. (probably undescribed).

Number	Locality	Temperature	Specific Gravity	pH
99	Duckwater, Nevada............	29.0°	1.0032	8.0

LANCIDÆ.

Lanx patelloides Lea.

Taken at Big Bend Hot Springs near Henderson, California (hot spring No. 123b), in water at 39.5°, specific gravity 1.0043, pH 7.4.

PLANORBIDÆ.

Three lots of Planorbis are all from only moderately warm, alkaline water.

Planorbis antrosus Conrad.

Taken in hot spring No. 111 near Gerlach, Nevada, at 33.7°, specific gravity 1.0042, pH 8.7.

This species occurs also in strongly brackish estuarine water.

Planorbis subcrenatus.

Taken in hot spring No. 98 at Sunnyside, Nevada, at 33.6°, specific gravity 1.0043, pH 8.3.

Planorbis sp.

An unidentified species was taken in hot spring No. 97, near Sunnyside, Nevada, at 30.8°, specific gravity 1.0029, pH 9.3.

PARAPHOLIGIDÆ.

Two forms of Parapholyx are included in our collections: *P. effusa* Lea from Big Bend Hot Springs, Henderson, California, in water of specific gravity 1.0043, pH 7.4 at 39.5°, and *P. effusa*, var. *costata* Stearns from Pyramid Lake, Nevada, in hot spring No. 105 in water at 32.0° with a specific gravity of 1.0064 and pH of 8.0.

PHYSIDÆ.

More series of this family than of any other were taken as Physa seems to be the dominate type in thermal waters.

Physa osculans Hald.

Number	Locality	Temperature	Specific Gravity	pH
145	Jacumba Hot Springs, Calif......	23.0°	1.0041	7.4

Physa johnsoni Clench.

I have received some specimens of this species which were collected by Mr. D. J. Zinn in a hot spring at Banff, Alberta, Canada. This is the type locality.

Physa propinqua Tryon.

Number	Locality	Temperature	Specific Gravity	pH
90	Lava Hot Springs, Idaho........	40°	1.0053	—

Physa spp.

The following records refer to species not yet specifically determined. Spring No. 98 yielded two species.

Number	Locality	Temperature	Specific Gravity	pH
51	Mary Bay, Yellowstone Park....	38.5°	1.0038	—
81	Tangle Creek, Yellowstone Park.	32.8°	1.0037	8.4
89	Indian Springs, Idaho...........	31.6°	1.0031	8.4
91	Cleveland, Idaho...............	32.8°	1.0062	7.5
92	Cleveland, Idaho...............	39.1°	1.0067	8.1
97	Sunnyside, Nevada.............	30.8°	1.0029	9.3
98	Sunnyside, Nevada (two species).	33.6°	1.0042	8.3
123a	Henderson, California...........	39.5°	1.0043	7.4
144	Paso Robles, California.........	32.4°	1.0051	7.4
146	Fish Springs, California.........	30.0°	1.0058	7.4
150	Paoha Isl., Mono Lake, Calif.....	32.8°	1.0045	8.3

PLEUROCERIDÆ.

Goniobasis nigrina Lea.

Number	Locality	Temperature	Specific Gravity	pH
123b	Henderson, California...........	39.5°	1.0043	7.4

Goniobasis placifera Lea.

Number	Locality	Temperature	Specific Gravity	pH
123b	Henderson, California...........	39.5°	1.0043	7.4

AMNICOLIDÆ.

Fluminicola nevadensis Walker.

Number	Locality	Temperature	Specific Gravity	pH
98	Sunnyside, Nevada..............	33.6°	1.0042	8.3
111	Gerlach, Nevada................	32.2°	1.0042	8.7

Fluminicola seminalis Hinds.

Number	Locality	Temperature	Specific Gravity	pH
123b	Henderson, California...........	39.5°	1.0043	7.4

Fluminicola sp.

Many young specimens of an undetermined species of Fluminicola were taken in hot spring No. 89, at Indian Springs, Idaho (specific gravity 1.0031, pH 8.4).

Pyrgalopsis nevadensis Stearns.

One series comes from the Pinnacles at the northern end of Pyramid Lake, Nevada (hot spring No. 105; specific gravity 1.0064, pH 8.0).

Amnicola sp. (probably undescribed).

A single locality, hot spring No. 99, at Duckwater, Nevada, yielded specimens of Amnicola. The water has a specific gravity of 1.0032 and pH of 8.0 with a temperature of 29.0°.

SPHÆRIDÆ.

A single series of an unidentified species of Sphærium represents the only pelecypod mollusc obtained. They came from Henderson, California, where they were taken together with a variety of other molluscs in hot spring No. 123b in water at 39.5°; specific gravity 1.0043 and pH 7.4.

It may be said of the entire collection of molluscs that none occur in very hot water as none have been taken at temperatures above 40.8°, and that their range with reference to hydrogen ion concentration is not greatly limited as they extend over a pH range of from 7.2 to 9.6 or over. This statement is made to include also a consideration of the data which we have previously collected (Brues '28) in the same general region.

Since my previous account was published two papers dealing with the occurrence of molluscs in brackish waters have appeared which are of interest here as they relate to the extension of fresh water species into brackish water. Richards ('29) has found that *Physa heterostropha* may be rapidly acclimatized to considerable salinities produced by the addition of sea water to the fresh water medium in which this species occurs. He added sea water at short intervals of a few days, producing consecutively mixtures containing 5%, 10%, 15%, etc., of sea water and these did not interfere with the activity and apparent health of the snails, at least in the weaker dilutions of 20–25% and less. These salinities are well above the concentrations encountered in thermal waters. He found the same conditions to prevail with two other common pond snails, *Lymnæa stagnalis appressa* and *L. palustris*, which could stand approximately the same concentrations of sea water ('29a). It thus appears that in the case of certain fresh water snails at least, the salinities present commonly in thermal waters are no barrier. Both of these genera have of course already been found in hot springs, Physa the most abundantly.

On the basis of field observations on the same *Physa heterostropha*, Bailey ('29) reports the occurrence of this species in brackish water with 50% of ocean concentration in Chesapeake Bay, together with *Goniobasis virginica* and *Congeria leucopheata*, the latter almost exclusively a brackish water form. The same observer records two other snails, *Amnicola limosa* and *Planorbis antrosus* in the bay water at 30% ocean concentration.

FISHES.

Only a few of the springs we visited were found to contain fish and none of these were living in water of high temperature. However, one form collected proved to be an extremely interesting type which has been described by Dr. Carl L. Hubbs who has also been so kind as to identify the few other specimens that we collected.

Two of the species are not native to the places where we obtained them. The first of these were numerous goldfish which were reared in one of the pools at hot spring No. 89 at Indian Springs, Idaho. These fish were living at 33.7° in water of moderate salinity (1.0031) and slight alkalinity (pH 8.4). The other introduced species is *Gambusia patruelis*, var. *holbrookii*, an Atlantic coast form which has been introduced into the west as an enemy of mosquito larvæ. It was present in hot spring No. 147 at Fish Springs in the Imperial Valley, California, in water at 30°, specific gravity 1.0058 and pH 7.4. In this spring we obtained larvæ of *Anopheles pseudopunctipennis* Theob.

Siphateles oregonensis Snyder ?

In hot spring No. 68b, 16 miles northwest of Gerlach, Nevada. Dr. Hubbs informs me that this chub falls within the limits of *S. obesus*, a species occurring in the Lahontan Basin, but that it agrees better with *S. oregonensis* that occurs in certain isolated Oregon Lakes. As this locality is near the Oregon border it seems probable that the fish reached there from Oregon, especially as this seems to be the case with frogs collected just north of the present place. The specimens were living in a large tule-bordered pool having a temperature of 33.7° in water of moderate salinity and alkalinity (1.0042; pH 8.7).

Apocope sp.

In hot spring No. 97 we found some young dace belonging to this genus. The water of this stream has a temperature of 30.8°, and rather slight salinity (1.0029) but is strongly alkaline with a pH of 9.3.

Crenichthys nevadæ Hubbs.

The specimens which form the types for this genus are one of the most interesting taxonomic finds made during the course of our investigations. According to Dr. Hubbs ('32) Crenichthys is a very distinct generic type of cyprinodont fishes related to the genus Empetrichthys which is known only from an isolated locality in Death Valley, California. It is also similar to Orestias, a genus known only from lakes in the high Andes of South America, whence numerous species have been described. Contrasted with the Death Valley type, Crenichthys is herbivorous, suggesting long isolation and certainly giving evidence of protracted activity either on the part of the spring in which it now occurs or at least of those in the immediate neighborhood.

The type locality is spring No. 99, Duckwater, Nevada containing water with a salinity of 1.0032 and pH of 8.0 at a temperature of 31.8°. The spring has an extensive flow and the fish were collected by Mrs. Brues near its source in the small stream which extends for a short distance before it is dissipated in a flat swampy area.

Some recent work which is of interest in connection with the fish fauna of thermal water has been presented by Agersborg ('30). He records a number of freshwater fishes as living in the polluted overflow from a corn-products plant at Decatur, Illinois in water at a temperature of 29°. These fishes were Carp (*Cyprinus carpio* Linn.), Sunfish (*Lepomis miniatus* Jordan), Bluegill (*L. pallidus* Mitchell), Crappie (*Pomoxis annularis* Raf.), Catfish (*Amieurus natalis* Le Seur), Chub (*Erimyzon sucetta oblongus* Mitchill), Steel-colored Minnow (*Notropis whipplei* Girard), Shiner (*N. atherinoides* Raf.), Gizzard-shad (*Dorosoma cepedianum*). He has also investigated the effect of sudden change from warm water of 28°–30° to water near the freezing point with reference to certain species of freshwater fish. He finds that in the case of fish living in winter in water thus warmed with waste from the corn-products plant at Decatur, that when brought into the cold water they manifest oxygen hunger and continued exposure is fatal. The dissolved oxygen content of the warm water is slightly less than half that of the cold water and he attributes their respiratory difficulties to vaso-motor changes causing non-functioning of the gills. Even sudden slight changes from 28° to 24° produced unbalanced movements and from 26° to 20° resulted in sudden death.

On the basis of these reactions to changes of temperature it is easy to see why fish are so rarely encountered in hot springs, even those of moderate temperature, since under the topographic conditions usually prevailing in thermal waters the water temperature is by no means constant throughout the pools or streams fed by the springs.

AMPHIBIA.

A few collections made during 1930 contain amphibians, mainly tadpoles, taken in hot springs. The tadpoles have been identified by Dr. G. K. Noble of the American Museum of Natural History and the adult forms by Mr. Arthur Loveridge of the Museum of Comparative Zoölogy. As the result of this kind interest on the part of these two gentlemen, I am able to present the following account of our material.

Although the data we have been able to obtain are not extensive they show that several toads and frogs live successfully in thermal water of moderate temperature and salinity, but not so far as our observations go, do they occur in highly acid or alkaline water as our pH records extend only from 7.0 to 8.8.

Bufo woodhouseii Baird & Girard (?).

Tadpoles of this toad were taken at three localities in Yellowstone Park, two in strictly thermal water and one in the cool, practically non-saline water of Indian Pond.

Number	Locality	Temperature	Specific Gravity	pH
44	Lewis Lake, Yellowstone Park....	27.6°	1.0023	7.8
47	Indian Pond, Yellowstone Park...	Cool	1.0001	—
69	Shoshone Basin, Yellowstone Park	31.2°	1.0036	8.2

In both cases the temperature is rather low and the reaction of the water is very slightly alkaline. The identification of the tadpoles is not absolutely certain, but Dr. Noble tells me that they are unquestionably a species of Bufo, probably *B. woodhouseii*.

Bufo boreas halophilus Baird and Girard.

Tadpoles of this species were found in abundance in hot spring No. 135 near the Sulphur Works in Mt. Lassen National Park, California in moderately saline (1.0034), very slightly alkaline (pH 7.4) water at 30.6°. This small stream flows rapidly along a rocky bed and is well supplied with algæ.

Adults of this species were observed commonly in Pyramid Lake which is quite noticeably saline. They may be seen swimming in the water at the edge of the lake which is practically devoid of shade or vegetation of any sort and most of their time, at least during the day must have to be spent in the water to avoid dessication. A number of individuals were seen also in damp niches just above water level on the massive pyramid of tufa which rises out of the body of the lake where they are washed by wave action which is considerable at all times in a lake of such large size. Due to the conditions prevailing here they appear to have become as highly aquatic as frogs. No frogs were seen in Pyramid Lake, however.

Bufo compactilis Wiegmann.

Adults of this species were taken in the seepage area of a water hole thirty miles north of Gerlach, Nevada in water evidently of thermal origin, but quite cool (19.2°; sp. gr. 1.0007; pH 8.8) together with three species of frogs. This spot which is isolated by miles of desert from the nearest permanent water has acquired a populous colony of amphibians.

Scaphiopus hammondii Baird.

Of the tadpoles listed here, Dr. Noble writes: "These tadpoles are certainly referable to Scaphiopus—the spadefoot toad—but they do not agree entirely with the published descriptions of *hammondii*, the only spadefoot toad known from this region. Their mouthparts are much nearer *S. couchii* which is not believed to occur in your locality. It is possible that *S. hammondii* exhibits much greater variation in tadpoles than we have hitherto believed." They were collected at Thermopolis, Wyoming in hot spring No. 35 in water of specific gravity 1.0056 and pH 7.9, at a temperature of 37°–38.4°. On being disturbed they would move into hotter parts of the pool as far as 45°, but soon returned to the cooler water as noted above. As the tadpoles were numerous, the species evidently breeds regularly in this spring.

Hyla regilla Baird and Girard.

This frog was previously collected by us in northern Nevada in hot spring No. 16 (Brues '28, p. 205) south of Denio, Oregon. In 1930 we found it again in numbers in hot spring No. 113 north of Gerlach, Nevada together with *Bufo compactilis* and *Pseudacris triseriatus*. Here the water is cool (19.2°) only slightly saline (1.0007) and moderately alkaline (pH 8.8). The salinity and hydrogen ion concentration of the water is similar in the two places (1.0015 and pH 8.5 in hot spring No. 16).

Tadpoles were collected at Paraiso Hot Springs, California in hot spring No. 143 living in water at 33.4°. At this place the water is more decidedly saline (1.0042) and it is not noticeably alkaline.

From our several observations we may say that this species occurs in water of slight alkalinity at moderate temperatures.

Pseudacris triseriatus Wied.

This frog was taken also in numbers with the preceding species, north of Gerlach, Nevada.

Rana pretiosa Baird and Girard.

On our first visit to Yellowstone Park in 1923 we secured tadpoles of this frog in a warm stream at Mary Bay near the northern shore of Yellowstone Lake. In 1930 we found adults near Tower Junction in hot spring No. 57A along the margins of the water and swimming about in water ranging from 39.2°–41.6°. The water here has a specific gravity of 1.0057 and pH of 7.0, almost exactly the same as the temperature we recorded previously for the tadpoles.

REPTILIA.

Although there are several references to the occurrence of turtles in the thermal waters of Europe, we have only once encountered them in the springs we have examined, at hot spring No. 131. This is really a rather cool stream with a temperature of 28.2°, at Clear Lake Park, California. Here we took a specimen of the Californian turtle, *Clemmys marmorata* Baird & Girard.

We learned, however, through conversation with Mr. Walti, owner of the ranch on which hot spring No. 103 is located, near Cortez, Nevada, of a very interesting and surprising experience which he had had with the common Florida alligator. In 1917 two very small alligators were received and exhibited at a local bank nearby, after which they were given to Mr. Walti. Being much interested in natural history and very well informed concerning biological matters he wished to see if they might be able to find a congenial habitat in the warm overflow from this hot spring. They were therefore given their freedom in the lower part of the stream where it enters a marshy area. For a time the alligators disappeared and had been well nigh forgotten till one cold, wintry morning in 1927, ten years later, a well-grown alligator about five feet in length was found dead in the marsh, presumably frozen to death. There seems to be no reasonable doubt that this was one of the small specimens placed there by Mr. Walti. I did not see the skin, but he showed me a photograph of the specimen. Not having seen the other animal, he supposed it to be dead, but of course there is a possibility that it may have still been alive at the time or that it may still survive.

I have absolutely no reason to doubt the accuracy of Mr. Walti's observation which furnishes an actual case of the opportunity which hot springs may offer for the extension of tropical animals into cooler regions.

ENVIRONMENTAL FACTORS.

As I have attempted to stress in previous discussions, there are several limiting factors which play a part in determining the composition of the fauna of thermal waters. Those which are most notable are the abnormally high temperature of the water, the highly variable hydrogen ion concentration and the variable salinity which is comparable to that of brackish water. It cannot be assumed that any of these act entirely independently of the others, although there can be no serious question either of the prime importance of temperature, or of its limiting action in many instances practically without reference to variations in other conditions. Hydrogen ion concentration is on the other hand only an indication of a series of factors in combination as it depends upon the chemical nature of the salts in solution, and also of the gases similarly present in solution. With reference to salts the composition of the water is quite stable, but as gases may be given off or absorbed with comparative ease and rapidity, their amounts are not at all uniform in the different parts of thermal pools or streams having the same origin. The principal reason for considering the relation of hydrogen ion concentration to the fauna is that it has actually been shown by observation to serve in many cases as an index of the suitability of waters to support animal life of various kinds. Variations in salinity as measured by the specific gravities of waters will be considered mainly because of the fact that, as I have pointed out previously, this serves as a basis for comparison of the faunas of thermal waters, fresh-water, brackish waters and in the sea.

TEMPERATURE.

In comparison with most non-thermal springs, and with fresh water pools, ponds and streams, and also with most parts of the ocean and its estuaries, thermal springs are characterized by two temperature peculiarities. The first and most obvious is their abnormally high temperature and the second is their nearer approach to uniformity in temperature throughout the year. In tropical regions the latter is of course more or less true of all non-thermal waters, but in temperate regions, including those where the hot springs under discussion are located, seasonal variations in temperature in waters of direct meteoric origin are very great. Ranges of temperature from well below the threshold of activity (0° (32° Fahr.)) to ordinary summer temperatures in the neighborhood of 20° C (68° Fahr.) represent the normal variation for small bodies of water in these regions. There is of course no such wide variation in the temperatures of hot springs nor ordinarily in the overflows from springs during the course of the seasons, and practically all springs with considerable flow undoubtedly maintain temperatures sufficiently high to permit of activity and development throughout the winter over a part of their course. This means continued activity for most of the organisms in the springs, except in the case of insects with aërial imagines where winter activity of the aërial stage is inhibited by the low air temperatures. Just how such abnormal conditions can be adjusted to the seasonal life-cycles of certain insects is not clear and must, I think, await observations during the cold season. Certainly one of three possibilities must prevail in each case. Either the imaginal stages must be capable of hibernation, or the life cycle of the aquatic stages must extend throughout the entire winter; or the springs must be repopulated each year from nearby waters. The last possibility could not occur in many isolated hot springs which are separated by many miles of desert country from the nearest water. Some isolated springs may maintain their fauna by the overwintering of individuals in the cooler parts of extensive overflows or along the margins of shallow pools, but this does not appear fully to account for the fauna of all springs. Herein must lie the reason for the absence, so far as we have been able to determine, of any insects with aërial adult forms that are entirely restricted to hot springs, and the only possibility for such to evolve would be through a synchronization of the life-cycle so that the aërial stage should always appear at the proper season for reproductive activity. Among animals with completely aquatic life-cycles such adjustments are not necessary, and it appears actually that the mite Thermacarus, and certain species of ostracod crustaceans are entirely restricted to hot springs. Incidentally these forms are the ones that have extended farthest upward the range of resistance to high temperatures.

On the basis of the observations made by ourselves and others it is now possible to make rather dogmatic statements concerning the extent to which animals are able to adapt themselves to high temperatures, and to establish quite accurately the upper lethal limit for animal life. This limit is quite definite for many types of animals that have been studied in this respect and so far as our experience extends and I think also that of other observers who have obtained accurate data. These all show that the upper temperature limit compatible with animal life is approximately 50° Centigrade or 122° Fahrenheit. Our highest actual record relates to the thermal mite, *Thermacarus nevadensis* which we have found living actively in water at 50.8° C., and two others to ostracod crustaceans in water at 50.0° C. In such cases there has been undoubtedly a very considerable acclimatization to heat as other species of insects and crustaceans die immediately if placed in water of that temperature. In a like manner body temperatures in the neighborhood of 40° C. (from 38°–42°) are known to cause death in numerous poecilothermic and homœothermic animals alike, while the great majority of cold-blooded animals are unable to withstand prolonged exposure to temperatures considerably lower than these. It may be said therefore in round numbers that a certain small number of animals have become in some way adapted so as to extend the upper temperature range compatible with life about 10 degrees in the centigrade scale, but that the process of adaptation is not capable of being indefinitely extended; rather it approaches with great difficulty a very definite limit. In the case of higher plants the upper limit is approximately the same as our highest record is 52.5° for the roots of a grass, *Distichlis spicata* which occurs abundantly about many of the hot springs in the regions that we have investigated.

As is well known such susceptibility to high temperatures does not obtain among the most primitive types of plant life, such as the bacteria and blue-green algæ. There are thus two types of organisms with fundamentally different reactions toward high temperatures, the one including all groups of animals and the higher plants, and the other represented by the lowest types of plant life. I have been lead to suggest (Brues '28) that inasmuch as the only known constant structural differences between the organisms of these types is the presence of mitochondria in the one and their absence in the other, that heat susceptibility is associated directly with these cellular structures. Such a conclusion is further borne out by the observable effects of heat near the critical temperatures of 40°–50° C. upon the mitochondria in several instances. The extended observations made by us since that time all agree with my previous statements concerning lethal temperatures and as was indicated in a postscript to my former paper ('28) the one apparent disagreement among certain Protozoa has been shown to be based on dubious and probably untrustworthy data.

However, since the state of our knowledge concerning the temperature relations of certain of the lower groups of plants will scarcely permit of generalizations, it does not seem wise at this time to go further than to state that the absence of mitochondria and the generally associated imperfect differentiation of nuclear structure are characteristic of those organisms that have been able to adapt themselves to life at temperatures above 50° or thereabouts. Conversely the other series of organisms are strictly limited to the lower temperature ranges.

From our discussion of temperature it is clear that we cannot draw any distinction between the adaptations of warm and cold-blooded animals to increases in body temperature.

In a recent book by Pearse and Hall ('28) they have stated that the development of homœothermism in the higher animals has necessitated the disappearance of certain proteins that occur in some pœcilothermic animals since the body temperatures of the former are so high that these proteins would undergo heat precipitation at the normal temperatures that prevail in the tissues in warm-blooded animals. It is thus assumed that an evolutionary change has occurred before homœothermic animals could come into existence. There are, however, as we have seen, numerous types of animals in hot springs that are able to withstand temperatures well in excess of those occurring in the tissues of mammals and birds. From these types no such changes or adaptations would be necessary for the development of homœothermism so that it cannot be said that the absence of such heat-susceptible proteins has necessarily been connected directly with the appearance of homœothermic animals. It must be admitted however that the types which are so far as we can determine most like the probable ancestors of the warm blooded animals (fishes, amphibians, reptiles) are at present not abundantly represented in hot springs. Such could not of course be expected of the reptiles as they are or have been mainly terrestrial. Of some of the early aquatic reptiles we cannot say what their relations to thermal waters may have been, but the present-day lizards and snakes are highly characteristic of hot, desert regions where abnormally high temperatures prevail. The paucity of records of turtles in hot springs, indicates clearly that these animals are not abundantly represented in thermal waters, and that as the most generally aquatic group of reptiles they show no tendency to adapt themselves to an environment where high temperatures prevail.

Certain observations by others have been published since my earlier paper ('28) was written which throw additional light on the extent of acclimatization that has occurred in some animals that are regularly or occasionally exposed to abnormally high temperatures. The first of these, relating to marine pelecypods seems clearly to give further evidence to indicate that littoral species are less susceptible to excess heat than those that live in the lower, more uniformly cooler parts of the sea.

Henderson ('29) has determined experimentally the lethal temperatures for a number of pelecypod molluscs by a gradual heating of the medium and determination of their ability to recover. He finds a wide variation in heat susceptibility among the species that he examined, ranging from 31.5°–48.5°. The more susceptible forms are in general those not normally subjected to heat, while those with limits of 40° or above include such species as *Mya arenaria* (40.6), *Mytilus edulis* (40.8), *Venus mercenaria* (45.2°) and finally *Ostrœa virginiana* (48.5°) which are at times exposed to the heat of the sun. The author finds these variations clearly correlated with the habitats of the several forms. Certainly the resistance of the oyster to heat is extremely great compared with that of other animals, although continuous heat of lower degree would undoubtedly destroy them.

The effects of high temperatures on certain bark-beetles, *Dendroctonus brevicomis* Lec. have been very recently examined experimentally by Miller ('31). Miller finds in the case of larvæ of this species which develop beneath bark that death from high temperatures is dependent to a considerable extent upon coincident dessication. Thus, in individuals removed and exposed to dry air in addition to heat a lethal temperature of from 35°–38° was determined for prolonged exposure, while from 38°–40.5° was the lethal point where evaporation was prevented by sealing up the larvæ in glass vials. At even higher temperatures of from 42°–43° recovery occurs after several hours exposure. Here it is evident that the higher limits must be regarded as indicating temperature effect unmodified by dessication which is a complicating factor in the case of terrestrial, but not of aquatic insects. Although this Dendroctonus must occasionally be subjected to excessive heat in its normal habitat beneath bark, it appears not to have developed any noticeable resistance to high temperatures.

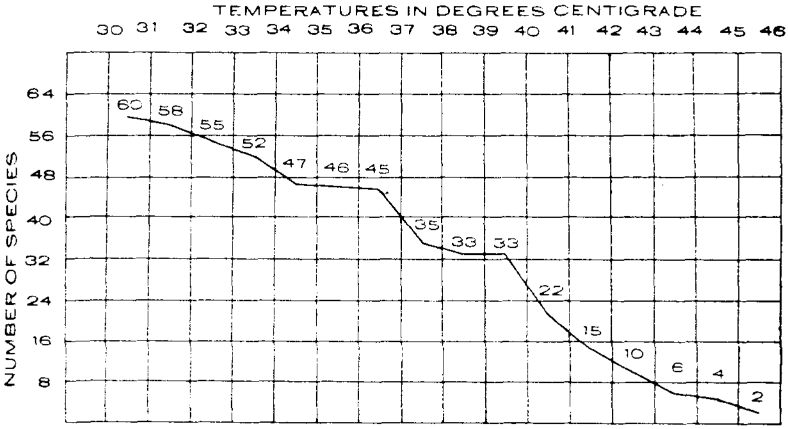

Figure 3. Graph illustrating the number of species of water-beetles collected in hot springs at temperatures of from 30° to 46°, showing the gradual elimination of species and reduction of the fauna as the temperature of the water increases.

Marcus ('29) states that certain Tardigrada will readily withstand temperatures of between 40° and 50° for short periods of time. He found movement to be inhibited on heating to 46°, but the animals all recovered when the medium was allowed to cool. On the other hand Heinis ('21) found that an exposure of two hours at 45° and at 47° destroyed all except certain particularly resistant species (*Macrobiotus hufelandi* and *Hypsibius oberhäuseri*). Observations by Rahm

('21) gave somewhat different results as he found that temperatures of more than 42° for a few minutes caused death.

In connection with the water-beetles which we have found living in thermal waters, I have tabulated the upper temperature limit to which each of the sixty species is known to extend. These data which are much more extensive than those formerly tabulated are combined in the accompanying graph (Fig. 3). It will be seen that there is a gradual and consistent decrease in the number of species as we pass to the higher temperatures. Those known to extend above 30° are tabulated by steps of one degree each, and the number of forms decreases gradually till only two extend into water above 45° and none above 46°. This does not form a very regular curve, but drops much more rapidly in the neighborhood of 37°, indicating that this point may be the critical temperature for a number of kinds of these beetles which belong to several related families. Otherwise the curve, which is based on approximately 300 separate records, is quite regular.

HYDROGEN ION CONCENTRATION.

In all the springs where we have collected material both in 1927 and 1930 the hydrogen ion concentration has been determined as indicated on a previous page. As would be expected from the wide range in physiographic structure of the lands adjacent to the several springs and the variety of rocks and soils through which the water passes before reaching the surface, the dissolved salts and gases in the water are highly variable both in proportions and constituents. It was found that the pH in the entire series of 149 springs where we made determinations ranges from 3.6 to 9.6 which encompasses the entire range of the indicators we had with us (see Fig. 4). At the upper and lower limits it is probable that had we been able to determine them, limits of 3.0 and 9.8 would have been found.

In spite of the fact that such determinations serve only as a crude guide in indicating similarities in composition and presumably the suitability of the several waters for particular species, the restriction of numerous species to certain ranges in pH indicates that this is actually an important factor in determining the distribution of some forms. This of course coincides with numerous observations made by others under the very different conditions that prevail in fresh-water streams, ponds, and lakes.

As may be seen from the accompanying graph (Fig. 4), there appear to be four or possibly five peaks, representing pH values of 3.6 or less

(11 springs), 7.4 (11 springs), 8.4 (14 springs), and 9.6 or above (9 springs). 54 springs, or almost exactly one-third, fall in the range of pH 7.4–8.5. Since only 17, or scarcely more than 10% of the entire number, are below pH 6.5 and only 32 or about 20% are below pH 7.0, we may class these hot springs in general as alkaline waters, remembering the notable exceptions with strongly acid waters. The springs of very low pH (3.6 to 6.2) are all either in Yellowstone Park or Mount Lassen Park, both of which are regions of notable volcanic activity and the particular springs are either ones closely associated with vents that are depositing crystalline sulphur or with certain geysers that are depositing siliceous sinter. One should suspect sulphur dioxide in the former case. In the case of geysers it has usually been assumed that excess of alkaline carbonates is the cause of the silica in solution but it would seem that this cannot be always true.

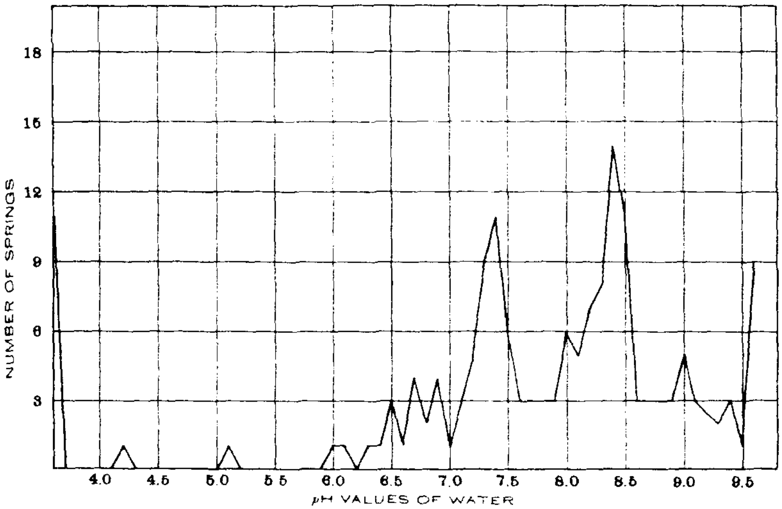

FIGURE 4. Graph showing range in hydrogen ion concentration of the waters in 149 of the thermal springs examined.

Since our collections of water beetles include a greater number of species than any other group, I have tabulated (Fig. 5) the distribution of this series which comprises in all 61 species. As many of these are represented by numerous records (given in detail on the preceding pages) their distribution with reference to the pH values of the waters is very well indicated. Thus combined the tendency for the number of species to decrease rapidly as the water becomes more acid or more alkaline is very evident. The most numerous records fall near pH 8 and slightly above, which represents the most favorable range for members of this group in general. As the number of records that we have accumulated for other groups are much more meagre, I have not attempted to tabulate them at the present time.

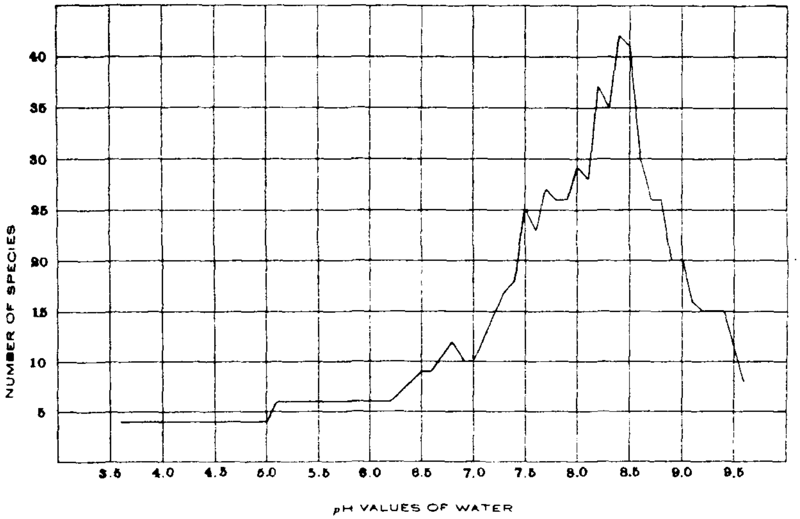

FIGURE 5. Graph showing range of 61 species of water-beetles in hot springs, with reference to the hydrogen ion concentration of the water. Each species is included in the graph over the complete range of pH values at which it has been collected.

The hydrogen ion concentration of the other waters from which animals might migrate into hot springs in the region where we have done our work is a matter which should be considered here, but I have been able to secure very few data relating to this. Shelford ('25) has published a number of pH determinations of waters in inland and coastal drainages in the west which seem to indicate generally a higher pH for the waters of the western plains and Great Basin. The averages of his tables are as follows:

Puget Sound and Columbia River Basins	7.3
Salt Lake and Colorado and Rio Grande Basins	7.8
Mississippi and Great Lake Drainage Basins	7.4

These differences are not very great, but nevertheless noticeable, although of course there are none of the great extremes that are encountered in hot springs and alkali ponds scattered through the west. Omitting the low records of 3.6 or below, we find that the mean of the springs we studied (pH 8.0) is not greatly in excess of the figures given by Shelford for non-thermal waters in the same general region. The majority of the springs are therefore similar in pH values to the waters from which migration from cooler water must have occurred while the more acid springs, where there is a great difference in pH from meteoric waters, have acquired a much more limited fauna.

An interesting comparison with the data just given is afforded by a series of observations on the distribution of fresh-water Copepoda with reference to pH that has been published by Lowndes ('28). These relate mainly to England but include also collections from Spitzbergen and Lake Lucerne, so the list may be considered as representative of conditions over an extended area. Lowndes finds that the pH values of the waters ranged from 3.0 to 9.8 and that none of the species of copepods were obtained over the entire range although *Cyclops langvidus* Sars (3.0–7.2) and *Cyclops vulgaris* Koch (4.6–9.8) show very wide ranges. In breeding experiments he found that the former species would develop through the widest range of pH, water of high pH being toxic only when calcium and magnesium salts were present to any great extent. He concludes that variation in hydrogen ion concentration can have no direct influence on this species and that probably its influence on other species is small.

I have plotted as a graph which is reproduced below (Fig. 6) the data given by Lowndes in his list of species and it appears from the standpoint of diversity that the fauna of the different waters is clearly correlated with the pH, showing a maximum (22 to 33 species) from pH 5.4 to 8.2 and decreasing very rapidly both above and below this range, particularly at the points of pH 4.5 and pH 8.7. Criticism of such a treatment may of course be made on the basis that there are more ponds falling in the middle ranges and consequently greater opportunities for obtaining longer lists for them. As many of the species (more than one-half) were common and represented by extensive records and as will be noticed, especially in the waters with high pH values, that a few species extend far beyond the median ranges, it is evident that pH values clearly indicate conditions in the medium which restrict the distribution of these copepods.

Schumann (1928) has shown that in certain waters Gammarid crustaceans cannot develop in water having an acid reaction (between pH 5.6 and 6.5) although the optimum appears to be between 7.2 and 7.8. In the case of these animals a considerable calcium content of at least 13 parts per million of $CaCo_3$ is necessary for their growth and development.

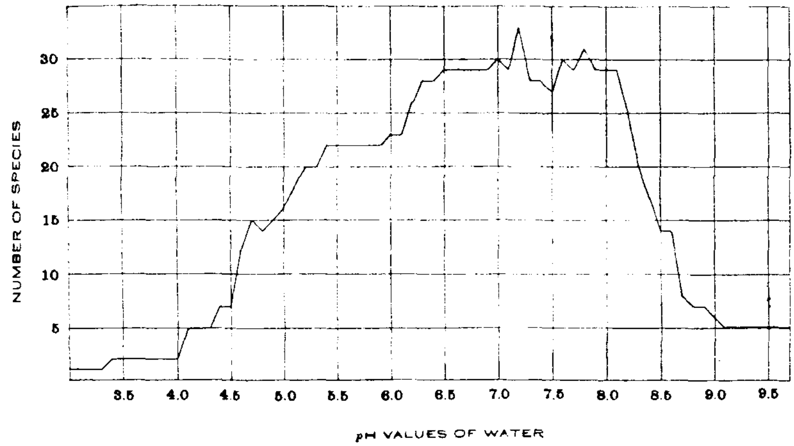

FIGURE 6. Graph showing range of European copepod crustaceans with reference to the hydrogen ion concentration of the water. Compiled from data published by Lowndes ('28).

The relation of hydrogen ion concentration to the development of mosquito larvæ, particularly those of malarial mosquitoes has engaged the attention of several entomologists. The European *Anopheles maculipennis* shows a definite range of tolerance with reference to the pH of the water, at least in peaty soils, as has been demonstrated by Sébenzow and Adowa ('29). They found that the first instar larvæ cannot withstand a pH of less than 5.0, although the third stage larvæ are able to tolerate water at pH 4.3, which point is the limit of tolerance for the developmental stages of this mosquito. Biriukov ('28) found that Anopheles in Russia did not develop in water of pH below 7.2, nor above 7.6 but he does not say what ranges he has investigated.

Similarly Senior-White ('26) has found several species of Anopheles in India extending from pH 5.8 to pH 8.6 which is rather toward the middle of the scale of the values that are ordinarily encountered in waters. The latter author has also made many observations on a variety of other mosquitoes which are embodied in the same publication. As his data are very similar to those of Lowndes for copepod crustaceans, I have plotted them in a similar way for comparison (Fig. 7). One sees in the graph that most of the 46 species crowd toward the range of pH 6.0–8.8, but with a much more pronounced peak at pH 6.6. One species *Stegomyia albopicta* extends over the entire range from pH 4.4 to 9.6. Here it is evident the peak of abundance is well below that shown for the beetles given on a previous page.

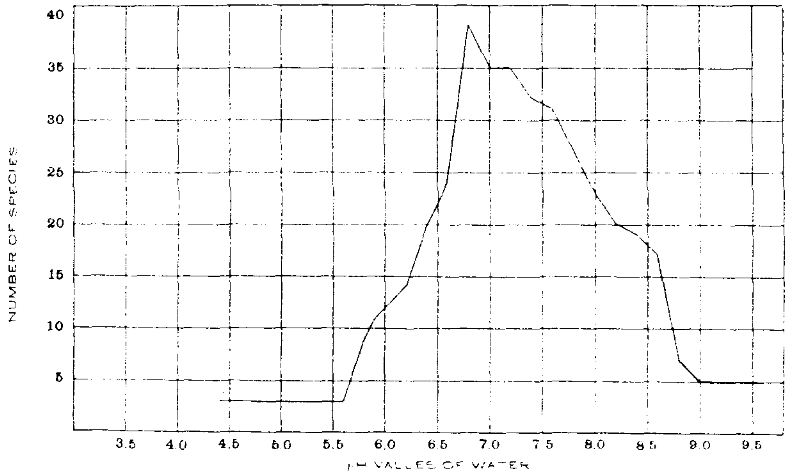

FIGURE 7. Graph showing range of 46 species of Indian mosquitoes with reference to the hydrogen ion concentration of the water. Compiled from data published by Senior-White ('26).

Among molluscs the tolerance of the snail, *Lymnæa truncatula* for water of varying hydrogen ion concentration has been determined by Walton and Wright ('26) who found that it ranged between pH 6.0 and 8.6 while another species of the same genus, *L. peregra* can withstand a somewhat wider range from pH 5.8 to 8.8.

Jewell and Brown ('29) who have studied extensively the fauna of certain bog lakes in northern Michigan come to the conclusion that pH is rarely a limiting factor in the occurrence of fresh-water fish in natural waters as they found fish in waters ranging from pH 4.4 to pH 10.0, no single species extending over the entire range, however. With molluscs they found no species in very acid water as the range for them was from pH 5.7 upwards. The lower figure approximates the point at which the deposition of lime shells would theoretically be inhibited.

Similarly with fishes, Schäperclaus ('27) finds from observations in Germany that the lower limit of tolerance of fresh-water fishes is sharply limited in the acid range of pH 4.8.

From the data just presented we are certainly not justified in concluding that the reaction of the medium as measured by its pH is without influence in determining the composition of its fauna. Whether this is a direct influence or whether it may depend upon the presence of suitable food cannot be stated, but the fact remains that the pH value in particular cases is one of the indices of the suitability of the habitat.

SALINITY.

Of undoubtedly lesser importance than temperature in determining the composition of the thermal fauna are differences in the salinity of the water in hot springs. Since water becomes a better solvent of nearly all salts as its temperature is raised we must expect on the average to find a greater amount of substances in solution in hot springs than in other natural waters, excluding of course the sea and certain smaller bodies of water without outlets in which the concentration of dissolved substances is continually increased through evaporation. In the region where the hot springs we have investigated are located there are numerous examples of this condition such as Great Salt Lake and Mono Lake which show much higher salinities than the ocean. There are even others where the dissolved salts are concentrated almost to the point of crystallization or where solution occurs only with the advent of periodic or occasional rains. Even Salt Lake and Mono Lake support a fauna and flora of meager extent while the many less strongly saline waters of the region are well populated by organisms adapted to life under the conditions that prevail so that there are numerous forms present elsewhere in the region which are living in waters of salinities comparable with those encountered in the thermal waters. That the salinity of the hot springs has been a really important factor in determining their fauna is shown indirectly, but I think quite conclusively, by the types of insects and other invertebrates found there.

As might be expected there is great diversity in the salinities of the 154 hot springs that I have reported upon. On the basis of specific gravities when these are reduced to a uniform temperature of 15° C. we find that the majority fall between 1.0010 and 1.0070 (Fig. 8) with a very clearly defined peak in the neighborhood of 1.0040. Practically none fail to show an appreciable amount of salts in solution and a few not included in the figure show very excessive amounts (1.0146–1.1181) due to high concentrations of NaCl and other salts. Only at the highest concentration of 1.1181 in hot spring No. 151 did we fail to find macroscopic forms of animal life.

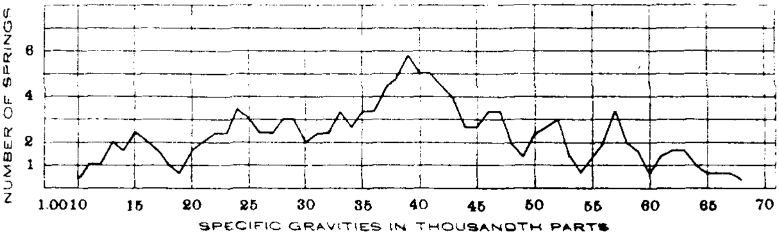

FIGURE 8. Graph showing range in specific gravity of the waters in 135 of the thermal springs examined. A few other springs falling beyond this range have been omitted.

In my previous account of the hot springs fauna I have called attention to the fact that the general constitution of this fauna with reference to groups, genera and species runs closely parallel to that of brackish water. From this I have drawn the obvious conclusion that those types which have been able to migrate from fresh water into brackish water without encountering insurmountable difficulties have been likewise able successfully to invade the generally strongly saline waters of thermal springs. If one views the series of types concerned it is evident that they are those which are able for some reason to live and reproduce readily water of greater salinity than that of the fresh-water medium from which they have come, since the hot springs fauna comprises almost entirely fresh-water animal types.

The physiological basis for this difference between types of animals must be dependent upon their actual ability to adjust themselves to the changed osmotic pressure which they encounter in the hot springs or saline habitat. Since the mechanism or mechanisms for osmotic control in animals whereby their body fluids are kept at uniform, or sometimes variable, concentrations with reference to the external medium seem never to have been clearly demonstrated, it would be idle to speculate further in the present connection. In fact, Schlieper ('30) in an elaborate consideration of this question comes to the conclusion that all aquatic animals have mechanisms for active osmotic control, that some (e. g. the crab, *Carcinus mœnas*) may consistently maintain a higher, or even lower, molecular concentration than that of the external medium. Undoubtedly, no simple explanation will finally be offered for the hot springs fauna as a whole, but it is difficult to believe that there are not differences inherent in certain specific types of animals in this respect, in view of the similarities between the thermal and brackish water fauna.

In at least one group of animals, the fishes, investigations by Smith, summarized in a recent paper ('32) and Keys ('31) have shown that in fishes the chlorides of sodium and potassium are excreted by the gills in solution hypertonic both to the blood and to sea water, thus enabling these animals to prevent the accumulation of these salts in the body fluids and to secrete a hypotonic urine. In marine invertebrates the body fluids are usually isotonic to sea water, although those of fresh water forms are strongly hypertonic to the external medium. Whatever the mechanism of osmotic control among these varied animals may be, it cannot always be like that in the fishes as organs similar to the gills of fishes are not uniformly present.

ADJUSTMENTS OF THERMOPHILOUS ANIMALS.

Investigation of the fauna of thermal springs shows conclusively that its components have been selected with reference to their ability to live and reproduce at temperatures which are higher than those which most animals can endure. This adaptation to high temperatures extends however over only a narrow range of about 5° to 20° Centigrade, as the animals live at temperatures of 40° to 50° C. instead of 30° to 40° which is the upper limit of existence for the majority of terrestrial and aquatic animals alike, including both pœcilothermic and homœothermic types. Such a condition of greater resistance to heat may have arisen either by a selection of forms naturally resistant, whereby such forms have found the environment of hot springs not detrimental to their welfare, or it may have come about through actual changes in the constitution of the animals whereby they have become more resistant to heat on account of actual changes in their protoplasmic composition or metabolic requirements. A third possibility that they represent remnants of a promordial fauna which have always lived at high temperatures may be dismissed as I have already indicated ('28) in the case of animals and the higher plants, although it may be true of some primitive types of plant life.

It is difficult to make any positive statement as to whether the fauna of thermal springs is wholly or in part made up of naturally resistant forms. Many observations on diverse animals show that the inhabitants of cooler environments are not all equally susceptible to heat when this is tested experimentally, some showing the ability to withstand temperatures similar to those met by the species that live in hot springs. Some such forms have probably become a part of the thermal fauna. On the other hand species in several groups show a great lack of heat susceptibility when compared with related forms from cooler environments and in such cases actual physiological adaptation to heat must have occurred. So far as known, however, in animals adaptation to heat never occurs beyond the critical point of 50°–51° centigrade.

Adjustment to the higher osmotic pressure of saline waters is of course not restricted to members of the thermal water fauna as the same adjustment is necessary for all fresh-water animals that invade brackish waters of the sea. That salinity is a factor in determining the composition of the thermal fauna has been shown by the similarities we have pointed out between it and the estuarine faunas.

Finally, in company with all non-marine faunas, the thermal fauna appears clearly to be selected to some extent with reference to the extremely variable hydrogen ion concentration of thermal waters.

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THE PRESSURE COEFFICIENT OF RESISTANCE OF FIFTEEN METALS DOWN TO LIQUID OXYGEN TEMPERATURES.

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INTRODUCTION.

IT WOULD be of great interest to determine the effects of high pressures on the properties of materials without any of the complications arising from thermal agitation, that is, at 0° Abs. Such measurements at absolute zero itself, or even within a few degrees of it, will probably, however, remain always impossible, not only because of the inaccessibility of the absolute zero, but because of other technical difficulties which will appear later, so that we will doubtless have to content ourselves with more moderate excursions into this field.

The measurements recorded in this paper are the first results of my attempt to combine high pressures and low temperatures. Technical difficulties were fully expected, but they proved to be even greater than anticipated, and the experiments have now been continued over several years. The net result is that I have now measured electrical resistance up to 7000 kg/cm^2 at liquid oxygen temperature, 90° Abs, and in this paper I present measurements on 15 metals over this range. The temperature limit has been set by the fact that only ordinary liquid air has been at my disposal; it is probable that the identical technique would permit measurements to several thousand kg/cm^2 at liquid hydrogen temperatures, were this available. The same apparatus should also serve with little modification for other sorts of measurement, such as thermal e.m.f.

In the following more attention than usual will be devoted to the technique used in entering this region.

TECHNIQUE.

General Technique. The most important difference between high pressure experiments at ordinary temperatures and at low temperatures is at once obvious in the restriction on the fluids which can be used for transmitting hydrostatic pressure. At room temperature, ordinary liquids, such as the lighter hydrocarbons, may be employed up to pressures of 12000 or 15000 kg/cm^2. The range is limited either by freezing or by too great an increase of viscosity, which is greatly affected by pressure. At liquid air temperatures, freezing occurs at much lower pressures, and an inspection of tables of melting points, as in International Critical Tables, will show at once that there is no known substance liquid under ordinary conditions which will remain liquid at liquid air temperatures over any pressure range worth considering. It is necessary, therefore, to have recourse to the so-called permanent gases, and even here the possibilities are very limited. Work by Simon and collaborators[1] on the solidification of gases under pressure shows that nitrogen, for example, solidifies at a pressure of 1500 kg/cm^2 at 90° Abs, and that the only possibilities are helium and hydrogen. Of these hydrogen can be at once eliminated because of its chemical attack on steel at high pressures,[2] it being impossible to retain without rupture a pressure of more than 9000 kg of hydrogen in a steel vessel, even for a short time, and after prolonged use, the pressure of rupture may be very much lowered, with accompanying danger. Helium, then, remains the only possibility. Simon's melting curves show by extrapolation that the freezing pressure of this at 90° Abs. is probably of the order of 15000 kg, thus making accessible a sufficiently wide pressure range. Fortunately helium is now sufficiently inexpensive so that it can be freely used in the laboratory, and an experiment which would have been absolutely out of the question several years ago, now at least enters the realm of possibility. The question had still to be answered, however, whether helium would penetrate steel like hydrogen; its chemical inertness offered ground for optimism.

The use of a gas as pressure medium demands the use of precompressors in order to get sufficient volumes. The design of these is sufficiently straightforward, and will not be described in detail. Two such precompressors were used. The first receives the helium at tank pressure, initially 1600 lb/in^2, which becomes less as the tank becomes exhausted. This first precompressor contains a single double-headed piston, with packing on both ends. Such an arrangement is better than the more usual mercury separator scheme, because there is no danger of mercury getting into wrong parts of the apparatus. One side of the piston is acted on by a suitable glycerine and water mixture from a hand pump, and the other end of the piston compresses the helium. The diameter of the piston is 1¼ inches, and the length of stroke 18 inches. The final pressure reached with this compressor is about 10000 lb/in^2. The pump is one of the hand pumps which I have used in all my other high pressure work for actuating the various hydraulic intensifiers. Since use of a gas demands the handling of large volumes, this pump has been fitted with a motor drive, which may be detached at pleasure, with much success. Beyond the first precompressor is the second, which is one of the 4 to 1 intensifiers used in my other work and described elsewhere.[3] There is a needle valve by which the first compressor is cut off during the action of the intensifier, and there are also valves by which the same pump which at first operated the separator may now operate the intensifier. The intensifier connects directly to the high pressure apparatus, through a bye-pass at the upper end of the working cylinder, exactly as in my other high pressure apparatus. The intensifier is operated to a pressure of about 2000 kg/cm^2, and then the piston of the high pressure apparatus comes down, shutting off the bye-pass. The remainder of the high pressure range is reached with a single stroke of the high pressure piston. The working high pressure cylinder, that is, the cylinder in which pressure is produced by the motion of the piston, is at room temperature, as in most of my experiments. This cylinder connects through a fine pipe with the lower cylinder, in which is the resistance to be measured, and which is immersed in a vacuum flask with liquid oxygen. The internal diameter of the working high pressure cylinder is 5/16 inch and the length of stroke 3 inches; the diameter of the lower cylinder containing the resistance specimen is 5/16 and its length 5/8 inches. These dimensions are evidently such as to permit building up a high final pressure if there is no leak.

Much time was spent in finding how to make tight connections and electrically insulated joints at low temperature. By far the simplest electrical set-up would be to take the leads of the resistance out through a plug screwed into the lower cylinder, as in my measurements at higher temperatures. The ordinary construction for the electrically insulated plug proved, however, absolutely unworkable at low temperature. the rubber by which mechanical tightness was secured becoming so hard and brittle as not to function. Efforts to modify the dimensions or to find other materials were unsuccessful, and finally the attempts to take the leads out through the cold cylinder were abandoned, and the leads taken out through the connecting pipe to the upper cylinder, at room temperature, as had once before been done when measuring resistance at a temperature so high as to char the rubber insulation. The problem of getting the insulated leads out of the upper cylinder will be discussed later.

The proper packing of the connecting pipe proved unexpectedly difficult. At higher temperature the problem of packing offers no difficulty at all when the principle of the "unsupported area" is used. Almost anything depending on this principle is successful; if rubber washers are used they may be whittled out in the crudest way with a jack knife, and even careless machine work on the metals parts is not fatal. But the viscosity of a gas is so low that it will escape at high pressure through almost molecular holes, and packings which are sufficiently plastic to close the ordinary imperfections and scratches to leakage by an ordinary liquid are freely permeable to compressed helium. My first connecting pipes were made for the ring type of packing of my ordinary high pressure apparatus, in which, after sufficiently high pressure has been reached to induce plastic flow, the eventual packing is provided by a soft steel ring forced by the pressure against a hardened conical shoulder on the pipe. This had to be given up, for although it could be made to work sometimes, it proved too uncertain. It was replaced by a lead packing, lead retaining its fluidity under pressure and at low temperature very much better than

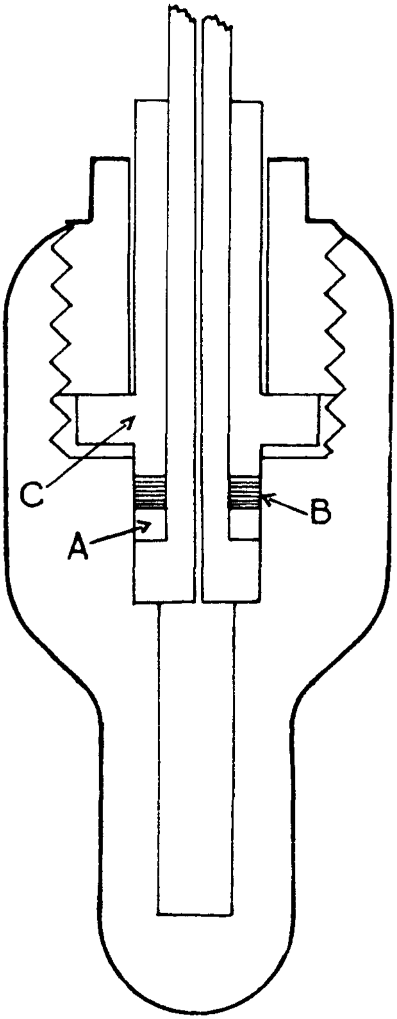

Fig. 1. The lower cylinder and connecting pipe and packing. A is lead cast around the pipe and B split discs of iron or nickel. The hardened retaining piece C is also split.

anything like rubber, for example. The packing is sufficiently indicated in Figure 1. It is of a type which I have used before and it has also been used in plugs for closing holes. The head on the end of the pipe has to be made integral with the pipe, and cannot be threaded on, because a thread always introduces weakness, the pipe being very likely to rupture by the "pinching off" effect at the bottom of the thread. The dimensions of the pipe were approximately as follows: inside diameter, 1/32 inch, outside diameter 7/32 inch, diameter of head 7/16, thickness of head 3/16 inch, length 12 inches. The pipe was drilled from the solid rod and then turned true to the hole. The drilling of so long a hole of such small diameter offers some difficulty, but nothing different in kind from difficulties already mastered in making similar drilled pipes of larger diameter. In drilling, the pipe was mounted in a special rotator, running at about 4000 R. P. M., and the drill was fed in by hand. The necessity for a solid head on each end of the pipe introduces obvious difficulties of assembly, which were overcome by various devious devices. The lead packing rings were cast in position around the pipe and turned to size. In the normal packing of this type for use at higher temperatures the lead is retained by a solid washer of mild steel, which in turn is retained by a washer of hardened steel, which again is retained by the screw plug. The washers of soft and hard steel evidently had to be modified because of the geometrical impossibility of slipping them over the heads on the two ends of the pipe. The soft steel washer was replaced by a number of washers of sheet iron or nickel 0.015 inch thick. Each of these discs was split along one radius. By means of this split it could be opened up, snapped over the pipe, and then pressed back to circular form. By staggering the splits of successive discs, the equivalent of a single solid washer was built up. The hardened steel washer of the normal packing was replaced by a split steel cylinder with a heavy flange. The hole in the screw plug was made large enough to slip over the head on the end of the pipe, as shown in the diagram. It will be seen that the entire outward thrust on the split cylinder gets itself transmitted to the screw plug through the medium of a shearing stress tending to cut off the flange at its roots.

This type of packing for the connecting pipe has proved entirely satisfactory, when used with proper precautions. Among these it is important that the bearing surface in the cylinder against which the lead is forced must be polished and free from scratches.

The order of the experiment was usually a series of measurements at CO_2 temperature followed by those at 90° Abs. It proved to be necessary to tighten the screw plug as much as possible after the run at CO_2 temperature to overcome the effect of the differential contraction between iron and lead on cooling to 90° Abs. At first it appeared that the only way to reach simultaneously low temperatures and high pressures without leak was to raise the pressure at room temperature to perhaps 5000 kg, a pressure so high that the lead is forced plastically into all the crevices, and then to lower the temperature. But this has disadvantages, it being impossible to make resistance measurements on the first application of pressure at low temperature in this way, and also there is considerable danger in the manipulations incidental to getting the liquid oxygen in place and the temperature adjusted while the pressure is in the apparatus. The procedure of tightening the packing between runs has none of these disadvantages, but on the other hand there are sometimes leaks. However, success was attained often enough to outweigh the disadvantages of the other procedure.

The dimensions of the lower cylinder are sufficiently indicated in the diagram. They were so chosen as to reduce the weight of metal to the minimum consistent with safety.

All the packings at the upper cylinder, which remains at room temperature, were made with rubber, there being no rigidity to hinder at this temperature as at lower temperatures. Care is necessary even under such apparently favorable conditions as these. The rubber washers must be smooth and geometrically perfect; the procedure was finally adopted of cutting them in the lathe with a special knife. The holes in the cylinder which receive the rubber washers must also be smooth and polished. It was found to pay to polish the holes before each assembly with a special polishing tool mounted on a dentist's flexible shaft.

The electrically insulated connections were all made in the top cylinder, one for the manganin gauge coil with which pressure is measured, and three connections to the specimen in the lower cylinder, the fourth current connection to this specimen being grounded to the cylinder. Hitherto the insulated connection has been made through a special plug, carrying on its axis an insulated lead. The plug could be made up separately and then screwed into place in the cylinder. This procedure was now abandoned in order to save room, since the entire apparatus had to be kept small in order to minimize the quantity of gas used. Figure 2 shows the new arrangement. The central stem is solid and insulated from the walls of the cylinder by bakelite, mica, and rubber as shown. As in the previous design of plug, the mica is so disposed as to carry the largest part of the mechanical stress. It is essential that the spaces between the

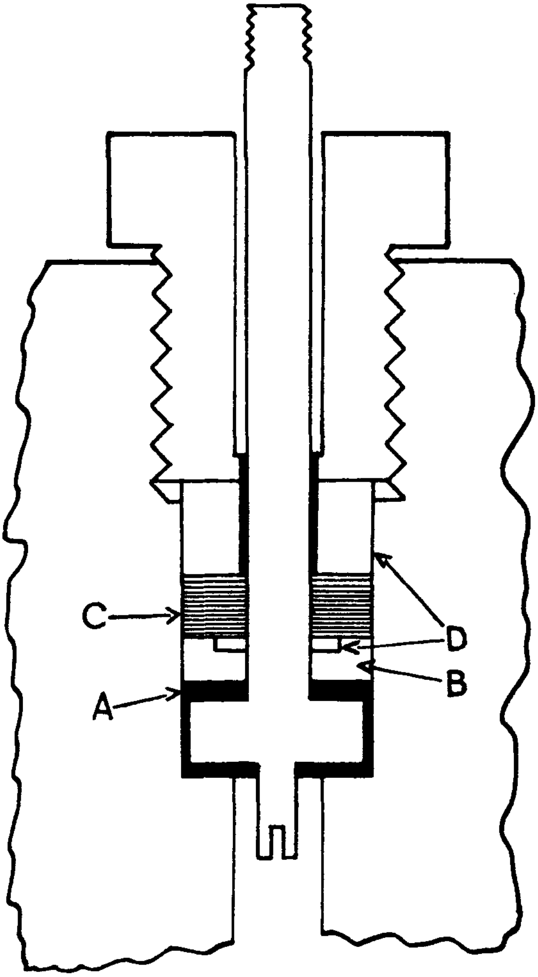

Fig. 2. New type of insulating lead adapted to be directly screwed into the cylinder. A is bakelite, B soft rubber, C mica, and D hardened steel.

stem and the upper hardened steel washer and between the head of the stem and the cylinder be as thin as is consistent with good insulation, or otherwise the soft rubber packing shears off a ring of mica. The diameter of the head must be a larger fraction of the diameter of the solid stem than when pressure is transmitted by a liquid in order that the excess pressure developed in the rubber by the principle of the unsupported area be sufficiently great to overcome the rigidity of the rubber and prevent the gas from leaking. In assembling it is necessary to take the same precautions as with the other type of assembly. The entire cylinder is baked at a temperature of 130° C immediately before assembling to get rid of moisture, the various washers are similarly baked, and the assembly is made with tweezers, taking care not to let the moisture of the fingers get on any of the washers. After assembly the insulation was tested on a bridge sensitive to 100 megohms, and the insulation required to be as good as this. It was checked by blank experiments that this insulation is maintained under pressure.

The three lead wires to the specimen in the bottom cylinder are each of enameled copper 0.005 inch in diameter. The three wires are twisted together into a single rope and then bound together with two or three coats of insulating enamel baked on. Considerable difficulty was found before a suitable method of dealing with the leads was developed. It is necessary that the wire be fine and that the insulation be perfect. The finer the wire, the greater the ratio of perimeter to cross section, and the greater the danger of rupture under the viscous drag of the gas as it passes through the pipe from the upper to the lower cylinder. Silk covered wire was found to be too clumsy; the viscous drag of the gas is so great that rupture frequently occurred, and also there were frequent short circuits due to the breaking through of the insulation where one wire rubbed on another. The use of insulating enamel instead of silk, and also the device of binding together the three wires with enamel, thus increasing the ratio of cross section to perimeter, solved the problem sufficiently well. However, a triple stranded lead made of this wire should not be used for more than two runs, the enamel cracking off at the bends. It is always necessary to carefully test the insulation of each strand before assembling.

The method of assembling was as follows. At the lower end of the central hole in the upper cylinder and just above the end of the connecting pipe there is an insulating block of bakelite bearing three brass ferrules, each of which is soldered to one of the strands of the triple conductor. Just below the ferrules there are three small pockets in each of which a short length of one of the strands of the conductor is coiled. The function of this is to allow some longitudinal motion of the conductor in the pipe as gas passes back and forth. If there is sufficient force to produce longitudinal motion, the conductor uncoils a little out of its pocket, instead of breaking off at the soldered connection. Each of the ferrules registers exactly with the center of one of the side holes through the cylinder and its insulated plug. This plug bears on its end a prolongation in the shape of a piece of brass wire, which is split and the prongs spread. When the plug is pushed home the prongs are forced into the brass ferrules, thus making a spring connection which has sufficient flexibility to allow for the outward motion of the plugs when pressure is applied, compressing the various washers. The resistance specimen is now ready to attach at the lower end. The specimen was usually in the form of wire about 30 cm. long, wound on an insulating spool, usually of "pipe stone." This spool is held in position against the lower end of the pipe with a hollow screw, threading into the end of the pipe, through which passes the triple conductor. The three strands of the conductor are bent around and soldered to the terminals of the specimen, and the fourth connection is made by soldering across to the pipe. The whole is finally protected by a layer of paper, and then the bottom cylinder slipped into place over the spool, and the screw plug turned home.

The manganin gauge was made of wire 0.003 inch in diameter obtained from the Driver Harris Co. The German manganin which I have used in all my other high pressure gauges was not advantageous because of its larger diameter, 0.005 inch, which would have resulted in smaller electrical sensitivity because of the lower resistance possible in the space available. The new manganin was calibrated over the entire pressure range against the old, and the relation found to be linear within 0.1%, approximately the accuracy of the measurements. This is very much more satisfactory than some manganin of American origin which I tried to use for gauges a number of years ago, and which had, among other defects, large wanderings of the zero. Evidently the quality of American manganin has by now sufficiently improved so that it may be used for pressure gauges with as satisfactory results as the German manganin. The composition of the American manganin cannot be quite the same as that of my German manganin, however, for its pressure coefficient of resistance is appreciably higher. $\frac{1}{R_0}\frac{dR}{dp}$ is 2.290×10^{-6} (pressure in kg/cm^2) for the German manganin, against 2.379×10^{-6} for the American.

Another feature not necessary when pressure is transmitted with a liquid is some method of checking each joint for leak during the progress of the experiment. It was found to pay to build around each joint a special leak tester. This was merely a small air tight chamber made of brass tubing, and pressed by a spring against a rubber washer which separated it from the cylinder, connecting through a small side tube to a nozzle opening under water. Leak was disclosed by bubbles rising through the water. The electrical connections were brought out through split rubber plugs in the ends of the leak testers. Five leak testers were used, four for the electrically insulated leads and one for the connection of the pipe into the top cylinder. The pipe connection to the lower cylinder was immersed in liquid oxygen and thus automatically provided its own test for leak. In addition to the stationary connections, the moving plug on the end of the piston might leak; leak here could be detected with a little Nujol painted on with a brush where the piston enters the cylinder.

Behavior of Steel under High Helium Pressures. With the technique outlined above measurements were made to a maximum pressure of a little over 7000 kg/cm^2. The pressure limit was set by the rupture of the steel. It was very soon found that helium under high pressures ruptures the containing vessels, as had been feared. A great deal of preliminary work was done in trying to find the cause of this rupture, and to devise means to prevent it. Special tests were made on about a dozen different grades of steel. Rupture usually took place in the neighborhood of 6000 kg/cm.^{2} This rupture might take various forms. Sometimes a fine seam was opened along a radial plane, so fine as to be hardly visible; at other times the cylinder was shattered into small pieces. The appearance of the ruptured surfaces was always different from the appearance when the rupture is produced with hydrogen, and the conclusion was soon drawn that the action was not chemical, as in the case of hydrogen, but mechanical. At first I thought that it might be connected with the hardening of the steel, since it is known that the density of hardened steel is less than that of untreated steel. Measurements of the densities of several of the specimens of steel showed a negative correlation between the change of density on hardening and ability to withstand helium pressure without rupture. Finally a steel was found, a Cr-Mo steel, which successfully withstood the action of 12000 kg without rupture. It is true that this pressure was not supported indefinitely, as would have been possible with a liquid, but instead, the pressure at constant volume dropped back slowly from 12000 to 10600. However, there was no evidence of leak, and this performance was so much superior to that of any other grade of steel tried that further search was discontinued, and a complete apparatus made of this grade of steel, comprising lower and upper cylinders and connecting pipe. The test cylinder, which had successfully withstood 12000, was 2 inches in diameter, whereas the upper cylinder of the final apparatus was 4.5 inches. The larger size was needed to provide for the screw connections of the insulating plugs, a feature not necessary in the test cylinder, which could therefore be made smaller. My disappointment was great when the upper cylinder ruptured on the first application of 6000. Various other cylinders were now made from the same grade of steel; one was cut from another bar of the same size, one was from a bar originally of smaller diameter forged up to the required size, and one was from a special forging made by the manufacturers with every precaution to obtain as homogeneous a piece as possible. All of these cylinders broke at about 6000 on the first or second application.

About this time I learned about the "hot acid" method of testing steels, now being used by a number of manufacturers to detect comparatively large scale imperfections, such as inclusions of slag, or particularly regions in which manganese sulfide is segregated. The hot acid test consists in immersing the steel for about 45 minutes in 50% HCl solution at a temperature of 160° F. Imperfections are shown by the more rapid attack of the acid, perfectly sound metal being almost untouched. Application of this test showed at once that the bars from which the 4.5 inch cylinders had been made had marked imperfections toward the center, and that the 2 inch bar, from which the successful test cylinder had been made, was very markedly more homogeneous. In general it appears that the smaller bars, which have been more reduced in diameter from the original billet, are more likely to be homogeneous, but this is not universally the case.

The hot acid test now provides the surest guide that I have found as to whether a given piece of steel is likely to be successful, and I now subject all bars to this test before machining them. Some manufacturers make a special point of providing steel homogeneous to this test, and much may be done by using only such steels, but this is not sufficient, and independent test is always desirable. Also, much may be done by using the proper grades of steel. The Cr-Va steel, of which I have constructed all my high pressure apparatus for use with a liquid as transmitting medium, is particularly likely to be "dirty," and I have had no luck in constructing gas apparatus from it, and have now discarded it entirely. Cr-Mo steel is much more likely to be clean. The cleanest appearing steel which I have found is a Cr-Ni steel, but this has not yet been adequately tested for high pressure gas use, as it is a recent discovery.

In obtaining the final measurements recorded below, at least six lower cylinders were used, and three or four connecting pipes. During all these breaks of the lower part of the apparatus I was exceedingly fortunate in using only a single upper cylinder. The design of this was modified from that described above, so that instead of a large cylinder 4.5 inches in diameter, a smaller one of 2 inch diameter could be used. This has at once the considerable advantage of starting from presumably sounder metal. In the new design the screw plugs for the insulating plugs were not carried in the walls of the cylinder itself, but in a heavy ring girdling the cylinder and a push fit for it. It is this feature of design which made the smaller diameter possible. The material of the 2 inch upper cylinder was "S-M" steel (silicon-manganese) from the Carpenter Steel Co. It is a water hardening steel; it was quenched from 1650° F and then drawn at 750° F to a Rockwell hardness of 51. It was then subjected to a preliminary pressure treatment, first to a pressure of 13500 kg at room temperature with petroleum ether as the transmitting liquid. There was slow yield at this pressure, as there always is, but pressure was maintained by pumping as necessary for nearly 24 hours, and after this time the yield had nearly ceased. It was then further seasoned by subjecting to pressure exerted by nitrogen at room temperature. My idea was that a preliminary treatment by nitrogen might plug the pores in the steel to penetration by helium, because of the much greater size of the nitrogen molecule as compared with the helium atom. The action during the treatment with nitrogen was entirely consistent with the hypothesis that gas was being forced into the pores. Marked dropping of pressure, but with no detectible leak, began in the neighborhood of 8000 kg, much lower than the pressure to which the steel had been accommodated by the previous treatment with petroleum ether. By slowly pushing in the piston, it was possible to build up the pressure, after every increase the pressure dropping back to a value higher than the previous maximum, until in the neighborhood of 10500 kg, after about 24 hours exposure, the dropping back after every increase became so much accelerated that I did not dare go any higher, and judged that the steel had about reached its limit for nitrogen. This decision was strengthened by a curious whistling noise at 10100, perhaps due to escape of gas through the cylinder. The permanent loss of volume during the seasoning, as shown by the permanent displacement of the piston, indicated that the gas had been forced into the pores of the steel. This agrees with previous results which I had found during measurements of the volume of gases at room temperature, and is a source of error in high pressure gas work which should be carefully guarded against.

The upper cylinder, treated as above, was surprisingly successful, and outlived, as already mentioned, six lower cylinders and several connecting pipes, the pressure with helium being regularly pushed to about 7500 kg/cm^2. In fact, from one point of view, the cylinder was too successful, because I had no opportunity to try several other cylinders of other grades of steel which I had already constructed against emergency, and which would have been instructive.

It can be understood that with rupture so frequent an occurrence there may be considerable danger in experiments of this kind. This was guarded against by surrounding the apparatus with heavy steel plates, arranging mirrors to see the critical parts of the apparatus, and taking care to stay on the right side of the plates while pressure was high. However, the manipulations of closing the valves of the bye-pass etc., had to be made at pressures of about 2500, and there were occasional ruptures at pressures even as low as this. These low-pressure ruptures, however, nearly always followed a previous application of a higher pressure. Various secondary lines of defense guarded against explosions at these low pressures, and care was always taken to make the necessary manipulations as quickly as possible.

Temperature control of the lower cylinder had to be fairly close, as otherwise the chance fluctuations of resistance with temperature mask the changes produced by pressure. Constant temperature baths at solid CO_2 and liquid oxygen temperatures proved adequate; liquid air itself proved absolutely out of the question. Commercial solid CO_2, "dry ice," was pounded to nuggets about the size of a pea, and packed around the lower cylinder in a thermally insulated vessel about 15 cm in diameter and 20 cm deep, and wet with the minimum amount of acetone, to facilitate heat transfer. The liquid oxygen was made by passing compressed oxygen from a commercial tank, about

99.5% pure, through a copper spiral immersed in liquid air. In the interest of economy the spiral was built on the principle of an interchanger, the escaping evaporated air precooling the oxygen as it enters from the tank. At first a great deal of trouble was encountered from stoppage of the oxygen pipe, but by taking out the liquid oxygen at the bottom and arranging the spiral so that flow always had a downward component, the condensed particles of ice or other solid matter were swept along with the oxygen current, and stoppage avoided. The liquid oxygen was placed in a vacuum flask of about 6 cm diameter and 25 cm length. The duration of the experiment was so short, usually about 90 minutes, that the initial supply of oxygen in the flask could be used without renewal, a feature desirable from the point of view of safety, if nothing else. The major source of heat leak was, of course, conduction along the pipe. The level of liquid oxygen was kept constant during the experiment by means of a diving belt arrangement attached to the lower end of the lower pressure cylinder. As oxygen evaporated, air was forced into this diving bell, raising the level of the oxygen in the flask. This manipulation was done by an assistant, who continually (during the experiment) watched the level through a peep hole.

It is an advantage of the potentiometer method of measuring resistance that the measurements are not sensitive to the resistance of the leads, that is, to the level of the oxygen around the pipe. In the preliminary work an attempt was made to measure the resistance by a bridge method with only two leads, and then great difficulty was encountered because of fluctuation of resistance due to the fluctuation of the level of the oxygen on the pipe. The adequacy of the final method of temperature control is shown by the self consistency of the readings and the good recovery of the zero, which never showed any displacement that could be ascribed to temperature drift.

The barometer was read during every run, and slight corrections, given in International Critical Tables (I. C. T.), were applied for the change with pressure of the boiling point of oxygen and the sublimation point of CO_2. The boiling point of O_2 at 760 mm was taken to be $-182.95°$ C, with a change of $0.32°$ for a one inch change of the barometer. The sublimation point of CO_2 was taken to be $-78.50°$ C, with a change of $0.405°$ for one inch change of the barometer. No allowance was made for the effect of acetone on the sublimation point of CO_2, as there do not seem to be adequate data on this subject. Landolt and Börnstein, Vol. I, p. 631, give figures from the old work of Cailletet and Collardeau for the temperature of solid CO_2 when mixed with various liquids, which are obviously wrong, and too high by from 5° to 10°, as could be shown by direct measurement with a toluol thermometer. In the absence of suitable data, and in view of the fact that the amount of acetone was comparatively small, it seemed safest to use the figures for pure CO_2.

In the preliminary work the question was investigated whether it would not be possible to replace the pure helium gas with a mixture of helium and nitrogen, the idea being that the freezing point of the nitrogen might be sufficiently depressed by the helium, and at the same time the nitrogen might plug the pores of the steel and so prevent rupture. The method used in this investigation was much the same as that used at Leiden[4] and also by Simon[1] and his collaborators in determining the freezing curve of pure helium. Both upper and lower cylinders were maintained at room temperature, and each was provided with a manganin pressure gauge. A receptacle for liquid air was built around the middle of the connecting pipe. Both gauges were read as pressure was increased by forcing the piston into the upper cylinder. Freezing of the transmitting medium of mixed helium and nitrogen produces a stoppage in the connecting pipe, with the result that the lower gauge remains stationary while the upper gauge rises. It was found in this way that a mixture of 44% N_2 and 56% He by volume is frozen solid at about 3000 kg/cm^2 in liquid air. No attempt was made to find whether stoppage occurred also at lower pressures, it being obvious that the mixture was not a practical transmitting medium for my experiments. The freezing pressure of pure nitrogen at 90° Abs. has been found by Simon to be about 1500 kg/cm^2. An interesting subject for further investigation would be to find whether the freezing of nitrogen-helium mixtures is governed by some sort of partial pressures law, a possibility which is not ruled out by these observations. It seems more probable to me, however, that the nitrogen separates out nearly pure, with little depression of its normal freezing point, because of the chemical inertness of helium.

Several times, particularly during the preliminary work, the connecting pipe clogged at 90° Abs at less than the maximum pressure of 7000. The reason for this was not always plain. One would suspect moisture or other impurities which might be in the helium, which was commercial helium of 99% purity obtained from the Matheson Co. An attempt was made to condense moisture out of the helium by passing it through a very fine capillary at liquid air temperature, but there was not enough condensation to produce stoppage, and this source of the difficulty is probably negligible. On a couple of occasions the difficulty was definitely traced to imperfect rubber washers in the separator by which the first stage of compression was produced, which allowed the merest trace of the mixture of glycerine and water used as a pump liquid to creep along the cylinder walls into the gas chamber, where it vaporized with enough vapor pressure to plug the connecting pipe on condensation. More careful oversight of the condition of the washers and more frequent renewal removed this difficulty. Toward the end of the experiments recorded in this paper the difficulty rather increased. The last measurements were made in June when the natural humidity of the air was high. Since the separator and intensifier usually contained at the start atmospheric air, this may have had something to do with the trouble, but flushing the system by a preliminary admission of helium did not eliminate the difficulty, and the explanation is not yet plain. It may be connected with the fact that toward the end of the experiments the helium pressure in the tanks was getting low.

Experimental Procedure. The order of experiment was usually as follows. First the lower cylinder was packed in ice and the resistance measured at atmospheric pressure at 0° C. Temperature was then lowered by packing the lower cylinder in solid CO_2 and acetone, and pressure applied, using nitrogen to transmit pressure at this temperature. Readings of resistance at this temperature were made at atmospheric pressure, the tank pressure of the nitrogen, at the end of the separator stroke, about 700 kg, at 1500 kg, at the end of the intensifier stroke after the bye-pass valves were closed, usually between 2000 and 3000, and then at intervals of 1000 kg up to 7000 and back again to 2000, the lowest pressure possible without uncovering the bye-pass and losing gas. The points were plotted as they were measured, and it was almost always found that there was much hysteresis between increasing and decreasing points. This is evidently the seasoning effect of the initial application of pressure which I have always found before in all measurements of the effect of pressure on resistance. A second application of pressure was then nearly always made from 2000 up to the maximum of 7000 and then back again. It almost always turned out that hysteresis disappeared on the second application of pressure, the up and the down points agreeing. If this was the case, pressure was then released beyond the bye-pass, back to zero, and the readings between 2000 and 0 obtained during the last decrease were accepted as correct. A few times a third application of pressure had to be made before the readings became reproducible. The zero reading was then checked in ice at atmospheric pressure, temperature was then lowered with liquid oxygen, and the sequence of readings above repeated, except that pressure was now transmitted with helium instead of with nitrogen. Another important difference was that the first seasoning application of pressure was hardly ever necessary, but on the first application of pressure at 90° Abs. accommodation was complete, the up and down points lying together, except for an important effect at the zero, to be discussed later. This difference was not entirely due to the previous pressure seasoning at CO_2 temperature, but was at least in part a specific effect of the lower temperature, for on several occasions the order was reversed, and the first run was made with liquid O_2 and the second with CO_2. On these occasions the first run with liquid oxygen was without hysteresis, while seasoning was necessary at CO_2 temperature in spite of the previous application of pressure at liquid oxygen temperature.

It is a great advantage of the small size of the apparatus and the increased thermal conductivity of metals at low temperatures and of gases at high pressures that temperature equilibrium was reached almost at once after every change of pressure, so that good readings could be obtained as fast as the adjustments of bridge and potentiometer could be made. This was especially the case at liquid oxygen temperature; at CO_2 temperature it was necessary to verify that a steady state had been reached before the reading could be accepted. At liquid oxygen readings could be made about every 4 minutes, while with CO_2 a minute or two more might be necessary.

In one other respect thermal effects are strikingly different at the temperature of liquid oxygen from what they are at room temperature. On several occasions there was a minute leak at the lower connection, so small that the bubbles could just be seen rising through the liquid oxygen, and much too slow to be detected by any drop of pressure on the gauge. On such occasions the resistance reading was off by a large amount, indicating an enormously increased Joule-Thomson effect under these conditions. This situation would be worth investigating further for its own sake.

There follows now the detailed presentation of data.

Detailed Presentation of Data.

With one exception the metals measured in this paper belong to the cubic system. They have, therefore, the same compressibility and pressure coefficient in every direction, so that measurements on polycrystal wire give without error a complete description of the behavior under pressure. The one exception is magnesium, which crystallizes in the hexagonal system; previous measurements on this have shown that it is very nearly isotropic with respect to compressibility and resistance, so that little could be gained by using single crystals of this metal. I intend to devote another paper to the markedly non-cubic metals; these must be measured as single crystals, and measurements made in different orientations. Such measurements have already been made on Zn and Bi.

In the following, the arrangement of the metals is the same as in my first paper on the pressure effect of resistance, namely in the order of melting points.

Lead. For this metal I am indebted to Professor G. P. Baxter, who prepared it in connection with atomic weight determinations. It was extruded into wire 0.009 inch in diameter, of the regular length of about 30 cm, and wound as usual on an insulating spool. The leads were soft soldered. Three sets of runs were made; the first two were incomplete, being terminated by the explosion of the upper cylinder; they do not differ within the limits of error from the results of the third and final run, to which alone the following description applies.

The seasoning effect of the first application of pressure at −78.5° amounted to an increase of relative resistance of 0.8% of the total resistance, or 8.8% of the total change of resistance produced by 7000. This comparatively large change of zero at liquid oxygen temperature was accompanied by a very much smaller permanent change of resistance at atmospheric pressure at 0° C, which was increased by less than 0.1% after the run at −78°. At −183° the permanent change of zero produced by the first application of pressure was inappreciable, so that a single run was adequate at this temperature. After seasoning had been completed, the points at both temperatures lay very smoothly, the maximum departure from a smooth curve at −78° being 0.4% of the total pressure effect, and twice this amount at −183°. The relation between pressure and change of resistance was distinctly not linear at both temperatures, there being a departure in the normal direction, that is, the pressure coefficient becomes less numerically with increasing pressure. The effect was small, however, and the second degree term in the following formulas must be recognized to be uncertain by perhaps 30%. The datum given by the measurements with the greatest accuracy is the mean pressure coefficient between 0 and 7000. The results are:

$$\text{At} - 78.5^\circ,\ -\Delta R/R(0, -78.5) = 13.58 \times 10^{-6}p - 9.9 \times 10^{-11}p^2.$$

$$\text{Average coefficient } 0\text{–}7000 = -12.88 \times 10^{-6}.$$

$$\text{At} - 183^\circ,\ -\Delta R/R(0, -183) = 13.81 \times 10^{-6}p - 15.0 \times 10^{-11}p^2.$$

$$\text{Average coefficient } 0\text{–}7000 = -12.76 \times 10^{-6}.$$

At atmospheric pressure the relative resistances at 0°, − 78.5°, and − 182.95° were 1.0000, 0.6880, and 0.2985. I. C. T. gives 1.0000, 0.6899, and 0.2953 as the mean of several observers.

Magnesium. I am indebted for this material to the courtesy of Dr. Zay Jeffries and Mr. L. W. Kempf of the Aluminum Co. of America. It had been specially purified by vacuum distillation in their Research Laboratories. The analysis showed Si 0.004%, Fe 0.007%. The Al content was not specially determined, but in other similarly prepared specimens it has been found to be seldom present in excess of 0.008%. I formed it into wire 0.009 inch in diameter by hot extrusion through a steel die. Considerable difficulty was experienced in making connections. Spot welding was not successful. The method finally adopted was to wrap about the wire a single turn of fine platinum, and then to pinch this together, practically cutting through the magnesium wire. The resistance of the contact was normally high, but by passing through it a high voltage current from a small magneto before each reading, the resistance was reduced to a sufficiently low and constant value. The irregularity of the readings was somewhat greater than usual, however, and doubtless the reason is to be found at the contact.

The runs were made in the conventional order, with no special incident. After the initial application of 7000 at − 78.5°, there was a permanent increase of resistance by 0.73% of its total value, or 21.3% of the pressure effect produced by 7000. The total resistance at the ice point was changed by only 0.18% after the runs at − 78.5°. The seasoning at this temperature was found to be practically all produced by the first 3000 kg. The second application at 7000 was without appreciable hysteresis. Within the limits of error the change of resistance is linear with the pressure, the maximum departure from a straight line being 2.3% of the total pressure effect. At − 182.86° (liquid oxygen) the first application of pressure was without seasoning effect, except that the zero points all lay high, the maximum deviation being by 7.3% of the maximum pressure effect. The maximum departure from a linear relation of any other single reading was 4.4% of the total pressure effect. The results are:

At − 78.4°, Average coefficient 0–7000 = -4.49×10^{-6},

At − 182.86°, Average coefficient 0–7000 = -5.89×10^{-6}.

The relative resistances at atmospheric pressures in terms of the resistance at 0° C were: at − 78.4°, 0.4649, and at − 182.86°, 0.1443. I. C. T. gives for practically the same temperatures 0.682 and 0.230, taken from a paper by Dewar and Fleming in 1893. Their magnesium was evidently much less pure than this sample.

Aluminum. For this metal, as well as the magnesium, I am indebted to Dr. Zay Jeffries of the Aluminum Co. of America. It was especially pure stock from the Research Laboratories, and had the following analysis: Si 0.008%, Fe 0.014%, Cu 0.011%, balance Al. It was extruded hot to wire of 0.0093 inch diameter through a steel die, and wound on an insulating spool of the usual length, 30 cm. The terminals were of fine silver wire, spot welded.

At CO_2 temperature the regular procedure was followed. The permanent change of zero after the first application of 7000 was 0.71% of the total resistance, or 23.5% of the maximum pressure effect. This change of zero at − 78.5° was followed by a relative change of resistance at 0° C of only 0.12%. Hysteresis practically disappeared after the first application of pressure at − 78.5°. The relation between pressure and change of resistance was sensibly linear, and the maximum departure from a straight line by any single reading was 2.4% of the maximum pressure effect. The first attempt at a run at liquid oxygen temperature was terminated by a rapid leak due to a crack in the lower cylinder. The lower cylinder was replaced by a new one, and a satisfactory run obtained without appreciable hysteresis. There was, however, a stoppage of the connecting pipe at the highest pressure. As usual, the zero lay high, by 5.7% of the maximum pressure effect. The other points were linear, with a maximum departure by any single point of 1.8% of the maximum pressure effect. The results were:

At − 78.4°, Average coefficient, 0–7000, = -4.71×10^{-6}.

At − 182.9°, Average coefficient, 0–7000, = -9.16×10^{-6}.

At atmospheric pressure the relative resistances in terms of the resistance at 0° C were: at − 78.4°, 0.6340, and at − 182.9°, 0.1521.

I. C. T. quoting Holborn, gives 0.6480 and 0.189 respectively. My sample is probably somewhat purer.

Silver. This was some of the identical metal with which my original pressure measurements were made in 1916.[5] It was obtained from the old U. S. Mint at Philadelphia, was "proof" stock, and had been drawn through diamond dies to avoid contamination with iron, and was doubtless of unusual purity.

At − 78.4° two complete runs were made, hysteresis disappearing on the second run. The maximum width of the hysteresis loop, which was also the displacement of the zero, on the first run was .24% of the total resistance, or 11.4% of the maximum pressure effect. On the second run the relation between pressure and change of resistance was linear, the maximum deviation from a straight line by any single point being 1.6% of the maximum pressure effect. The readings at liquid oxygen temperature gave difficulty. Twice there was leak at the lower connection at comparatively low pressure, so that the oxygen bath had to be removed, the cylinder warmed to room temperature and the joint tightened. After this the leak apparently stopped, and readings were made to 7000 and back. But these readings were unique in that there was very large curvature in the abnormal direction, and also marked hysteresis, but only a small displacement of the zero. In view of this unusual behavior, the run at liquid oxygen temperature was repeated with the identical sample after two months, but with a different lower cylinder. The results were now entirely canonical; the relation was linear with a maximum deviation from a straight line by any single point of 1.3% of the maximum pressure effect, and the zero lay exactly on the curve, rather unusual. There was stoppage of the pipe at the maximum pressure, 7400, but a good reading was obtained at 7000. It seems highly probable that the anomalous results obtained with the first set-up were due to a minute leak, too small to detect by direct measurement, combined with a very high Joule Thomson effect in He, as has been verified by direct observations on other occasions. The results of the first run at − 183 were therefore discarded. The final results were:

At − 78.4°, Average coefficient 0–7000, $= -3.46 \times 10^{-6}$.

At − 182.9°, Average coefficient 0–7000, $= -4.09 \times 10^{-6}$.

At atmospheric pressure the relative resistances in terms of the resistance at 0° C were: at − 78.4°, 0.6820, and at − 182.9°, 0.2466 (the two values on the two different runs differed by 0.13% from the mean). I. C. T. gives at the same temperatures from data by Clay, 0.685 and 0.250 respectively, rather close agreement.

Gold. This, like the silver, was the identical piece of wire which was used in the 1916 resistance measurements. The source of the material was the same, and it doubtless is of the same high purity.

At − 78.4° two applications of pressure were necessary, as usual, to give regular results. The effect of the seasoning was unusual in that the resistance was decreased rather than increased. The magnitude of the change of zero was 0.15% on the total resistance, or 9.3% of the maximum pressure effect. On the second application of pressure the relation between pressure and change of resistance became linear, with a maximum deviation from a straight line by a single reading of 2.4% of the maximum pressure effect. At liquid oxygen temperature it was not possible to get beyond 6000 because of leak at the lower connection, and the maximum pressure at which good readings could be obtained was 4300. In this range the relation was linear and there was no hysteresis; it did not seem worth while to set up again to increase the range to 7000. The maximum deviation from a straight line at this temperature was 2.5% of the maximum pressure effect, and the average deviation of the seven readings, 1.1%. As usual, the zero lay a little high, by 1.8% of the pressure effect. The results were as follows:

At − 78.4°, Average coefficient 0–7000, $= -2.97 \times 10^{-6}$.

At − 182.9°, Average coefficient 0–4300, $= -3.27 \times 10^{-6}$.

At atmospheric pressure the relative resistances in terms of the resistance at 0° C were: at − 78.4°, 0.6917, and at − 182.9°, 0.2784. I. C. T. gives 0.6904 and 0.2708 respectively from the best work at Leiden; their gold is evidently a little purer than mine.

Copper. This material is electrolytic copper from the Bureau of Standards, and is the identical material as that used for the measurements in 1916.

At − 78.4° only a single excursion to 7000 and back was made instead of the usual two. It was evident that the seasoning process was practically all accomplished by the first 1500 kg, and the points obtained with decreasing pressure all lay on a straight line with a maximum departure of 0.017% of the total resistance, or 1.6% of the maximum pressure effect. The permanent change of zero was a decrease by 0.32% of the total resistance. The ice point was changed by only 0.012% after the run at − 78.4°. At − 182.9° more points were taken than usual, because it was necessary to remove the bath and tighten the lower connection after about 3000 had been reached. Except for one discarded point, not explained, and which may have been due to a blunder in reading, there was no hysteresis and the relation between pressure and change of resistance was linear with a maximum departure of 3.7% of the maximum pressure effect. The points with decreasing pressure were markedly more uniform; here the maximum deviation was 1.4%. As usual, the initial zero lay a little high, by about 4.5% of the pressure effect, or 0.073% of the total resistance. The results were:

At − 78.4°, Average coefficient 0–7000, $= -2.14 \times 10^{-6}$,

At − 182.9°, Average coefficient 0–7000, $= -3.09 \times 10^{-6}$.

At atmospheric pressure the relative resistances in terms of the resistance at 0° C were: at − 78.4°, 0.6692, and at − 182.9°, 0.1866. I. C. T. gives, quoting from Henning, 0.6524 and 0.1890 respectively, lower at the first temperature and higher at the second. Such a reversal is not usual, and does not seem to permit any conclusion about the relative purity of the materials.

Nickel. This wire, of 0.0035 inch diameter, had been obtained from the Leeds and Northrup Co. in 1921, and was a product of their Research Laboratory, made especially to have the highest possible temperature coefficient of resistance, and therefore of high purity. The terminals were fine platinum, spot welded.

At CO_2 temperature the behavior was highly abnormal, and recalled the known great hysteresis and related effects when nickel is stretched by a one-sided tension. The readings obtained during the several applications of pressure are reproduced in Figure 3. Not only is there great difference between the results with increasing and decreasing pressure, but there are also time effects, the resistance creeping for a while after release of pressure. There seems to be a distinct preference for the points to lie on a straight line of definite slope, the decreasing points after the first and second applications and the increasing points of the third application all lying on a line of this slope. Furthermore, there is a distinct tendency for the increasing and the decreasing points to come more closely together as the runs are repeated. The heavy line in the diagram was taken to be that corresponding to complete seasoning, and was assumed to be correct in the final computations. The total change of zero at − 78.4° was 0.91% of the initial resistance at this temperature, whereas the change of zero at 0° C found after the runs at − 78.4° was only 0.068% of the initial resistance at 0°. The seasoning effect, as has been found so many times before but less strikingly, is obviously a local effect, characteristic of the particular temperature, and is not carried as a permanent change to other temperatures. At − 182.9° the abnormalities peculiar to − 78.4° had almost entirely disappeared, the only remnant being a slight permanent increase of resistance at the maximum pressure, 7400. All the points with increasing pressure lay on one straight line, and those with decreasing pressure on another line of the same slope and higher by 0.06% of the total resistance, or 6.0% of the maximum pressure effect. The maximum departure (except for the zeroes) of a single point from one or the other of these lines was 0.02% of the total resistance, or 2.4% of the maximum pressure effect. The zero, as usual, lay high, but by an unusually

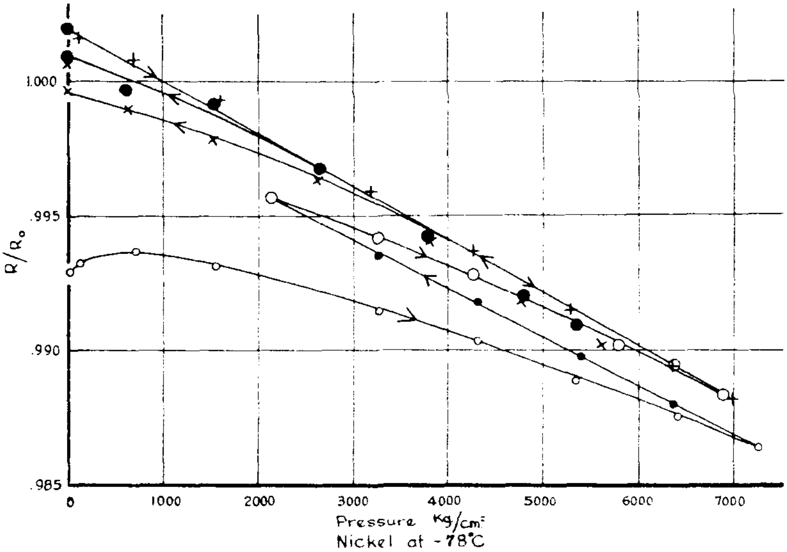

FIG. 3. The relative resistance at solid CO_2 temperature of nickel as a function of pressure. The small open circles show the first application of increasing pressure, the small solid circles the first decreasing pressure, the large open circles the second increasing pressure, the large solid circles the second decreasing pressure, the perpendicular crosses the third increasing pressure and the diagonal crosses the third and last decreasing pressure.

large amount, 0.27% of the total resistance. The results assumed to be best justified by the data are:

At $-$ 78.4°, Average coefficient 0–7000 = -2.00×10^{-6}.

At $-$ 182.9°, Average coefficient 0–7000 = -1.88×10^{-6}.

At atmospheric pressure the relative resistances in terms of the resistance at 0° C were: at $-$ 78.4°, 0.5938, and at $-$ 182.9°, 0.1771. I. C. T. gives, from papers by Fleming in 1900 and Dewar in 1904, 0.613 and 0.208 respectively. My material is evidently considerably purer.

Iron. The same piece of wire was used which had been previously measured for the effect of pressure on resistance in 1916[5] and on thermal e.m.f. in 1918.[6] It was originally drawn down from a bar of American Ingot iron, and was stated to have less than 0.03% total impurity.

There were various disturbances during the measurements, so that the results for iron are perhaps less satisfactory than for any other metal; however, the results seemed sufficiently secure not to justify repeating the experiment. The new type of upper cylinder was used for the first time for this metal, and on the first run at CO_2 temperature the final form of the CO_2 container and the technique of handling it had not been reached. The most efficient combination of resistances for the potentiometer also had not been worked out. Considerable irregularities on the first attempts at this temperature are doubtless due to inadequacy of temperature control. The last run at $-$ 78.4°, when the final technique for handling the CO_2 had been worked out, showed no hysteresis, as was to be expected because of the previous applications of pressure, but a leak at the lower cylinder set the maximum pressure at this temperature as 4300 instead of the usual 7000. Except for the zero, the points all lay on a straight line with a maximum departure by any single point of 0.13% of the total resistance or 3.0% of the maximum pressure effect. The zeroes were low by 0.07% of the total resistance. It is perhaps worth remarking that after the first and before the final run at $-$ 78.4° the apparatus had come to room temperature. The fact that the seasoning effect had been retained is significant. At liquid oxygen temperature a maximum pressure of 6000 was reached. Here a short circuit developed in the manganin gauge, making further readings impossible. The gauge was repaired, and another run made to a maximum of 4400. Here the stem of the manganin plug pinched off, allowing pressure to leak back nearly to zero. The run was not repeated, but the final results were made up from a combination of the results of the two runs. The relation is linear within the limits of error if one point of the first run is disregarded. Attaching weight to this one point would have demanded slight curvature in the normal direction. Except for the discarded point the maximum departure of a single reading from a straight line is 0.05% of the total resistance, or 4.0% of the maximum pressure effect. The discarded point is too low by 6.0% of the pressure effect. The zeroes did not deviate appreciably from the line. The final results are:

At $-78.4°$, Average coefficient, 0–4000 $= -2.27 \times 10^{-6}$.

At $-182.9°$, Average coefficient, 0–4000 $= -2.44 \times 10^{-6}$.

At atmospheric pressure the relative resistances in terms of the resistance at 0° C are: at $-78.4°$ 0.6029 and at $-182.9°$ – .1639. I. C. T. gives, from Holborn, 0.5770 and 0.120 respectively, considerably lower than my values, and indicative of higher purity. It has been known for some time that the effect of minute impurities on the resistance of iron is unusually high.

Palladium. This was obtained from Baker and Co., their purest. Before use it was heated to 1000° C for 8 hours in a diffusion pump vacuum in order to remove any occluded hydrogen. The pressure coefficient of resistance has been previously measured at ordinary temperatures, but the sample then used was presumably not of high purity, as shown by its low temperature coefficient of resistance. At atmospheric pressure the average temperature coefficient of this new sample between 0° and 95° C was 0.00376, against 0.00318 of the previous sample. This new sample is therefore presumably so much purer that it seemed worth while to redetermine the pressure coefficient at ordinary temperatures, in addition to the regular measurements at lower temperatures. Runs were therefore made in the regular high pressure apparatus to 12000 kg/cm^2 with petroleum ether and kerosene as the transmitting media at 0° and 95° respectively. At 0° the regularity of the readings was sufficient to establish a slight departure from linearity in the normal direction, but at 95° the error was greater and the relation had to be assumed to be linear. At 0° the average deviation from a second degree curve of a single one of the 13 readings was 0.12% of the pressure effect, and at 95° the corresponding deviation from a straight line for 14 readings was 0.30%. The numerical results are given below.

At $-78.4°$ the first application of 7000 produced permanent effects, as usual, but hysteresis disappeared on the second application, and the points were linear with a maximum departure by any single point of 0.029% of the total resistance, or 2.3% of the maximum pressure effect. The permanent change of zero produced by the applications of pressure at $-78.4°$ was 0.15% of the total resistance at this temperature, which on warming to 0° resulted in an increase of 0.08% of the total resistance at 0°. At $-182.9°$ the only incident was the stoppage of the pipe at the maximum pressure; the other points, except the zero, lay on a straight line with a maximum deviation by any single point of 0.06% of the total resistance, or 4.1% of the maximum pressure effect. The zero lay high by about 0.08% of the total effect.

The numerical results are as follows:

At 95° C, Average pressure coefficient, 0–12000, -2.052×10^{-6}.

At 0°, $\Delta R/R_0 = -2.166 \times 10^{-6}p + 5.4 \times 10^{-12}p^2$. (Valid to 12000).

Average pressure coefficient, 0–12000 $= -2.101 \times 10^{-6}$.

At $-78.4°$, Average pressure coefficient, 0.7000 $= -2.32 \times 10^{-6}$.

At -182.9, Average pressure coefficient, 0–6000 $= -2.82 \times 10^{-6}$.

The previous sample of palladium gave at 0° an average pressure coefficient between 0 and 12000 of -1.95×10^{-6}, materially lower numerically than the above, bearing out the universal rule with respect to the effect of impurity on pressure coefficient.

At atmospheric pressure the relative resistance of this new sample of palladium in terms of the resistance at 0° C was: at $-78.4°$, 0.6893, and at $-182.9°$, 0.2339. I. C. T. gives from Holborn 0.684 and 0.229 respectively, slightly lower than my figures, and indicating somewhat greater purity.

Columbium. This was obtained from the Fansteel Co. in Chicago. It was 0.0078 inches in diameter, and of the conventional length, 30 cm. The terminals were platinum, spot welded. The pressure coefficient of columbium at ordinary temperatures has been previously measured; the material was obtained from the same source, but since the specimens were not identical, it is probable that the results are not strictly comparable. The purity, as indicated by the temperature coefficient, was doubtless of a lower order than that of any other of the metals measured.

The usual procedure was followed. At −78.4° the first application of pressure produced permanent changes, but the effects disappeared on the second application. The relation between pressure and resistance is linear; the maximum deviation from a straight line of any single reading was 5.% of the maximum pressure effect, or 0.027% of the total resistance. The total resistance was permanently increased by 0.32% of itself by the first application of pressure. At −182.9° there were no appreciable seasoning effects, but the relation was linear on the first application, with a maximum deviation of any single reading of 8.3% of the maximum pressure effect, or 0.036% of the total resistance. The zero did not deviate more than the other points. The results are:

At − 78.5°, Average coefficient, 0–7000, = -0.98×10^{-6}.

At − 182.9°, Average coefficient, 0–7000, = -0.80×10^{-6}.

At atmospheric pressure the relative resistances in terms of the resistance at 0° C were: at −78.4°, 0.9344, and at −182.9°, 0.8099. The abnormally low temperature coefficient indicates inferior purity, as already remarked.

Platinum. This was obtained from Baker and Co., their "c.p." brand, for use in resistance thermometry. It was 0.007 inch in diameter, and of the conventional length, 30 cm. It was supplied in the dead soft condition, so that temperature annealing was dispensed with. It was, however, pressure seasoned by an exposure to 12000 at room temperature.

At −78.4° the seasoning effect of pressure was less than usual, doubtless because of the preliminary seasoning at room temperature, and the points with decreasing pressure all lay on a good straight line, with a maximum deviation by any single point of 1.1% of the maximum pressure effect, or 0.012% of the total resistance. It seemed safe to accept this as the correct effect, and a second run at this temperature was not made. The zero was permanently raised by 0.23% of itself by the application of pressure at −78.4°. At −182.9° the usual order of things was reversed, and the points on the first application of pressure were more irregular than usual. This, however, was probably not an abnormal seasoning effect, but was doubtless the effect of a slight leak, which was large enough to be perceptible by dropping of the gauge at the lower pressures, but which automatically stopped when pressure reached the maximum, 7400. The run was repeated, and now the points lay on a good straight line, with a maximum deviation by any single point of 3.3% of the maximum pressure effect, or 0.042% on the total resistance. The zero was high by 0.06% of the total resistance. The results were:

At $-$ 78.4°, average coefficient 0 $-$ 7000, $= - 1.97 \times 10^{-6}$.

At $-$ 182.9°, average coefficient 0 $-$ 7000, $= - 2.34 \times 10^{-6}$.

At atmospheric pressure the relative resistances in terms of the resistance at 0° C were: at $-78.4°$, 0.6842, and at $-182.9°$, 0.2488. I. C. T. gives an elaborate set of tables for the resistance of platinum, from three different samples. The purest of these was "P. T. R. No. 29," measured by Henning, and gave the values 0.6844, and 0.2461 respectively; the first value is practically the same as mine, and the second somewhat lower. The resistances of the two other samples of "pure" platinum quoted in I. C. T. are higher than mine. The purity of this specimen of mine must therefore be judged to be high, although not quite as high as absolutely the best that can be produced.

Rhodium. This was obtained from Baker and Co., was 0.0007 inches in diameter and 30 cm long. Connections were made with platinum terminals by spot welding. Before use it was annealed to a white heat in a gas furnace, and then seasoned by two applications of 12000 at room temperature.

There were various vicissitudes which make the experimental error larger than usual. On the first application of pressure at $-78.4°$ there was leak at the maximum, which later became worse and made impossible a run with liquid oxygen. The leak was traced to a flaw in the lower cylinder, which was replaced by another. The seasoning effects were unusually large at CO_2 temperature, and persisted somewhat more than usual during the second application of pressure. After the termination of the run at liquid oxygen temperature, which went smoothly enough, an attempt was made to repeat the run at $-78.4°$, but the lower cylinder split on release of pressure at 5000, after readings had been obtained at 7000 and 6000. The initial irregularities were great during this second attempt at CO_2 temperature, showing again that seasoning is a more or less local matter at each temperature. The second run of the first set-up at CO_2 temperature was used in making the final calculations. The points are linear, the maximum deviation of any point, except the zero, being 5.9% of the maximum pressure effect, or 0.049% of the total resistance. Both zeroes lay low by 0.085% of the total resistance. At $-182.9°$ the maximum deviation from a straight line of any single reading except the zero was 4.0% of the maximum pressure effect, or 0.049% of the total resistance. The zero lay high by 0.075% of the total resistance. The results are:

At $-78.4°$, average coefficient $0-7000 = -1.86 \times 10^{-6}$.

At $-182.9°$, average coefficient $0-7000 = -2.26 \times 10^{-6}$.

The relative resistances at atmospheric pressure in terms of the resistance at 0° C were: at $-78.4°$, 0.6573, and at $-182.9°$, 0.1935. I. C. T. gives from Holborn 0.658 and 0.201 respectively. My sample is therefore probably a little purer.

Molybdenum. This was the identical piece of wire which was used for my measurements of the effect of pressure on thermal e.m.f. in 1918[6], and was obtained originally from the General Electric Co., Its mean temperature coefficient of resistance between 0° and 100° is 0.00461.

The usual order of the experiment was reversed, and the first run was made at liquid oxygen temperature. This run was terminated after a maximum pressure of 6000 had been reached by leak due to the pinching off of the moving plug. The points up to then were fairly good, however. The next run was at CO_2 temperature, and here there was no apparent seasoning effect, but on the first application the points lay on a straight line. The maximum deviation of any point, except one discard, was 4.8% on the maximum pressure effect, or 0.025% on the total resistance. The zero lay on the curve as closely as any other point. The run at liquid oxygen temperature was then repeated, this time successfully, with increasing and decreasing pressure. The points were linear, the maximum deviation of any single reading being 16.4% of the maximum pressure effect, or 0.16% of the total resistance. Both zeroes lay high by an average of 0.12% of the total resistance. The points of the first run at liquid oxygen were much smoother than those of the second, the maximum deviation being only 0.03% of the total resistance. It seemed, therefore, that these should be given more weight in the final result, in spite of the fact that they were obtained only with increasing pressure, and to a maximum of 6000, instead of 7300. The average coefficient from the first run was -1.90 and from the second -1.93×10^{-6}. The adjusted mean was taken to be 1.91. The results follow:

At $-78.4°$, average coefficient $0-7000, = -1.29 \times 10^{-6}$.

At $-182.9°$, average coefficient $0-7000, = -1.91 \times 10^{-6}$.

At atmospheric pressure the relative resistances in terms of the resistance at 0° C were: at −78.4°, 0.6333, and at −182.9°, 0.1582. I. C. T. gives, from a paper by Holborn, 0.665, and 0.230 respectively. The material of this paper is perceptibly purer.

Tantalum. Two samples of this were used. The first was an old sample originally obtained from the General Electric Co., the pressure coefficient of which had been measured in 1917. At atmospheric pressure its mean temperature coefficient between 0° and 100° was 0.00297, and its average pressure coefficient at 30° between 0 and 12000 kg/cm^2 -1.47×10^{-6}. This sample was quite hard mechanically, and could not be bent to a sharp angle without breaking. The greatest difficulty was experienced in spot welding to it the platinum terminals, the tantalum becoming so brittle that it almost always broke in handling afterwards. Finally, by using a platinum terminal larger than the tantalum, 0.010 against 0.007 inches, the temperature was kept sufficiently low during the welding so that the brittleness did not become excessive. A single run at liquid oxygen temperature was made with this sample, using one of the primitive lower cylinders, and before the final details of technique had been elaborated. The maximum pressure was only 5000, and the run was terminated by the blowing out of the manganin pressure gauge. The results were quite irregular, the average deviation from a straight line of the six readings being 10% of the maximum pressure effect. The mean coefficient to 4000 was -1.08×10^{-6}. At atmospheric pressure the relative resistance at this temperature in terms of the resistance at 0° C was 0.406.

The second specimen was obtained from the Fansteel Co. Mechanically, it was entirely different from the first specimen, being soft, easily bent to sharp angles, and furthermore it was easy to spot weld the platinum terminals without producing brittleness. A rough exploration was made of its pressure and temperature coefficients. At atmospheric pressure the mean temperature coefficient between 0° and 100° was 0.00302, only a trifle higher than that of the first specimen, and at 30° the mean pressure coefficient to 12000 was -1.42×10^{-6}, slightly lower than the other. The accuracy of this pressure coefficient was not high, however, readings being made only at 0 and 12000.

The regular sequence of readings at low temperatures was made on this sample, and the results were found to be much like those for other metals. At −78.4° the first application of pressure produced marked permanent effects, but afterwards the points lay on a straight line with a maximum deviation of 2.7% of the maximum pressure effect, or 0.027% of the total resistance. The zero was permanently raised by 0.27%. At −182.9° the points lay on a straight line on the first application of pressure with a maximum deviation by any single point, except the zero, of 2.3% of the maximum pressure effect, or 0.018% of the total resistance. The zeroes lay high, as usual, by 0.04% of the total resistance. The results are:

At − 78.4°, average coefficient 0 − 7000 = -1.42×10^{-6}.

At − 182.9°, average coefficient 0 − 7000 = -1.17×10^{-6}.

At atmospheric pressure the relative resistances in terms of the resistance at 0° C were: at −78.4°, 0.759 and at −182.9°, 0.416. I. C. T. gives from Holborn 0.729, and 0.332, materially lower, and indicating inferior purity in my sample. It is not probable, however, that the purity of either sample was very high.

It is to be remarked that in spite of the great difference in the mechanical properties there is not much difference in the electrical properties of the two samples of tantalum used by me.

Tungsten. This was obtained from the General Electric Co., their so-called "C" wire, free from thorium, and was the identical wire which had been used in the investigations of Dr. Blodgett on thermionic emission.

The runs followed the conventional course. No seasoning effect was evident after the first application of pressure at −78.4°. The maximum deviation of any single point, except for the low pressure points below 1000, from a straight line was 1.8% of the maximum pressure effect, or 0.015% of the total resistance. The low pressure points at 0, 100 and 700, however, lay distinctly below the curve, the zero lying off by 0.055% of the total resistance, and on a smooth prolongation of the curve through the 100 and 700 points. If weight is given to the low pressure points, the relation between pressure and resistance departs slightly from linearity, the curvature being in the abnormal direction, and the pressure coefficient increasing numerically with increase of pressure. At −182.9° there was no hysteresis within experimental error, and a single run was sufficient. Here the relation was distinctly not linear, the curvature being evident over the entire pressure range, and being abnormal in direction, as at −78.4°. The experimental points are reproduced in Figure 4. Except for the two divergent points with increasing pressure at 700 and 1500, which may have been a seasoning effect, the maximum departure of any point from a second degree curve is 2.1% of the maximum pressure effect, or 0.018% on the total resistance. The curvature is too great to be ignored at this temperature, and the measurements are accordingly reproduced by a second degree formula. The results are:

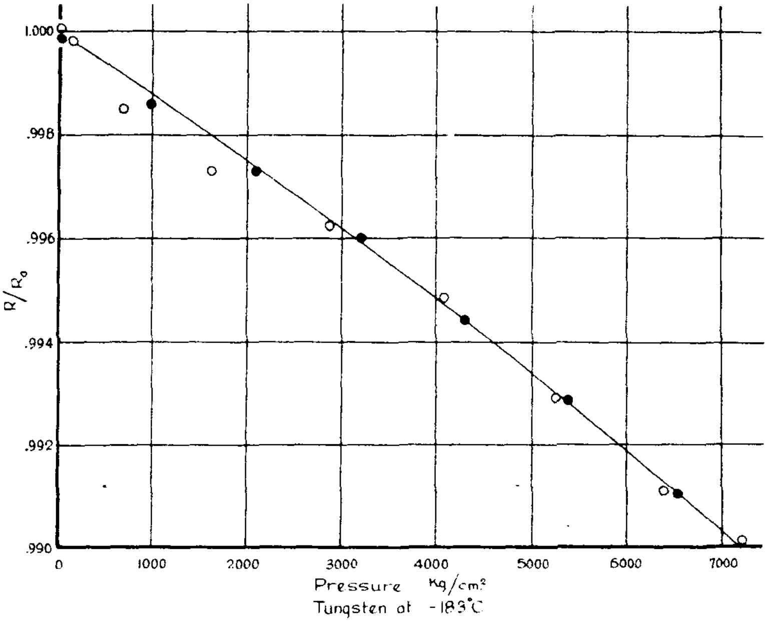

FIG. 4. The relative resistance at liquid oxygen temperature of tungsten as a function of pressure. The feature is that there is marked curvature in the abnormal direction.

At − 78.4°, average coefficient, 0 − 7000 = − 1.42 × 10^{-6}.

$$\text{At } -182.9° \begin{cases} \text{average coefficient, } 0 - 7000 = -1.36 \times 10^{-6}, \\ \Delta R/R(0, -182.9) = -1.071 \times 10^{-6} p - 4.1 \times 10^{-11} p^2. \end{cases}$$

The relative resistance at atmospheric pressure in terms of the resistance at 0° C was determined only at −182.9°, the resistance at −78.4° not being calculable because of an oversight in not recording one of the essential resistances. At −182.9°, R/R_0 was 0.3132.

I. C. T. gives from Holborn 0.195. My specimen, therefore, evidently has considerable impurity.

DISCUSSION.

There were two principal possibilities which I had in mind before beginning the experiments. The first has to do with the minimum of resistance at high pressures shown by the alkali metals.[7] The probability was recognized that all metals may have a similar minimum of resistance at very high pressures, and that the pressure of the minimum would be a function of temperature. The question was therefore, whether the pressure of the minimum moves so as to come within reach as temperature is lowered. If such a minimum pressure should prove not to be actually attainable, then evidence as to its probable location would be obtainable from the curvature of the relation between pressure and resistance. Increased curvature in the normal direction as temperature is lowered would mean that the minimum of resistance is displaced to lower pressures at lower temperatures. The second possibility has to do with the temperature of supraconductivity. The temperature at which supraconductivity appears must be a function of the pressure, and the possibility was to be kept in mind that this temperature might be raised as high even as that of liquid oxygen by sufficiently high pressure. There seems at present no general method of telling whether the supraconducting point will be raised or lowered by the application of pressure; it is known that tension raises this point in some metals and lowers it in others, but there have been no measurements on the effect of a hydrostatic pressure.

These two possibilities can be dismissed very briefly. In the first place, no suggestion of supraconductivity was detected. One would expect, perhaps, to find the greatest upward displacement of the supraconducting temperature in those metals which have the highest supraconducting temperature under normal conditions. This was my reason for trying columbium, which is known to have the highest supraconducting temperature of any simple metal, about 8.5° K. The specimen of Cb used in these measurements was not pure, but high purity is of no particular importance, contrary to the first ideas, and Meissner and Franz[8] found supraconductivity in a sample of Cb impure with 1.5% Sn. In the list of 15 metals measured above, three are known to become supraconductors, Pb, Cb, and Ta, and it is probably significant that the pressure coefficient of these three metals becomes less numerically at low temperatures.

With regard to the other possibility, a minimum resistance within reach at liquid air temperature, there is the striking fact that nearly all the metals examined show a linear relation between pressure and resistance. The only instances of definitely established curvature are by lead, in which there is a slight curvature in the normal direction, and tungsten, in which the direction is abnormal, the relative decrease of resistance increasing at high pressure. Appreciable curvature would be expected in the case of lead because of the large numerical value of the coefficient. It does seem, however, that in the case of lead the curvature becomes somewhat less at lower temperatures, the second degree term in the two power expansion for the relative resistance being 19.0×10^{-11} at 0° C, 9.8 at $-78.4°$, and 15.0 at $-182.9°$. The same is probably true of the other metals, namely, that there is a decrease of relative curvature at low temperatures, although the accuracy is not high, and the effect of the expected curvature is not far from the limits of error. Thus in the case of magnesium, it may be calculated that if the curvature previously found at 0° C also occurs at $-78.4°$, then a maximum deviation from a straight line is to be expected equal to about the maximum deviation of any of the observed points and about four times the average deviation. It is to be remembered that the accuracy with which deviation from linearity can be determined increases with the square of the pressure range, so that the previous work to a maximum pressure of 12000 gave this term about three times as accurately as the present work to only 7000, to say nothing of other reasons for greater accuracy in the previous work.

The other metals after lead are linear, except tungsten, which shows a distinct curvature in the abnormal direction. One is tempted to see some significance in this in view of the fact that tungsten is the metal with highest melting temperature and highest characteristic temperature.

Finally, discussion must be made of the previous measurements of Ulrich Fischer.[9] He measured lead, copper, iron, molybdenum, tungsten and the alloy, constantan, at temperatures much lower than mine, in fact, down to liquid hydrogen temperature, but over a much lower pressure range, 150 kg/cm.^{2} His pressure was the tank pressure of his hydrogen supply. The pressure coefficients were determined by making a great many measurements of the resistance, first without the tank pressure and then with it. Fischer summarizes his results by saying that the pressure coefficient of resistance increases greatly with decreasing temperature, and by a greater amount the higher the characteristic temperature of the metal. This summary, however, does not quite reproduce his results; it is true that his coefficient for lead does not begin to increase appreciably until hydrogen temperatures are reached, whereas those of tungsten and molybdenum have begun to increase perceptibly already at CO_2 temperatures, and at liquid air temperatures have doubled, but on the other hand, copper, which has a much lower characteristic temperature than either tungsten or molybdenum, shows a slightly greater increase of pressure coefficient with decreasing temperature. Iron, Fischer does not plot in his final diagram because its purity was not high. Fischer sees in his results confirmation of what he has been led to expect from Grüneisen's empirical expression for the resistance of a metal. Grüneisen[10] found that the temperature dependence of the resistance of a metal at atmospheric pressure can be reproduced by an expression of the type $R = C\tau F(\tau/\Theta)$, where F is the Debye function, and Θ the characteristic temperature. F approaches a horizontal asymptote for values of τ as large or larger than Θ, but approaches zero for small τ's. Now Fischer expects the effect of pressure to consist mainly in an increase of Θ, the elastic restoring forces and so the general stiffness of the metal becoming greater as the atoms are pushed closer together. Hence the effect of pressure on resistance for a given τ will be large if the Θ is such that $F(\tau/\Theta)$ lies in the region in which it is capable of experiencing large changes, and this obviously means the region in which the argument τ/Θ is small or Θ large. Tungsten and molybdenum have a large Θ and lead a small Θ, so that from this point of view Fischer finds his results exactly what he would expect. But these considerations of Fischer seem to take no account of the fact that the C in the formula of Grüneisen may well be a pressure function, and also take no account of the fact that a given increment of pressure may well produce a greater proportional change of Θ in a soft metal like lead than in a hard one like tungsten, and thus compensate for the effect of the favorable absolutely high value of Θ in tungsten.

But entirely apart from these theoretical considerations, it seems to me that Fischer's experimental values cannot be correct. The effects which he finds are much greater than I find. We agree essentially with regard to lead, which shows very little change of coefficient down to liquid oxygen temperature. But for the other metals Fischer has at liquid oxygen temperature an increase in the pressure coefficient of tungsten compared with its value at 0° C of 77% against my decrease of less than 1%; for molybdenum an increase of 100% against my 47%, for copper 94% increase against my 65%, and for iron 59% increase against my 4%. It is true that the coefficients of most of my metals increase numerically at low temperatures, as shown in Table I, but this increase is not universal by any means.

TABLE I.

Average Pressure Coefficient of Resistance up to 7000 kg/cm² as a Function of Temperature.

	Temperature		
Metal	0°	−78°.4	−182°.9
Pb	-12.99×10^{-6}	-12.88×10^{-6}	-12.76×10^{-6}
Mg	−4.39	−4.49	−5.89
Al	−4.28	−4.71	−9.16
Ag	−3.45	−3.46	−4.09
Au	−2.94	−2.97	−3.27*
Cu	−1.88	−2.14	−3.09
Ni	−1.85	−2.00	−1.88
Fe	−2.34	−2.27	−2.44
Pd	−2.13	−2.32	−2.82
Cb	−1.18	− .98	− .80
Pt	−1.93	−1.97	−2.34
Rh	−1.64†	−1.86	−2.26
Mo	−1.30	−1.29	−1.91
Ta	−1.45	−1.42	−1.17
W	−1.37	−1.42	−1.36

* Max. press. 4300.

† On a less pure sample.

and furthermore, there is no correlation with the characteristic temperature, by far the greatest increase being for aluminum, with a low characteristic temperature, whereas the metals with the two highest characteristic temperatures, tungsten and tantalum, show decreases.

It seems to me that an adequate explanation of Fischer's results can be found in his low pressure range. This I believe to have been too low to sufficiently season the metals. Seasoning effects were always present in my measurements, and in most cases did not disappear until pressures of several thousands of kilograms were reached, twenty times or so higher than the pressures of Fischer. Furthermore, in spite of the fact that complete regularity could be given to the high pressure readings by proper seasoning, there remained in the majority of cases an irregularity in the zero reading. If the detailed presentation of the results of the seasoning applications of pressure is consulted, which I have purposely made very full in order to permit examination of this point, it will be found that in many cases the zero lies consistently high. It will be recalled that my readings were made at atmospheric, and then at tank pressure, which would be Fischer's maximum pressure. If the zero lies high, a pressure coefficient calculated from the first two readings will obviously be high, and I believe that this is exactly what has happened in the case of Fischer's results. Furthermore, such an effect would be expected to be greatest in a mechanically hard metal, like tungsten, which has also a high characteristic temperature, thus accounting for the apparent correlation with the characteristic temperature, and to be much less in so soft a metal as lead, for which Fischer and I have essentially the same results.

It does not seem to me that the time is yet ripe to speculate on the theoretical significance of these results, since the explanation of the effect of pressure under ordinary conditions, and in particular the minimum resistance, is not at all understood. These results must be taken in the spirit of an addition to our experimental knowledge which will later do their part in assisting to understand the conduction mechanism.

In general, the results may be summarized by the statement that the pressure coefficient of resistance increases somewhat numerically at low temperatures. It is significant that with the exception of nickel, which is anomalous in other respects, the metals of this paper which have shown a numerical decrease of coefficient at low temperatures are those which are known to be least pure, columbium and tantalum in particular being of inferior purity and investigated only because they exhibit the phenomenon of supraconductivity. With regard to the other metals, there is no correlation which strikes the eye between the magnitude of the change of pressure coefficient with temperature and other properties.

I am indebted to my mechanic Mr. Charles Chase for assistance in setting up the apparatus and making the runs, and to the Milton Fund of Harvard University for financial assistance in procuring some of the apparatus and other supplies.

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THE COMPRESSIBILITY OF EIGHTEEN CUBIC COMPOUNDS.

By P. W. Bridgman.

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Introduction.

In this paper measurements of linear compressibility are extended to 18 new solid compounds by the method already described and extensively applied.[1] The pressure range is that usual in this work, 12000 kg/cm². The compressibility of a number of these compounds has already been measured by other observers over a pressure range of a few hundred kg/cm², but such measurements cannot give the change of compressibility with pressure, for which a fairly extensive pressure range is necessary. All of these 18 substances crystallize in the cubic system. Since the linear compressibility of cubic substances is the same in all directions, the cubic compressibility may be computed from the linear compressibility in a single direction, which exhaustively characterizes the changes of form under hydrostatic pressure. In addition to geometrical simplicity, measurements of cubic substances have the convenience that much less exacting demands are placed on the perfection of the specimens, so that a much larger proportion of the total number of substances crystallizing in the cubic system may be measured as compared with substances crystallizing in other systems.

Because of the equality in all directions of the compression of cubic substances, the compressibility in any direction of an aggregate of crystal grains oriented at haphazard with respect to each other is the same as the linear compressibility in any direction of a single crystal of the same substance. There are many compounds which cannot easily be obtained in large single crystals, such, for example, as those which are normally formed as fine precipitates from solution, many of which cannot be melted without decomposition. If such a substance is cubic in structure its compressibility may be completely determined from a measurement of a massive block of the haphazard aggregate, whereas if the substance is non-cubic, not only does measurement of the aggregate not give any definite information, but often the measurements themselves, when made to pressures as high as those used here, cannot be simply interpreted because of hysteresis or other effects arising from the large differential compression of the grains in different directions, with resultant relative slip and plastic deformation.

Most of the substances of this paper were originally in the form of fine powders, which were formed by suitable means into massive cylindrical blocks or slugs for the compressibility measurements. This is not the first time that I have measured the compressibility of compressed slugs. The technique of making them has, however, been improved, so that now substances can be successfully molded into massive form which before I could not have handled. The material, usually in the form of powder, is formed by compressing it in a mold by a pressure sufficient to weld it into a homogeneous whole, of the same density as the massive material. Since the properties of the pure material are the subject of investigation, no binder can be used. Not all powdered materials can be formed in this way into massive blocks without binder. In general, substances with high melting points, like some of the oxides, or substances of great mechanical strength, like many of the metals, are difficult to form, but no universal rule can be given, and each new material must be tried for itself.

The mold is cylindrical in form and the compression is exerted by the thrust of a hardened piston; the forming stress is therefore not a hydrostatic pressure, but there is great distortion and plastic flow, and it is doubtless this fact which is responsible for welding together the powder. I have not been successful in producing satisfactory massive slugs by the application of truly hydrostatic pressure. One is reminded in this connection of the early work of Spring on the fusion of powders and the considerable controversy that arose in connection with it.

The greatest difficulty in the formation of the slugs is in getting them out of the mold after compression without rupture. Almost always there is some sticking to the side of the mold; if the compressed slug is pushed through and out of the mold by the piston, the friction produces such a distribution of stress in the slug that it very often breaks on emerging into laminae, the plane of the laminae being perpendicular to the axis of the cylinder. This can be avoided in many cases by making the mold split, so that the slug can be removed without pushing it the length of the mold. Removal of the slug is further facilitated by greasing the mold with graphite or graphite and vaseline. The graphite, which is nearly all confined to the outer layer of the slug, can be removed by turning the slug down to smaller diameter. But even this method does not work for all substances. The arrangement which I finally adopted for the most difficult substances, such as the copper halides, was as follows. The mold was made in two parts; the inner mold was of a tube of soft steel, ¼ inch inside diameter, and ⅜ inch outside diameter. The outer part of the mold was a cylinder of heat treated Cr Va steel, 2 inches in outside diameter, and ⅜ inch inside diameter, into which the inner mold just fitted. The powder was lightly tamped into the inner mold, and then the whole combination, powder, inner and outer molds, was heated to about 450° C, or a lower temperature if decomposition took place. The piston was then introduced, and the powder rapidly compressed in an arbor press, before the piston had time to heat sufficiently to become soft. The pressure thus exerted was about 20000 kg/cm^2. When cold, the inner mold was pushed out of the outer block, and turned off in the lathe until the slug of compressed powder was free. The advantages of this method are that the high temperature facilitates welding of the powder to a coherent mass, and at the same time the internal stresses are not so great as when compression is done cold; the slug does not break on removal from the mold, since no new stresses are introduced by the process of removal, and there is no contamination, even by a small amount of graphite. The slugs as finally compressed were usually between 3/8 and 1/2 inch long; if one attempts to make much longer slugs, too large internal stresses are likely. The mold must be of such a length as to give the required final length in a single stroke of the piston; the mold cannot successfully take two charges, even if only moderately compressed, as by a blow of a hammer, because the line of demarcation between the two charges is always evident.

By choosing a method of formation for the slug adapted to the particular material, slugs can be produced entirely massive in character, betraying no trace of their method of formation. The slugs were shaped to final dimensions by turning off the cylindrical sides in a jeweller's lathe, and by grinding the ends, which must be perfectly flat.

The measurements themselves always contain internal evidence as to the satisfactory homogeneity of the material; perfectly homogeneous material should show no permanent changes of dimensions even after the first exposure to hydrostatic pressure. This extreme requirement was seldom met, and in fact is seldom met by any material except single crystals, but in practically all cases a completely seasoned condition, with no hysteresis or set, was attained by one or more applications of pressure before the measurements were begun. The permanent deformations produced by the preliminary applications of pressure were usually a very small fraction of the elastic deformations due to the pressure; the details will be found under the data for the individual substances.

The measurements of this paper have been so conducted as to permit a fairly good quantitative estimate of one quantity not hitherto attempted, the thermal expansion. The apparatus is not especially well adapted to this sort of measurement, the changes of dimensions under pressure being much greater than the changes due to temperature in the range of this work from 30° to 75° C. Nevertheless the zero readings at atmospheric pressure at 30° and 75° should allow the thermal expansion to be calculated with some accuracy. Early attempts to make such calculations did not yield very satisfactory results, for reasons which were somewhat puzzling. I have therefore not given in my previous papers the values of thermal expansion which could be calculated from the original data. It now appears that the reason for the unsatisfactory character of many of the previous results is that many substances experience permanent changes of dimensions after their first exposure to a change of temperature, so that satisfactory measurements of thermal expansion can be obtained only on materials seasoned for temperature, just as satisfactory compressibility measurements can be obtained only on materials seasoned for pressure. Another measurement was therefore added in the series recorded in this paper to those usually made. The order of the measurements is: first, a compressibility run at 30°, starting from atmospheric pressure, raising pressure to 12000, and then back to atmospheric; temperature is then raised to 75° at atmospheric pressure, and measurements made at 75° up to 12000 and back to atmospheric. To these measurements is now added a return check reading at 30° at atmospheric pressure. If this agrees fairly well with the initial 30° reading, then one may be sure that the temperature seasoning has produced no permanent distortion, and the data may be expected to give good values for the thermal expansion at atmospheric pressure. If the difference between the 30° readings is important, the thermal expansion thus computed is not so reliable.

In computing the thermal expansion the most important correction is for the temperature coefficient of resistance of the nichrome wire of the lever piezometer; this was determined by independent experiment, and in fact this correction had already been determined for use in calculating the compressibility. The difference of thermal expansion between the nichrome and soft steel, which also enters as a correction, was neglected, the data in International Critical Tables showing an imperceptible difference between the thermal expansion of these two substances. The linear expansion of soft steel was assumed to be 12×10^{-6}. The results given by the measurements are essentially the difference between the expansion of the substance in question and soft steel. An idea of the accuracy of these thermal expansion measurements is given by comparison with known values in some cases where such comparison is possible. Thus the expansion of pure gold was found to be 14.4×10^{-6}, which agrees exactly with the value in Smithsonian Tables at 40°. The mean expansion of RbCl between 30° and 75° was 121×10^{-6}, whereas International Critical Tables (abbreviated hereafter as I. C. T.) gives 126×10^{-6} as the mean between 0° and 50°. In general, one may expect the values to be correct within a few per cent. The results will not be so good for those substances with large seasoning effects; details will be found under the individual substances.

DETAILED DATA.

CaS. This was "c.p." material obtained from Eimer and Amend. It was formed into a slug by cold compression in the split mold. After formation it was exposed to two seasoning applications of 12000 kg/cm² hydrostatic pressure, during which no measurements were made of the dimensions. It was then machined to final size. A final preliminary application of 12000 to season before the actual measurements produced a permanent distortion of 0.4% of the elastic deformation under 12000. This is less than the error of several of the single readings, and is without significance. Runs were made in the regular way at 30° and 75° with the following results:

$$\text{At } 30^\circ, \; -\Delta V/V_0 = 22.80 \times 10^{-7}p - 38.8 \times 10^{-12}p^2.$$

$$\text{At } 75^\circ, \; -\Delta V/V_0 = 20.22 \times 10^{-7}p - 25.8 \times 10^{-12}p^2.$$

Mean linear thermal expansion between 30° and 75° at atmospheric pressure, 17×10^{-6}.

At 30° the average departure from a smooth curve of a single one of the 13 readings was 0.72% of the maximum effect, and at 75°, 0.94%. The distinctly smaller compressibility at 75° than at 30° is a noteworthy feature.

SrS. This was from the same source and the manner of formation and the preliminary seasoning were the same as for CaS. The final pressure seasoning before the measurements resulted in no larger permanent set than for CaS. The return check reading at 30° after the measurements showed, however, so large an effect of the temperature seasoning that it did not seem worth while to attempt to compute the thermal expansion. At 30° the average arithmetical departure from a smooth curve of a single one of the 12 readings (one discard) was 0.65% of the maximum effect, and at 75° nearly five times as much. The much greater error at the higher temperature is unusual, and is doubtless connected with the same permanent effects which made impossible a calculation of the thermal expansion. In making the calculations of compressibility, by far the greater weight was given to the points obtained with decreasing pressure, which lay comparatively smoothly, so that the final results are probably much less in error than the figures given would suggest. The results were:

$$\text{At } 30^\circ, -\Delta V/V_0 = 23.83 \times 10^{-7}p - 38.6 \times 10^{-12}p^2.$$

$$\text{At } 75^\circ, -\Delta V/V_0 = 23.18 \times 10^{-7}p - 43.5 \times 10^{-12}p^2.$$

BaS. This was from the same source and formed and treated in the same way as the two preceding sulfides. For one reason or another the results were not as satisfactory as usual. The insulation of the electrical connections made much trouble, the work being done at a particularly humid time in the middle of summer. The pressure range of the usable readings was only 10000 kg at 30° and 8000 at 75°. The permanent distortion of the seasoning application of 12000 before the first run at 30° was of the order of 25% of the maximum pressure effect. The results of the first run at 30° were so irregular that this run was discarded, and repeated, somewhat more satisfactorily, after the run at 75°. At 30° the average arithmetical departure from a smooth curve of a single one of the 15 readings was 1.78% of the maximum effect at 10000 kg, and at 75° the departure of a single one of the 10 readings was 0.80% of the maximum effect at 8000 kg. The results were:

$$\text{At } 30^\circ, -\Delta V/V_0 = 28.93 \times 10^{-7}p - 52.1 \times 10^{-12}p^2.$$

$$\text{At } 75^\circ, -\Delta V/V_0 = 27.86 \times 10^{-7}p - 61.7 \times 10^{-12}p^2.$$

The average thermal expansion at atmospheric pressure between 30° and 75°, calculated from the best mean readings, is 34×10^{-6}; the extreme variation in the data for different runs would allow values ranging from 25 to 47×10^{-6}.

In general comment on the values for the three sulfides of the alkali earths, the progression in the series is in the direction to be expected, compressibility increasing with the atomic weight of the alkali element, as is already known to be the case with the alkali metals. The compressibility of two other series of compounds of the alkali earths has been determined by Madelung and Fuchs.[2] For $CaCO_3$ (arragonite), $SrCO_3$, and $BaCO_3$, all orthorhombic, they find the compressibility at 0° C to be 15.3, 17.4, and 20.2×10^{-7} respectively. The progression is in the same order, and there is also a larger jump between Sr and Ba than between Ca and Sr, as with the sulfides above. The same authors have also measured the compressibility of the sulfates, also all orthorhombic, and find for Ca, Sr, and Ba sulfates values for the initial compressibility ranging from 17.6 to 19.0, 15.7 to 16.2, and 17.4 to 17.8 × 10^{-7} respectively, depending on the source of the material. Here the progression is not at all regular, the compressibility of the Sr salt being less than that of the Ca and Ba salts; this effect is not obscured by the fact that materials from different sources give somewhat different values.

The unusual decrease of compressibility with rising temperature of the three sulfides is to be noted. The fact that it is shown by all three makes it fairly certain that it is a legitimate effect.

MgO. This material does not occur crystalline in nature, and all massive specimens of natural occurrence are badly flawed. I was most fortunate to obtain a water-clear transparent crystal of artificial origin, which I owe to the kindness of Professor C. Palache of the University Museum, who in turn obtained it from Mr. B. W. St. Clair of the General Electric Co. It was made at the Pittsfield Works of the General Electric Co., and was a by-product of the manufacture on a large scale of MgO for commercial purposes. Commercial MgO is an electric furnace product; after long operation of the furnaces a transparent deposit of MgO is laid down in the neighborhood of the electrodes as a condensate from the vapor. The piece at my disposal was not only perfectly clear and free from flaws, but was also practically all a single crystal grain, as could be proved by the cleavage pattern, which is simple cubic, exactly like NaCl.

The specimen of the clear crystal was cut from the block with a diamond saw, and ground to shape in the usual way. Its length was about 1.0 cm. After the very first application of pressure the length at atmospheric pressure was decreased by about 0.25% of the total elastic change of length under 12000 kg/cm², that is, by about 6 × 10^{-6} cm. This change cannot properly be described as a permanent change of length, however, because the subsequent zeroes wandered about in a random fashion in this range. Throughout, the behavior was like that of a material not in a perfect state of ease. Not only was there some wandering of the zero, but there was hysteresis at both temperatures between points with increasing and decreasing pressure, the width of the hysteresis loop being roughly the same as the wandering of the zero. The averages of the increasing and decreasing points, which give the differential compressibility between MgO and iron, were linear. Neglecting hysteresis, and using the mean values, the results were:

$$\text{At } 30^\circ,\ -\Delta V/V_0 = 5.904 \times 10^{-7}p - 2.22 \times 10^{-12}p^2.$$

$$\text{At } 75^\circ,\ -\Delta V/V_0 = 5.979 \times 10^{-7}p - 2.22 \times 10^{-12}p^2.$$

The mean thermal expansion between 30° and 75° was 13.3×10^{-6}, the extreme values being 13.9 and 12.7.

Before the sample of clear crystal MgO was obtained, measurements had been carried through in the regular way on a compressed slug of powdered MgO, made from Kahlbaum's MgO, "Zur Analyse." The results were perfectly regular and without appreciable hysteresis, and from this point of view were better than the results for the transparent crystal. The absolute value of the compressibility was of the order of twice that of the crystal, however. In attempting to find the explanation of this large discrepancy, the densities were measured. The transparent crystal at 23.8° has a density of 3.573. The compressed slug was proved by the weighings to be very porous in character, for on standing under water it absorbs such an amount as to indicate that about one quarter of the apparent volume is composed of minute cavities. Water, and therefore the pressure transmitting liquid, freely permeates these cavities, since it was possible to obtain consistent weighings repeatedly, both in the dry and the saturated condition. The slug must be thought, therefore, as in somewhat the condition of a skeleton frame-work, so that the compressibility measurements should give the compressibility of the material of which the framework is composed. The density of the framework was found to be 3.01, much lower than that of the clear crystal, and indicating that the material to a large extent must be in the amorphous condition. It is natural to suppose that a structure similar to that of the slug occurs in the rocks of the earth's crust, so that there is sufficient interest in the numerical values obtained with the slug to justify recording them, although the figures do not pertain to a physically well defined substance.

For the compressed slug:

$$\text{At } 30°,\ -\Delta V/V_0 = 9.86 \times 10^{-7}p - 10.8 \times 10^{-12}p^2.$$

$$\text{At } 75°,\ -\Delta V/V_0 = 9.97 \times 10^{-7}p - 12.2 \times 10^{-12}p^2.$$

The mean linear thermal expansion between 30° and 75° was 12.0×10^{-6}; data were not obtained from which the possible limits could be set. It is to be noticed that this expansion is somewhat less than that of the crystal.

Madelung and Fuchs give for the compressibility of three samples 6.9, 7.9, and 6.3, mean, 7.1×10^{-6}, much less than for the compressed slug and materially higher than for the crystal. Their material is not described further than as being of artificial origin. Their third figure, 6.3, was obtained on four fragments selected as the best from those with which the first two measurements were made. These fragments are described as small crystals. The densities of the first two samples were 3.476 and 3.466; the value for the third sample is obviously misprinted. It seems highly probable that the rather wide range of the values of Madelung and Fuchs means that their specimens included in an irregular way both crystalline and amorphous material.

CaO. This was formed from powder from Kahlbaum in the same way as the successful slug of MgO. It appeared hard and well polished, but did not function as well as nearly all the other materials. At both temperatures only the readings with increasing pressure were regular enough to retain, the decreasing readings lying off the curve by increasingly large amounts, the final zero shift at 30° and 75° being respectively 7.0% and 3.9% of the maximum effect. The average numerical departure from a smooth curve of the 12 readings retained at 30° was 0.52% of the maximum pressure effect, and at 75° the corresponding figure for 11 readings was 0.08%. It is to be noted that in all respects the behavior was better at the higher temperature, not a usual effect. The results were as follows:

$$\text{At } 30^\circ, \ -\Delta V/V_0 = 45.7 \times 10^{-7}p - 58.2 \times 10^{-12}p^2.$$

$$\text{At } 75^\circ, \ -\Delta V/V_0 = 46.4 \times 10^{-7}p - 58.5 \times 10^{-12}p^2.$$

Mean linear thermal expansion at atmospheric pressure, 30° to 75°, 21.0×10^{-6}. The extreme values allowed by using the values obtained with increasing and decreasing temperature are 19.2 and 22.8×10^{-6}.

The compressibility of CaO appears not to have been determined previously. The enormously greater value than for MgO is noteworthy, and would seem to demand explanation. The crystal structure of MgO and CaO is the same and of the NaCl type. It is highly probable that if clear crystallized CaO could be obtained the same differences as compared with the compressed slug would be found as were shown by MgO.

Cu_2O. This material was from Kahlbaum. Some difficulty was experienced in making it into a slug, there being a tendency to split on an axial plane when the split mold was opened. This was before the development of the technique using the soft steel mold turned off from the specimen. Finally, the difficulty was overcome by the use of sufficient lubrication of graphite and vaseline. Since the

specimen was turned down from 0.80 to 0.64 cm, there should have been no contamination of the final specimen by the lubrication. Two seasoning applications of 12000 were made as usual before machining, and then a further seasoning application after mounting in the lever piezometer. The change of zero during the final seasoning was 3.0% of the elastic effect of 12000. The average arithmetical departure from a smooth curve of a single one of the 14 readings at 30° was 0.41%, and at 75° the corresponding figure was 0.80%. There was a large permanent change of the zero on changing temperature from 30° to 75° at atmospheric pressure, indicating an unusually large temperature seasoning effect in this substance. After the pressure run at 75° temperature was lowered to 30° at atmospheric pressure, then raised to 75°, and then down to 30° again, in the attempt to get a good thermal expansion. The increasing and decreasing readings were not very consistent on a fractional basis, although the absolute difference was not greater than for many other substances. The only statement that appears justified about the thermal expansion is that it is very close to that for pure iron, namely 12.0×10^{-6}. The results for compressibility follow:

$$\text{At } 30°, \; -\Delta V/V_0 = 19.09 \times 10^{-7}p - 19.92 \times 10^{-7}p^2.$$

$$\text{At } 75°, \; -\Delta V/V_0 = 20.03 \times 10^{-7}p - 22.74 \times 10^{-7}p^2.$$

Cu_2O crystallizes in a type of structure which is shared only with Ag_2O. Similar behavior of the two substances is to be expected. Ag_2O is, however, entirely different, and experiences an internal change of some sort when exposed to pressure, with enormous hysteresis. The behavior of Ag_2O has been made the subject of a special paper.[3] Because of the hysteresis the compressibility of Ag_2O has no unique meaning at low pressures; at the upper end of the pressure range, however, an approximate value can be found, which is 23×10^{-7}. This is not far from the value just given for Cu_2O, and differs in the direction to be expected, metallic Ag being more compressible than metallic Cu by 3×10^{-7}.

As_2O_3. The powdered material was from Eimer and Amend. It was prepared in the split mold and seasoned by an application of 12000 kg/cm² before machining, like the previous materials. The final seasoning application of 12000 resulted in a permanent change of zero of 0.8% of the maximum effect. This may not all have been due to permanent distortion, but may have been partly due to disarrangement of the electrical connections because of too great a

pressure, for on making the measurements it was found that the compressibility was so great that 12000 carried the lever of the piezometer beyond its range, so that pressure during the measurements had to be restricted to 9000. At 30° the average arithmetical departure from a smooth curve of a single one of the 9 readings was 0.03%, and at 75° the corresponding figure for 15 readings was 0.41% of the maximum effect. The return check reading could not be made because of a connection which broke at the psychological instant, so that a good value for the thermal expansion was not obtained. The results for compressibility were:

$$\text{At } 30°,\ -\Delta V/V_0 = 92.49 \times 10^{-7}p - 272.4 \times 10^{-12}p^2.$$

$$\text{At } 75°,\ -\Delta V/V_0 = 92.88 \times 10^{-7}p - 250.5 \times 10^{-12}p^2.$$

The high compressibility and the unusually large curvature (large second degree term) are to be noted for this substance. The crystal structure of As_2O_3 is known to be molecular rather than ionic in character.[4] Furthermore, there are large open spaces in the structure, so that a high compressibility is to be expected.

$Sr(NO_3)_2$. This somewhat complicated substance, of a type unlike that of any of the others studied, was measured because the crystal structure is known and because it was easily formed into a slug. The powdered material, which was obtained from Eimer and Amend, on heating to near the melting point, showed a very small but perceptible amount of water. This water was doubtless present in the compressibility sample. It would have been dangerous to try to get rid of it, because of decomposition.

The final seasoning application of 12000 produced a permanent change of zero of 0.1% of the maximum pressure effect, less than the error of many of the individual readings. The average departure from a second degree curve of a single one of the 14 readings at 30° was 0.20% of the maximum effect, and the corresponding figure at 75° for 13 readings was 0.50%. The results were:

$$\text{At } 30°,\ -\Delta V/V_0 = 32.49 \times 10^{-7}p - 43.2 \times 10^{-12}p^2.$$

$$\text{At } 75°,\ -\Delta V/V_0 = 33.66 \times 10^{-7}p - 41.1 \times 10^{-12}p^2.$$

Mean linear thermal expansion 30°–75° at atmospheric pressure, 32.2×10^{-6}. The extreme range of values is from 31.7 to 32.7×10^{-6}.

The crystal structure of $Sr(NO_3)_2$ is somewhat complicated,[5] there being four molecules in the unit. A rather large compressibility is to be expected; as a matter of fact it is only 37% greater than the sulfide.

The Copper Halides. A great deal of trouble was experienced with these materials. They were obtained in the form of fine powder "c.p.," from Eimer and Amend. In spite of their low melting point they were particularly hard to get into coherent slugs, either sticking to the mold, or else breaking up into thin laminae during machining. After many trials a specimen of CuCl was obtained of good enough appearance to justify trying for compressibility measurements. This specimen was produced in the split mold; sticking and breaking were avoided by using only half the normal amount of material, thus making the slug only half the normal length, and decreasing the effects due to internal stresses. For the compressibility measurements it was necessary to make special holders of copper, previous experience during formation of the slugs having shown that these salts decompose even in the dry state on contact with iron. The compressibility measurements on CuCl now disclosed the new phenomenon of a very large plastic change of dimensions during the application of pressure; this effect was much larger at 75° than at 30°, and at first I was inclined to attribute it to some actual change in the material. It turned out, however, to be due to plastic creep under the small compressive force exerted by the lever of the piezometer. This was established by blank experiments, in which the slug of CuCl was maintained at atmospheric pressure in an oil bath at 105° for a week or more under a compressive force of 1 kg. The force exerted by the lever of the piezometer is 0.95 kg, and the usual cross section of the specimen is 0.28 cm^2. The length of the specimen was measured at intervals during the week with a micrometer, and found to be shortening continuously, until brought to a stop by receiving support from the walls of the container.

To meet the situation, the method of forming the slugs by compressing them hot into a soft steel shell, which was then turned off, was evolved. The temperature was not raised above 300° with the copper halides in order to avoid the polymorphic transitions which occur at higher temperatures at atmospheric pressure. Slugs formed in this way were free from internal strains and could be machined to shape without difficulty. Furthermore, they are much less subject to plastic deformation under the one-sided compression of the lever of the piezometer. This plastic deformation was eliminated as a source of error in the compressibility measurements by subjecting the specimens, in addition to the regular seasoning under 12000 kg/cm² hydrostatic pressure, to a preliminary application of 1 kg one-sided compression at 105°, following the change of length with a micrometer, and continuing the treatment until the motion had come to its asymptotic resting point. Since the flow increases at higher temperatures, it was assumed that if the seasoning was complete at 105° it would be adequate at 75°.

The plastic effects are greatest in the chloride, intermediate in the bromide, and indetectible in the iodide when the slug was made by the final method, although there was measureable plastic flow of the iodide when made by the first method. The order of magnitude of the effect is shown by the results for the chloride. This shortened at 105° under 1 kg load from 0.3246 inches to 0.3208 inches in 72 hours, and the shortening was about one half accomplished in the first 20 hours. A further study of the phenomena of plastic flow in these materials would doubtless be interesting for its own sake; there may be some connection with the mechanism responsible for the polymorphic transitions.

CuCl. In spite of the preliminary treatment of the slug, the permanent changes of zero were greater than for most substances. Two preliminary seasoning applications of 12000, instead of the usual one, were necessary before the readings became sufficiently steady at 30°, and even then it was necessary to discard the initial point, the others, both with increasing and decreasing pressure, lying on the same smooth curve. The run at 75° also showed a permanent effect; here the permanent change seems to have taken place at a single pressure near 8000 kg on increasing pressure, the points with decreasing pressure below 8000 differing by the approximately constant amount of 1.7% of the maximum effect from the points obtained with increasing pressure. The average arithmetical departure from a smooth curve of the 7 points retained at 30° (discarding one) was 0.15% of the maximum effect, and at 75° the corresponding figure for all 14 readings, making no discards, was 0.46%. In computing the compressibility correction had to be made for the fact that part of the gross compressibility was due to the copper holder; the necessary compressibility of copper had been previously measured. The results were:

$$\text{At } 30°,\ -\Delta V/V_0 = 24.63 \times 10^{-7}p - 14.1 \times 10^{-12}p^2.$$

$$\text{At } 75°,\ -\Delta V/V_0 = 27.16 \times 10^{-7}p - 24.2 \times 10^{-12}p^2.$$

Mean linear thermal expansion at atmospheric pressure between 30° and 75°, 21.8 × 10^{-6}. In spite of the zero shifts incident to the applications of pressure, the wandering of zero during changes of temperature at atmospheric pressure was small, and the value for thermal expansion should be fairly good. The values given by the first excursion from 30° to 75° and by the second excursion from 75° back to 30° were 22.8 and 20.8 × 10^{-6} respectively.

CuBr. Two specimens of this were used; the measurements recorded below were obtained with the second specimen, the first being unsatisfactory for the same reason as the first specimen of CuCl. Furthermore, the second specimen, like the second specimen of CuCl, was exposed to two seasoning applications of pressure, but not by intention. On the first application, which should have been to 12000, a defective washer prevented reaching more than 4000. A large zero change was found after this application. The second successful seasoning application was to 12000, and produced only a negligible further change of zero. At 75°, the first zero was off, all the other points, both with increasing and decreasing pressure lying approximately on a smooth curve. At 30° the average arithmetical departure from a smooth curve of a single one of the 14 readings was 0.15% of the maximum effect, and at 75° the corresponding figure for the 14 readings left after discarding the first was 0.17%.

CuBr departs from most of the other substances of this paper in that the relation between pressure and change of dimensions distinctly departs from the second degree, the curvature being greatest at the lower pressures. For this reason the results are given in the following Table.

TABLE I.

$-\Delta V/V_0$ OF CUBR.

	2000	4000	6000	8000	10000	12000
30°	.00574	.01090	.01562	.02016	.02461	.02888
75°	632	1162	1660	2135	2604	3064

The initial compressibilities are 28.7 and 31.6 × 10^{-7} at 30° and 75° respectively.

The mean linear thermal expansion between 30° and 75° at atmospheric pressure was 20.7 × 10^{-6}. This value is not quite as good as usual, the difference between the two values with increasing and decreasing temperature being somewhat more than usual. These two values were 22.2 and 19.2 × 10^{-6} respectively.

CuI. Measurements were made on only one sample, which was prepared by the final method, and was at once satisfactory. Preliminary compression under the one-sided application of 1 kg to test for plastic flow gave no set of as much as 0.001 mm in 24 hours at 105°. A single preliminary seasoning application of 12000 produced only a small change of zero. Many more readings were taken on this substance than on the other materials of this paper, since it seemed at first that there might be some small scale pattern like that suggested for MgO and actually demonstrated to exist for a number of other substances. Careful examination convinced me that there was no such pattern, however, the first suspected effect being simply incidental to a somewhat larger irregularity than usual. This was particularly marked at 30°; at 75° the points were as regular as usual. At 30° the average arithmetical departure from a smooth curve of a single one of the 25 readings was 0.43% of the maximum effect, and at 75° the corresponding figure for 31 readings, but with one discard, was 0.25%. The results were:

$$\text{At } 30°, \ -\Delta V/V_0 = 27.52 \times 10^{-7}p - 25.1 \times 10^{-12}p^2.$$

$$\text{At } 75°, \ -\Delta V/V_0 = 29.57 \times 10^{-7}p - 33.6 \times 10^{-12}p^2.$$

Mean linear thermal expansion between 30° and 75° at atmospheric pressure, 24.5×10^{-6}. The extreme values with increasing and decreasing temperature were 25.5 and 23.5×10^{-6} respectively.

Comparing now the three cuprous halides, the bromide occupies an anomalous position with regard to its behavior at low pressures. Its initial compressibility lies higher than that of the iodide, whereas it is to be expected that it will lie between that of the chloride and the iodide. The compressibility of iodide is higher than that of chloride, as is normal. The sequence of values of thermal expansion is also abnormal. The expansion of the iodide is higher than that of the chloride, but that of the bromide is lower than that of the chloride. At high pressures, however, the normal sequence is restored, the changes of volume at 30° and 12000 kg/cm^2 of the chloride, bromide, and iodide being 0.02752, 0.02888, and 0.02937 respectively. The return of the bromide to normal is evidently connected with the departure of the volume from a second degree relation in the pressure, the deviation being in such a direction that at low pressures the compressibility is higher than would otherwise be the case, and at high pressures lower. There is no very obvious reason in the crystal structure for the exceptional behavior of the bromide; all three salts have the same lattice structure, that of ZnS. The atomic spacing in the bromide is, however, somewhat greater than in other halogen series, and this somewhat looser structure may be responsible for the greater initial compressibility. Thus the spacing of CuBr is .428 of the way from the spacing of CuCl to the spacing of CuI, whereas in the series NaCl, NaBr, NaI, the spacing of the bromide is only 0.375 of the way from chloride to iodide.

The Alkali Halides. Probably more work has been done on the comparative properties of the alkali halides than on any other series of compounds, but in spite of this there have still remained a number of gaps in our knowledge of compressibility, the most striking being with regard to the Cs salts, none of which have been hitherto measured to high pressures. The data presented here complete the compressibilities for the 20 salts of this series, with the exception of RbF. With regard to this, X-ray analysis has shown that it is exceedingly difficult to get the simple fluoride, the salt as it is usually obtained being a mixture of various fluorides. With my limited chemical resources it would have been foolish to attempt the preparation of the pure fluoride, and since there was no other source of supply available, I have had to omit it.

LiI. This substance has been previously measured by Richards and Saerens[6] up to 500 kg, but their value lies so far off the sequence of the other salts, lying between Na and K instead of below Na as is to be expected (see Figure 2 in my paper in These Proceedings 66, 266, 1931), that it was desirable to repeat their measurements in order to correct, if necessary, the initial compressibility, as well as to extend the pressure range.

The great difficulty with LiI is that it is enormously hygroscopic. The first sample appeared dry enough, and I unsuspectingly made a slug in the regular way in the split mold, and completed a run with apparently good results at 30°. At 75°, however, something happened, and opening the apparatus the specimen was found completely melted. It seems evident that this first specimen was the cryohydrate $LiI3H_2O$. Since the results on this were so regular, they are recorded below, although they doubtless are not of great significance, because $LiI3H_2O$ crystallizes in the hexagonal system, and the method of formation of the slug was such that there may well have been a preferred orientation. The second sample was made with every precaution to keep the water out, but was not entirely successful. Immediately before formation of the slug, the material, which was from Kahlbaum, was heated to the melting point to remove water. During the turning to final dimensions in the lathe it was kept covered with Nujol applied with a brush held against the rotating specimen. Three days elapsed after the machining before measurements could be made. During this time it was kept in a stoppered bottle under Nujol but enough moisture got in through the Nujol from the air to result in an increase of length of 0.0008 inches on .4000 inches. Just before the compressibility measurements it was turned again in the lathe, shortened and made smaller in diameter so that most of the water should have been removed with the outer layer. The measurements at 30° were good, but at 75° there was some plastic flow, and the zero readings were not entirely steady, which was taken to mean that there were still slight traces of water remaining. For this reason, a pressure run was not attempted at 75°.

The average arithmetical departure from a smooth curve of a single one of the 14 readings for $LiI3H_2O$ was 0.06% of the maximum effect, and the corresponding figure for the 14 readings of LiI was 0.10%. Both substances, therefore, give unusually regular readings, as is to be expected when the compressibility is high. The results follow:

For the cryohydrate, $LiI3H_2O$, assuming equal compressibility in all directions:

$$\text{At } 30°, \ -\Delta V/V_0 = 65.1 \times 10^{-7}p - 117 \times 10^{-12}p^2.$$

For LiI plus a trace of water:

$$\text{At } 30°, \ -\Delta V/V_0 = 58.89 \times 10^{-7}p - 107.4 \times 10^{-12}p^2.$$

The value of Richards and Saerens[6] was 71×10^{-7}. The new value is much lower and brings LiI about the distance below Na that is to be expected. The high value of Richards and Saerens can evidently not all be explained by the presence of water, as is shown by my value for the cryohydrate, and also by the fact that their value for RbCl, which does not absorb water, is also too high by approximately the same amount. Some difficulty with their method is indicated. There is no reason to think that this same error applies to other papers in which Richards collaborated with other students.

The comparatively small difference between the compressibility of LiI and of $LiI3H_2O$ is worthy of comment; it is evident that water in the form of water of crystallization has a much smaller compressibility than when it is free. It is to be remembered, however, that the definiteness of this result is somewhat obscured by the fact that

$LiI3H_2O$ does not crystallize in the cubic system, so that its true compressibility may be different from that given above.

NaI. This has been previously measured by Richards and Jones[7] but since their pressure range was only 500 kg, it was desirable to repeat the measurements over a wider pressure range. The material was "c.p." stock from Eimer and Amend. It was formed in the split mold in the regular way, after heating to remove moisture. This salt is much less hygroscopic than LiI and does not form a cryohydrate, but it is nevertheless much more hygroscopic than most of the other salts of the series. A regular run was made at 30°, and then the run at 75° was attempted, but there was some plastic yield, which was taken to indicate the presence of a slight amount of moisture. Although the yield and the probable amount of water were both small, it did not seem worth while attempting the measurements at 75°, since the only purpose of these is to obtain the temperature coefficient of compressibility, and it is probable that the effect of slight moisture on the temperature coefficient is much greater than on the compressibility itself.

The relation between pressure and change of volume is distinctly not of the second degree, the compressibility at low pressures being greater and at high pressures less than such a relation would demand. The results are therefore given in a table rather than by the usual formula. The average arithmetical departure from a smooth curve of a single one of the 14 readings was 0.36% of the maximum effect.

TABLE II.

$-\Delta V/V_0$ OF NaI AT 30°.

Pressure, kg/cm²,	2000	4000	6000	8000	10000	12000
$-\Delta V/V_0$	.01299	.02471	.03525	.04513	.05462	.06385

The initial compressibility, which may be obtained graphically from the data in this table, is 69.36×10^{-7}. This is not in bad agreement with the mean value of Richards and Jones, between 100 and 500 kg, which is 70×10^{-7}. My values are, if anything, a little high because of moisture; the effect of this would be to accentuate the difference between the value of Richards and Jones and mine.

RbCl. This had been previously measured by Richards and Saerens,[6] but their value lies so far out of the sequence of the other salts (see Figure 2 of my 1931 paper already quoted), that the same situation exists as with regard to LiI, and repetition is desirable.

The RbCl used for these measurements was obtained originally from Professor Richards, who had prepared it for atomic weight determinations. I had previously used this sample in measuring the pressure transitions, which occur between 5000 and 6000 kg. During these manipulations it became contaminated with a minute quantity of iron. This iron was kindly removed electrolytically by Dr. P. A. Anderson. It was then fused and formed into a single crystal by the technique already used and described in connection with measurements of the elastic constants of a number of the alkali halides. The resulting crystal was perfectly clear and transparent, and appeared faultless in every way; in view of the results presently to be described it was fortunate that the measurements could be made on so perfect a specimen.

In order to avoid the known transition, the pressure range of the measurements was restricted to 4500. Although the high pressure modification is also cubic, it was to be anticipated nevertheless that if the transition had been allowed to run the specimen would not have recovered its single crystal condition on release of pressure. Satisfactory measurements were made at 30°, the points lying on a smooth curve with about the usual amount of irregularity, except for one point which was markedly divergent. The run at 75° also appeared at the time of making the readings to be satisfactory, but on plotting the points on a much larger scale for the final computations, there appeared a distinct break in the curve between 900 and 1000 kg, corresponding to a discontinuity in the volume of 0.0005. Although small, this seemed too definite to ignore, and the run at 75° was repeated four months later, with again unmistakable evidence of a discontinuity at the same place. In Figure 1 the deviations of the observed changes of length from linearity are plotted, showing the discontinuity. The discordant point of the 30° run was now seen to fall in with the existence of a discontinuity, this point being no more irregular than the others when such a discontinuity is assumed. The precision is not high enough to permit the calculation of accurate values of the discontinuity; $\Delta V/V_0$ has already been stated to be approximately 0.0005 at 75°, and at 30° it is perhaps 25% less.

The initial compressibilities at 30° and 75° were found to be respectively 65.2 and 67.5×10^{-7}. The compressibility doubtless decreases with increasing pressure, as is normal, but the pressure range was so low, less than 1000 kg, that no good values could be found for the decrease of compressibility with pressure of the low pressure form. The range from 1000 to 4000 is large enough, however, to allow an estimate to be made of the curvature. At 30° the average arithmetical deviation from a smooth curve with single discontinuity of the 10 readings was 0.19% of the maximum effect,

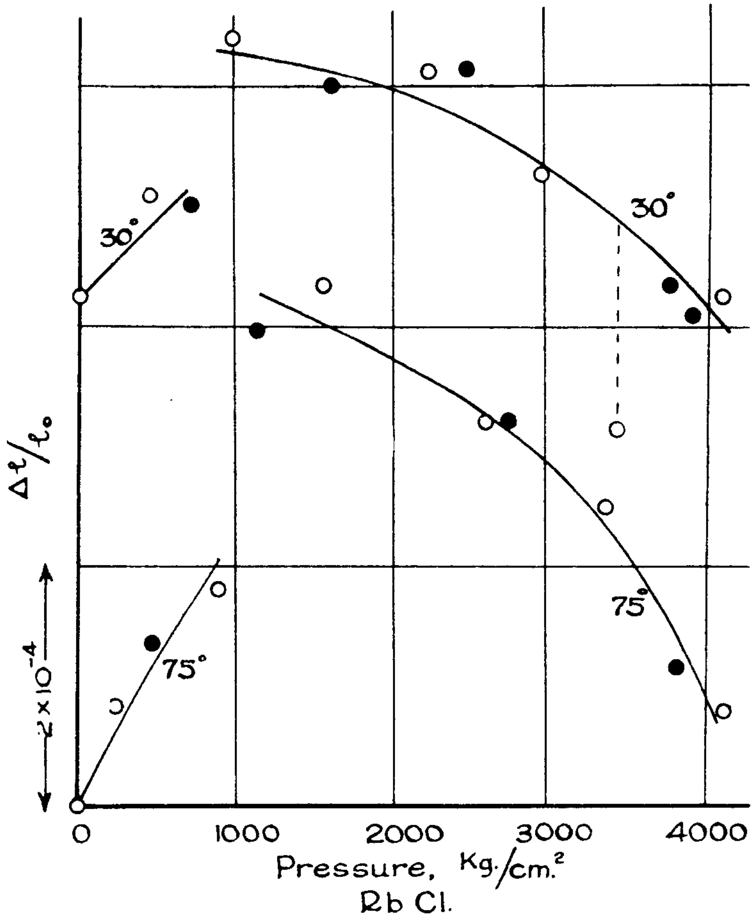

FIGURE 1. The deviations from linearity of the uncorrected differential changes of length of RbCl with respect to iron at 30° and 75° C. Points with increasing pressure are shown by open circles, and those with decreasing pressure by solid circles.

and at 75° the corresponding figures for the two runs of 11 and 14 readings were 0.20% and 0.14%. The absolute values of the two runs at 75° differed from their mean by 0.75%. Compared with the other substances of this paper, the figures for the average deviation from a smooth curve should be decreased by a factor of 3 to compensate for the smaller pressure range, indicating a regularity in the results materially better than usual. Between 1000 and 4000 the change of volume is represented by the formulas:

$$\text{At } 30^\circ, \ -\Delta V/V_0 = .00692 + 58.8 \times 10^{-7} (p - 1000) - 57.3 \times 10^{-12} (p - 1000)^2.$$

$$\text{At } 75^\circ, \ -\Delta V/V_0 = .00725 + 61.9 \times 10^{-7} (p - 1000) - 81.3 \times 10^{-12} (p - 1000)^2.$$

The constant terms in these formulas, that is, the initial ΔV's at 1000, do not lay pretense to much accuracy, in fact it is doubtful if the first significant figure could be defended. However, merely as a matter of computation, the same number of figures was given as in the other terms, so as to allow the relative volume changes above 1000 to be found with no formal change in the manipulations of computation.

The mean linear thermal expansion at atmospheric pressure between 30° and 75° was 121 × 10^{-6}. I. C. T. gives 126 × 10^{-6} as the mean for the range 0°–50°. The agreement is fairly good.

In comment on the numerical values it is to be said in the first place that the initial compressibility is markedly lower than that of Richards and Saerens,[6] 65 against 73. The new value brings RbCl into a smooth sequence with the other alkali halides, as may be seen from Figure 3 at the end of this paper.

The surprising and significant feature is the mere existence of the discontinuity in the volume; the crystal structure is so simple that there does not seem to be place for a transition of this character. The volume change is so small compared with that of the transition from simple cubic to body centered cubic, which occurs between 5000 and 6000 kg, 0.0005 against 0.146, that the mechanism must be entirely different, and is probably not a change of lattice structure at all, but some change of a quantum character in the constituents of the lattice, such as a passage of an internal electron from one shell to another. The ionic structure of the lattice is so simple that one cannot look for rotation effects, which may well be involved in such transitions as those of NH_4Cl.

One would expect discontinuities of this sort in other salts of the series. We owe our knowledge of most of these to Slater,[8] who used a lever piezometer and general technique similar to that used here. Professor Slater, at my request, examined again his data for RbBr and RbI but could find no trace of such a discontinuity. Of course it may well be that if this effect occurs at all for the other salts it may be at negative pressures, for, after all, 1000 kg is small compared with the internal pressures which are supposed to be characteristic of all these salts.

CsF. This material I owe to the kindness of Dr. R. W. G. Wyckoff; it is in fact the identical sample which he had previously loaned me for my exploration searching for a transition at high pressure[9] (which was not found), and had been kept by him undisturbed for five years in the sealed bottle in which I had returned it to him. I fused it in a platinum crucible, and then broke up the fused mass, picking out the clear parts, the parts which solidified last being somewhat discolored with an unknown impurity. The colorless parts were then recrystallized by slow lowering from the furnace in a cylindrical mold of platinum foil 0.001 inch, by the same technique as that used in making single crystals.[10] The resulting product, although colorless and without flaws, was not transparent, but cloudy, and was doubtless not of the same high purity as the RbCl.

At 30° the relation between ΔV and pressure was distinctly not of the second degree, although the deviations were slight. The formula given below is to be used with corrections, as indicated. There was also evidence of the same effect at 75°, but it was too slight to attempt to reproduce numerically. At 30° the average arithmetical departure from a smooth curve of a single one of the 14 readings was 0.08% of the maximum effect, and the corresponding figure at 75° for 13 readings (one was discarded) was 0.17%. The results are:

$$\text{At } 30°, \; -\Delta V/V_0 = 41.55 \times 10^{-7}p - 57.9 \times 10^{-12}p^2.$$

(to be used with corrections as described)

$$\text{At } 75°, \; -\Delta V/V_0 = 43.47 \times 10^{-7}p - 61.8 \times 10^{-12}p^2.$$

At 30° the compressibility is greater at low pressures and less at high pressures than indicated by the formula. The greatest deviations are at 3000, where the actual $-\Delta V/V_0$ is 0.000186 greater than given by the formula, and at 9000, where it is 0.000325 less. At 0, 6000, and at 12000 the formula gives the correct ΔV. By drawing a sort of sine curve through these corrections, which are thus known at 5 points, values of $-\Delta V/V_0$ may be found at any pressure accurate within the limits of error of the measurements. I find from a curve of this kind that the initial compressibility is 43.95×10^{-7}, instead of 41.55×10^{-7} as given by the formula. The sign of the temperature coefficient of compressibility indicated by this would be negative, but this cannot be accepted as proved, because practically all the irregularity in the 75° points occurred at the low pressure end of the curve, the points above 4000 not departing appreciably from a smooth curve.

The average linear thermal expansion at atmospheric pressure between 30° and 75° was 31.7×10^{-6}. The two values with increasing and decreasing temperature were 32.1 and 31.3 respectively, much better agreement than usual.

CsCl. This material I owe to the kindness of Professor L. M. Dennis of Cornell University. It was compressed into a slug in the split mold. The seasoning application of 12000 produced a change of zero of 2% of the elastic effect of 12000. At 30° the average departure from a smooth curve of a single one of the 14 readings was 0.12% of the maximum effect, and at 75° the corresponding figure, also for 14 readings, was 0.17%. The results were as follows:

$$\text{At } 30°, \; -\Delta V/V_0 = 58.29 \times 10^{-7}p - 96.9 \times 10^{-12}p^2.$$

$$\text{At } 75°, \; -\Delta V/V_0 = 60.60 \times 10^{-7}p - 101.1 \times 10^{-12}p^2.$$

The mean linear thermal expansion between 30° and 75° at atmospheric pressure was 49.5×10^{-6}, the extreme values corresponding to increasing and decreasing temperature were 51.0 and 47.0×10^{-6} respectively. I. C. T. gives for the density the formula $d_t = d_0 [1 - 15.9 \times 10^{-5}t + 4 \times 10^{-7}t^2]$, between 0° and 70°. The first term is not far from three times the linear expansion just given, as it should be, but the second degree term is so large that at 70° the expansion is 38% less than at 0°. It would seem that there must be some error in the formula.

The compressibility of CsCl has been measured by Richards and Saerens[6] in the range between 50 and 200 kg. Their compressibility in this range at 20° is 58×10^{-7}, agreeing with the above within the limits of error.

CsBr. This material I also owe to the kindness of Professor Dennis, who specially prepared it from his stock of other caesium salts. It was compressed to a slug in the split mold, like CsCl. Due to an oversight in setting up the apparatus it was not possible to measure the permanent set produced by the seasoning application of 12000; the subsequent changes of zero were of the normal order of magnitude. The average arithmetical departure from a smooth curve of a single one of the 14 readings was 0.39% of the maximum effect at 30°, and at 75° the corresponding figure for 13 readings was also 0.39%. The results follow:

$$\text{At } 30°,\ -\Delta V/V_0 = 69.18 \times 10^{-7}p - 144.9 \times 10^{-12}p^2.$$

$$\text{At } 75°,\ -\Delta V/V_0 = 72.00 \times 10^{-7}p - 141.6 \times 10^{-12}p^2.$$

The mean linear thermal expansion at atmospheric pressure between 30° and 75° was 59.8×10^{-6}. The maximum and minimum values for this, obtained with increasing and decreasing temperature, were 60.8 and 58.8×10^{-6}. I. C. T. gives 46.7×10^{-6} as the mean between 0° and 50°. The difference is larger than it should be.

Richards and Saerens found for the average compressibility between 50 and 200 kg at 20° 69×10^{-7}, agreeing with the above within the limits of error indicated by their significant figures.

CsI. For this material I am indebted to Professor G. P. Baxter, who had a supply of the pure material already on hand. It was formed into a slug in the split mold like the chloride and the bromide. The seasoning application of 12000 produced a permanent change of length of 2.5% of the total elastic change under 12000. At 30° the average arithmetical departure from a smooth curve of a single one of the 14 readings was 0.35% of the maximum effect, and at 75° the corresponding figure for 13 readings, after discarding one, was 0.09%. The results follow:

$$\text{At } 30°,\ -\Delta V/V_0 = 84.03 \times 10^{-7}\text{p} - 201.9 \times 10^{-12}\text{p}^2.$$

$$\text{At } 75°,\ -\Delta V/V_0 = 86.61 \times 10^{-7}\text{p} - 195.6 \times 10^{-12}\text{p}^2.$$

The mean linear thermal expansion at atmospheric pressure between 30° and 75° was 54.9×10^{-6}. The extreme values obtained with increasing and decreasing temperature were 57.3 and 52.5 respectively. I. C. T. gives for the mean expansion between 0° and 50° 48×10^{-6}. The difference is in the direction to be accounted for by the difference of temperature range, and perhaps is not too large, if the very large change of thermal expansion of CsBr with temperature indicated by I. C. T. is near the truth.

Richards and Saerens[6] give for the mean compressibility between 20 and 500 kg at 20° 91×10^{-7}, considerably higher than the above, and greater by an unexpected amount from what might be extrapolated from the figures for chloride and bromide.

DISCUSSION.

Most of the necessary comment on these figures for compressibility has already been given in connection with the presentation of detailed data. It is worth while, however, to consider the values for the alkali halides. Figure 2 shows the initial compressibilities at 30° of the 19 compounds which have been measured, RbF being the only missing member, and Table III contains the numerical values. This figure is to be regarded as a revision and extension of Figure 2 of my 1931 paper. In that figure the value for NaI was shown by a cross, the implication being that it had been determined by Slater with the lever piezometer. This was an error; Slater did not measure the compressibility of NaI, and the value there shown should have been designated by a circle instead of a cross, and originated from Richards and Jones.[7] The new value of this paper for NaI agrees very closely with that of Richards and Jones, the difference being hardly noticeable in the figure.

TABLE III.

INITIAL COMPRESSIBILITIES, AT 30°, OF THE ALKALI HALIDES.

	F	Cl	Br	I
Li	11.5×10^{-7}	33.4×10^{-7}	42.2×10^{-7}	58.9×10^{-7}
Na	20.7	41.8	49.8	69.4
K	32.4	55.2	65.7	83.7
Rb		65.2	77.8	93.8
Cs	41.6	58.3	69.2	84.0

It is in the first place to be noticed that the new values for LiI and RbCl bring these substances down into the sequence of values of the other compounds.

CsF is of the same crystal structure as the other fluorides, so that it is fair to interpolate between the values for KF and CsF and estimate that the compressibility of RbF is about 40×10^{-7}. The curves assume a regularity which makes this interpolation much more certain if one reverses the order of Figure 3, spreading along the axis of abscissas the alkali metals, spaced according to their ionic radii (not uniformly, as in Figure 3), and draws curves through the points with a common halogen ion. Both the values for CsF and RbF are

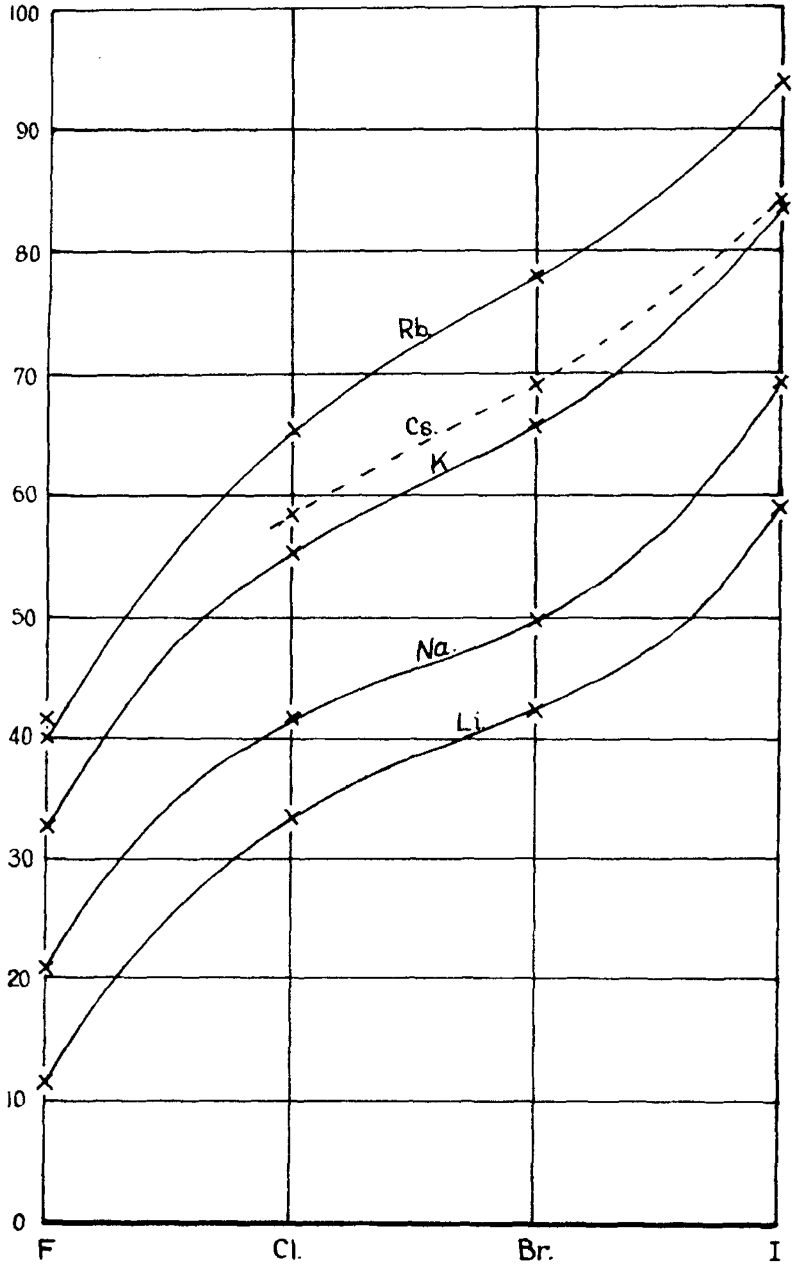

Figure 2. The first degree term in the compressibility at 30° C multiplied by 10^7 for the alkali halides.

lower than would probably have been estimated from an extrapolation of the three values for LiF, NaF, and KF, which were the only fluorides previously measured.

The other caesium salts, CsCl, CsBr, and CsI, crystallize with the body centered type of structure instead of the simple cubic type of the other alkali metals, and a break in the sequence is therefore to be expected. This is very strikingly shown in the diagram, these caesium salts falling between K and Rb, instead of lying beyond Rb, as the sequence would demand. The increments of compressibility on passing from chloride to bromide and to iodide is approximately the same as with the other type of crystal structure.

It is instructive to plot the second degree terms in the compressibility, that is, the term which determines the change of compressibility with pressure. The numerical values are given in Table IV, and they are plotted in Figure 3, which is the exact analogue of Figure 2. The curves are the same in general character as the curves for the first degree terms, but the tendencies are exaggerated. Thus the first degree term for the Li salts increases by a factor of 5.1 on passing from the fluoride to the iodide, whereas the second degree term increases by a factor of 12.5.

TABLE IV.

SECOND DEGREE TERMS, AT 30°, IN THE COMPRESSIBILITIES OF THE ALKALI HALIDES.

	F	Cl	Br	I
Li	8.6×10^{-12}	32.4×10^{-12}	50.2×10^{-12}	107.4×10^{-12}
Na	17.7	50.4	62.1	136.
K	32.0	73.5	102.5	150.7
Rb		156.9 57.3	133.4	197.7
Cs	57.9	96.9	144.9	201.9

Several points with regard to the second degree terms require comment. In the first place, the value 136×10^{-12} is given for NaI, although it was stated in the text that the results for NaI could not be represented by a second degree formula. The value 136×10^{-12} is the value which, when combined with the initial compressibility of 69.4×10^{-7}, reproduces by a second degree formula the observed change of volume at 12000. It seemed that this method of procedure gave the fairest comparison with the other substances. The second

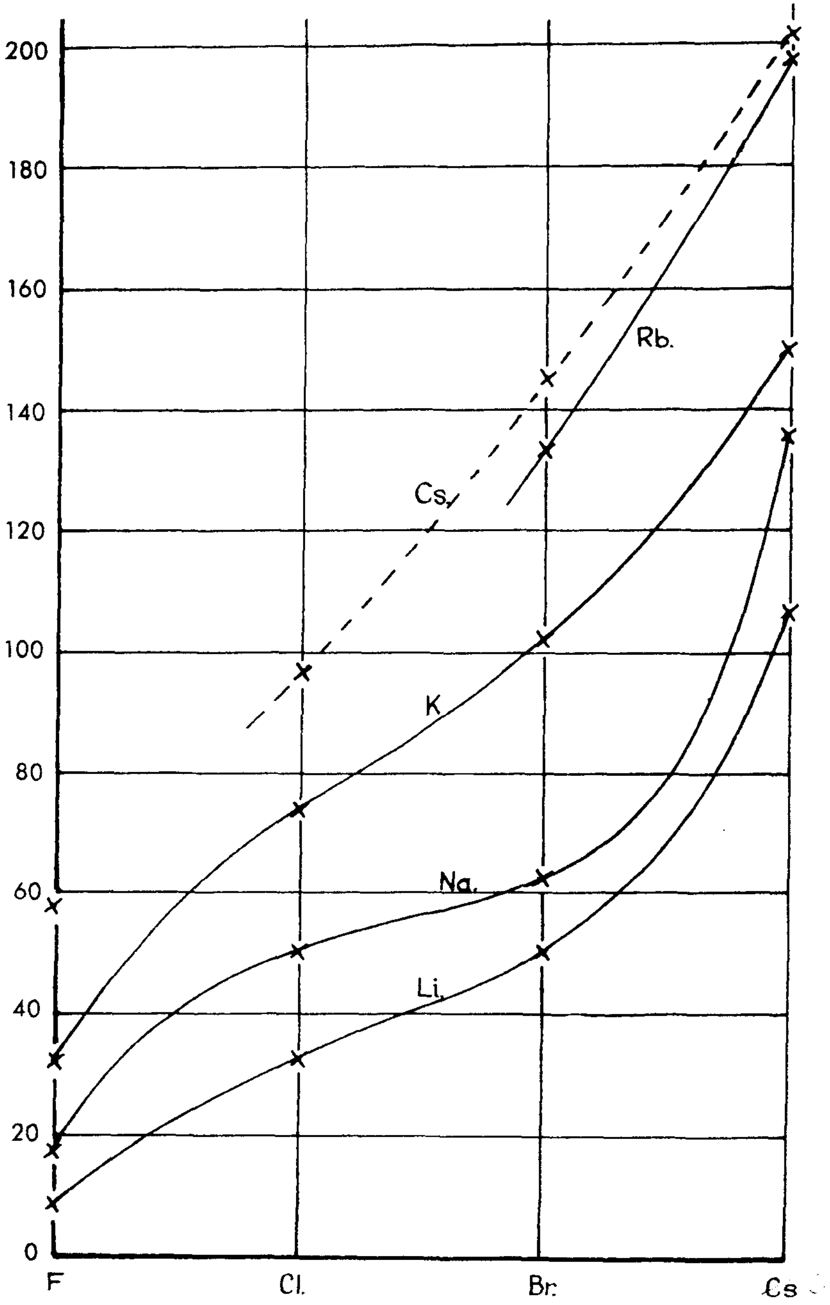

FIGURE 3. The second degree term in the compressibility at 30° C multiplied by 10^{12} for the alkali halides.

degree terms for the body centered caesium compounds lie above the corresponding values for the simple cubic rubidium compounds, instead of below them, as did the first degree terms. The difference, however, is much less than would be expected from the sequence of the other members of the series, so that we have here again a definite effect of the modified crystal structure. In Table IV two values are given for RbCl, neither of which is plotted. It will be remembered that RbCl was found to have a small break at about 1000 kg, indicating some kind of internal transition. The second degree term that one would like to plot and that would be expected to fall smoothly with the values for the other compounds would be that given by the experimental points below 1000 kg, but the experimental accuracy was not sufficient to allow this to be determined. The first value of the table, 156.9×10^{-12}, was obtained by ignoring the break, and drawing the best possible smooth curve through all the points below 4500 kg. This value is so high as to be impossible. The other value, 51.3×10^{-12}, is the second term in the formula which reproduced the experimental results between 1000 and 4500. It is obviously much too low. The fact that one of these values is too high and the other too low very considerably increases our assurance that this transition is real. Furthermore, the fact that none of the other salts, whether of rubidium or the other alkali metals, show any such anomaly in the second degree term makes it highly probable that the transition of RbCl is unique.

I am indebted to my assistant, Mr. L. H. Abbot for making the readings of this paper. I am also indebted for financial assistance in purchasing supplies to the Rumford Fund of the American Academy of Arts and Sciences and for financial assistance in procuring the services of Mr. Abbot to the Francis Barrett Daniels Fund of Harvard University.

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THE SALTATORIAL RODENT DIPODOMYS: THE FUNCTIONAL AND COMPARATIVE ANATOMY OF ITS MUSCULAR AND OSSEOUS SYSTEMS.

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INTRODUCTION.

As an important detail in the program of the writer to investigate and report upon the morphology of diverse groups of mammals especially adapted for particular modes of progression, work upon saltatorial and fossorial rodents was started some ten years ago. After making considerable progress these projects were laid aside until a greater variety of material could be secured. Many rodents of diverse affinities, in many parts of the world, have become independently adapted in high degree for progression by saltation, and it was desired especially to obtain for study such forms as the 4-toed jerboa (*Scarturus*) and the Australian jerboa mice *Notomys* and *Ascopharynx*, the last with its peculiar external gular pouch. After a number of years, however, this desire seems no nearer realization, and rather than delay the project indefinitely it was determined to offer the present contribution as a study of the anatomy of one of the most interesting of New World rodents, the kangaroo rat *Dipodomys*, in comparison with such forms of the Old World jerboas as could be secured. For the reason that comprehensive reports of the myology and osteology of the more interesting of our small mammals are so seldom undertaken, the present work is offered in considerable detail, with especial reference to the saltation features and the convergence exhibited by the two groups. Some of the myological details of the jumping mouse *Zapus* are also mentioned, and osteological ones of the large African saltator *Pedetes*, as well as comparison of external features of a number of other genera available. Only a partial, and rather meager, discussion of results is offered, however. The conclusions resulting from this study will be presented in a separate paper, discussing in detail the comparative saltatorial and cursorial modifications of marsupials, insectivores, primates, rodents, lagomorphs, and ungulates, with special reference to the basic principles involved.

In the accompanying drawings no uniform scale was employed. For such features as the shank and thigh, the different details appearing on a single plate were all drawn the same actual size so as to facilitate comparison.

MATERIAL.

With the exception of a single skeleton of *Scirtopoda* originally donated by me to the Museum of Vertebrate Zoology of the University of California several years ago and later borrowed, all the material upon which the present study was based is in the collections of the United States National Museum. In this connection I wish to express appreciation for the helpful coöperation of the U. S. Biological Survey, through the courtesy of Dr. E. W. Nelson and Dr. W. P. Taylor, in securing for my use numbers of spirit specimens and of skeletons of *Dipodomys spectabilis*, which is one of the largest (and hence most desirable for dissection) forms of this genus. I have had access to two spirit specimens, one complete skeleton, and numerous skins and skulls of the 5-toed, Asiatic jerboa (*Allactaga mongolica longior*), obtained some years ago by A. deC. Sowerby. Of the 3-toed, Egyptian jerboa (*Scirtopoda orientalis*), I have had one complete and one incomplete skeleton and two spirit specimens, the latter originally secured in the vicinity of Alexandria, Egypt, by the late John Hornung. I purchased these specimens years ago and later presented them to the National Museum. Of further specific interest are a skeleton of the African *Pedetes*, skins and skulls of the Asiatic 3-toed jerboa *Dipus sowerbyi*, the type skin and skull of the rare *Euchoreutes naso alashanicus*, and a dissected specimen of the genus *Zapus*. Specimens, including skeletons, of other genera and species of the Heteromyidae (pocket mice and kangaroo rats) have also been used to some extent, as well as of the wood rat *Neotoma*, various sorts of squirrels, and many miscellaneous mammals of more or less saltatorial habits.

In the following pages where *Dipodomys*, *Allactaga*, and *Scirtopoda* are mentioned without statement of the species, reference is to the forms dissected, as mentioned above.

AFFINITIES.

No historical résumé of the nomenclatural changes experienced by the animals discussed will here be offered, but a few words regarding relationships will be in order. The genus *Dipodomys*, constituting the chief subject of the present paper, belongs to the family

Heteromyidae. *Perodipus*, long recognized as a genus on the basis of its possession of a hallux, is considered to be a synonym, as suggested by Grinnell (1922), for the reason that in certain forms the hallux is present in some individuals and absent in others. *Dipodomys* is the most saltatorial genus of this family, but the little kangaroo mouse *Microdipodops* has more specialized cranial features.

Of more generalized form are the genera *Perognathus* (with subgenera *Perognathus* and *Chaetodipus*), *Heteromys* and *Liomys*, some features of which are herein discussed. This family, as pointed out more specifically by Miller and Gidley (1918), belongs to the superfamily Sciuroidae, and hence it is fitting that the features of *Dipodomys* should be compared with those of some of the squirrels of the family Sciuridae, which has been done from time to time. Of interest in this connection also are some of the characters of the pocket gophers, of the family Geomyidae, belonging to this superfamily, for geomyids are the closest living relatives of heteromyids, and the two groups agree in being the only living mammals with capacious, bilaterally symmetrical, external cheek pouches.

Comparisons have frequently been made in the following pages with features of the wood rat *Neotoma* simply for the reason that a study of this animal, as representing a fairly generalized rodent, has previously been made. Actually, however, its relationship to the kangaroo rat is rather distant, for *Neotoma* belongs to the family Cricetidae (or Sigmodontidae if one prefer to consider the New World members as constituting a separate family) of the superfamily Muroidae.

To the superfamily Dipodoidae belongs the family Dipodidae or jerboas. In this superfamily is also *Zapus*, of the family Zapodidae, and *Pedetes* of the family Pedetidae, both of which are herein mentioned. The jerboas that have been dissected in this connection are the 3-toed Egyptian animal long known as *Dipus aegyptius* but termed in later years *Scirtopoda orientalis*, and the 5-toed Asiatic jerboa of the form *Allactaga mongolica longior*. Proportions and skull characters of the 3-toed Asiatic species *Dipus sowerbyi* have also been given, but the comparative material is too scanty for me to have attempted to pass upon the generic status of this animal. The jerboa of the genus *Euchoreutes* is discussed elsewhere. A single poorly preserved specimen of this mammal has been examined, but none of the interesting 4-toed *Scarturus* from Africa, the remarkable little *Salpingotus* from northern Asia, or the genus *Cardiocranius* of China.

HABITS OF THE KANGAROO RAT.

In order that the bodily form and the functions of the different structures of a specialized mammal may be evaluated with some intelligence it is necessary that one consider its habits and habitat.

The genus *Dipodomys* is distributed over the open country of western North America from Montana to the Valley of Mexico, and from Kansas and Texas to the Pacific Ocean. On the whole it is found in open country but is not confined to this type of habitat. In prairie country it is at home in the short grass lands, often devoid of brush, but over most of its range there are at least scattered bushes of sage, greasewood, or other shrubs characteristic of an arid region; and the genus is often particularly well represented in some sections of California quite densely grown to chaparral. It seems highly probable, however, that the latter type of habitat has been secondarily invaded from contiguous areas of a more open character. Invariably, however, the preferences of the genus are for surroundings with arid tendencies. In parts of its range the genus is found where the winter rains may be relatively copious, and there may even be rather low summer temperatures accompanied by frequent fogs (as in the case of *D. m. goldmani*, near Monterey, Calif.), but always the immediate habitat is of an arid character, and individuals will never be found, in any part of the range of the genus, in low, swampy areas amid lush vegetation.

In the main, also, kangaroo rats will not be found in areas where the ground is covered by large rocks, nor are the animals partial to hard ground. There are exceptions to the latter rule and occasionally they occur where the ground is both hard and covered with small, loose rocks. But preference is very definitely shown for soil that is deep and loamy, and especially for the more sandy type of soil. One species (*D. deserti*) is practically confined to deposits of aeolean or drifting sands, and as these are usually very light in color, the paleness of the coat of this species may be considered to be correlated therewith.

When conditions chance to be favorable kangaroo rats may occur in great numbers so that dozens may be caught in the collector's traps during a single night. It is therefore indicated that in their normal range they are fully as successful a mammalian type as the ground squirrel, meadow mouse, and other rodents which are ecologically aggressive.

Most species of *Dipodomys* do not construct elaborate burrow systems, possibly because of their weak digging equipment. Mostly the home consists of a rather short and shallow burrow with two entrances and a side chamber for a nest, or at times (possibly in the case of a vagrant male), a tunnel but little more than a foot long will be constructed. But in most cases there will be employed an added safety factor in the shape of a loose earthen plug, closing the entrance from the inside. It is probable that even when this is lacking at the entrance the sleeping chamber is so closed.

Some of the species which are more particular in their habitat preferences may be gathered by the exigencies of their surroundings into small colonies, or some sorts may actually be colonial to a slight extent. It seems that this may be the case with *D. deserti*, which often constructs series of galleries that are more elaborate than usual. This may be partly attributable to the fact that the burrows are usually in sand that is so loose that the tunnels collapse in part upon the slightest provocation, when a new gallery must be initiated.

Dipodomys spectabilis, the species upon which the present article has been based, most often occurs in a type of loam more clayey than sandy. Its habits are remarkable in that it constructs a "mound" of extensive tunnels. Where the animals occur these are dotted over the landscape, each mound "ranging from 6 inches to 4 feet in vertical height, and from 5 to 15 feet in diameter" (Vorhies and Taylor, 1922). They are occupied for years, the earth excavated being deposited about the entrances, thus raising the latter above their surroundings. Whether constructed with this purpose or not, the fact remains that this slight elevation serves to protect many of the burrows on flat ground from the flooding that they would otherwise experience during the occasional torrential rains of summer.

At first thought it is somewhat surprising that rodents having the weak fore feet of *Dipodomys* should construct fairly elaborate burrow systems in ground which at times is not at all soft. But weak tools are effective when wielded with proper industry, and the latter is undoubtedly an ingrained character, for the reason that proper underground retreats are certainly more necessary for the survival of this dweller of the open spaces than for most other rodent genera.

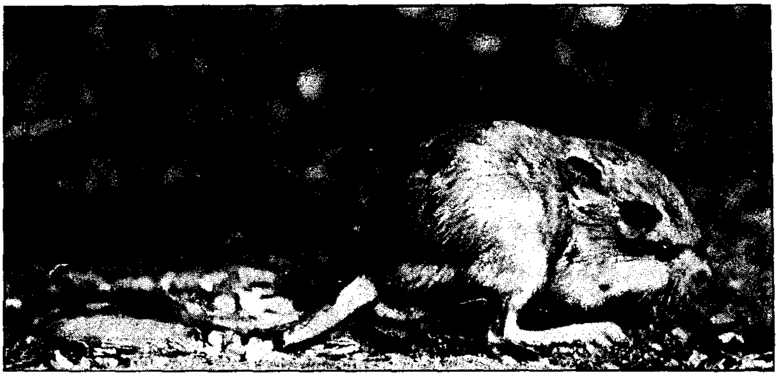
The characteristic digging posture is for the animal first to elevate its posterior quarters (as in fig. 3) and to scratch rapidly with the fore feet. This is followed by vigorous kicking with the hind feet, for the purpose of ejecting to the rear the earth loosened by the fore feet. The animal may then back for a short distance before repeating the latter action and in this manner the earth may be transported throughout the length of a burrow. I have never observed any other method of moving earth, so I am unable to state how a burrow entrance is plugged from the inside. Some tamping action by the fore feet must be involved in this, however.

The species of kangaroo rat which inhabit the more elevated or rigorous part of the genus' range must either hibernate during the winter months or else remain torpid for considerable periods. In the colder parts of the range of *D. spectabilis*, as about Flagstaff, Arizona, it is likely that there are winter spells of suspended or at least greatly reduced activity lasting for many days at a time, but in the milder sections, as about Tucson, the only suspension of surface activity is experienced during periods of storms or unusual cold, lasting for not more than two or three days at a time. Nevertheless food is stored in the burrows more industriously than is perhaps the case with any other rodent of the region.

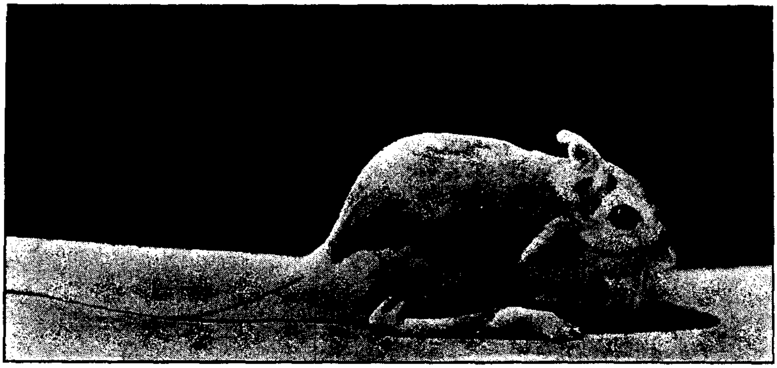
This habit of food storing is a deep-seated instinct and must be classed as one of the major behavioristic urges. Even an animal that was freshly removed from a trap, with an injury that must have caused considerable inconvenience, feverishly set to work without delay moving a pile of grain from a corner of its cage to better concealment at the opposite end. I have watched long and intently in an effort to learn the actions involved in the use of the external cheek pouches, but unsuccessfully. Food is rapidly passed, by means of the fore feet, into first one and then the other pouch. The fore feet also aid in rapidly emptying the pouches of their contents.

Vorhies and Taylor (1922) have demonstrated that this species of kangaroo rat is partial to a long list of plant foods, but the principal items consist of grass seeds and root stocks of grasses, which are transported in the cheek pouches and stored in side chambers of the burrows until needed during periods of food shortage. But little green food appears to be taken, even when available, although a variety of juicy roots is consumed. It is doubtful if the animals ever deliberately seek water, although some free water is probably taken into the system when this occurs in the form of dew, or vegetation has been wet with rain.

Certainly the ability of at least some species of the Heteromyidae to dispense with free water is most remarkable. It is true that in captivity many sorts will regularly drink considerable quantities of water if this be supplied, but the same species appears to thrive equally well without this item. Kangaroo rats have upon a number of occasions been kept in captivity for many months upon an exclusive diet of dry grain and without water. I myself have kept a pocket mouse (*Perognathus*) for thirteen months in a cage situated in my garage. During this time it was fed dry rolled oats and nothing else, either liquid or solid.



Vernon Bailey photo.



A. deC. Sowerby photo. courtesy T. F. Unwin.

FIGURE 1. Convergence of form for saltation, as exhibited by the American kangaroo rat Dipodomys spectabilis (above), and the Chinese jerboa Dipus sowerbyi (below).

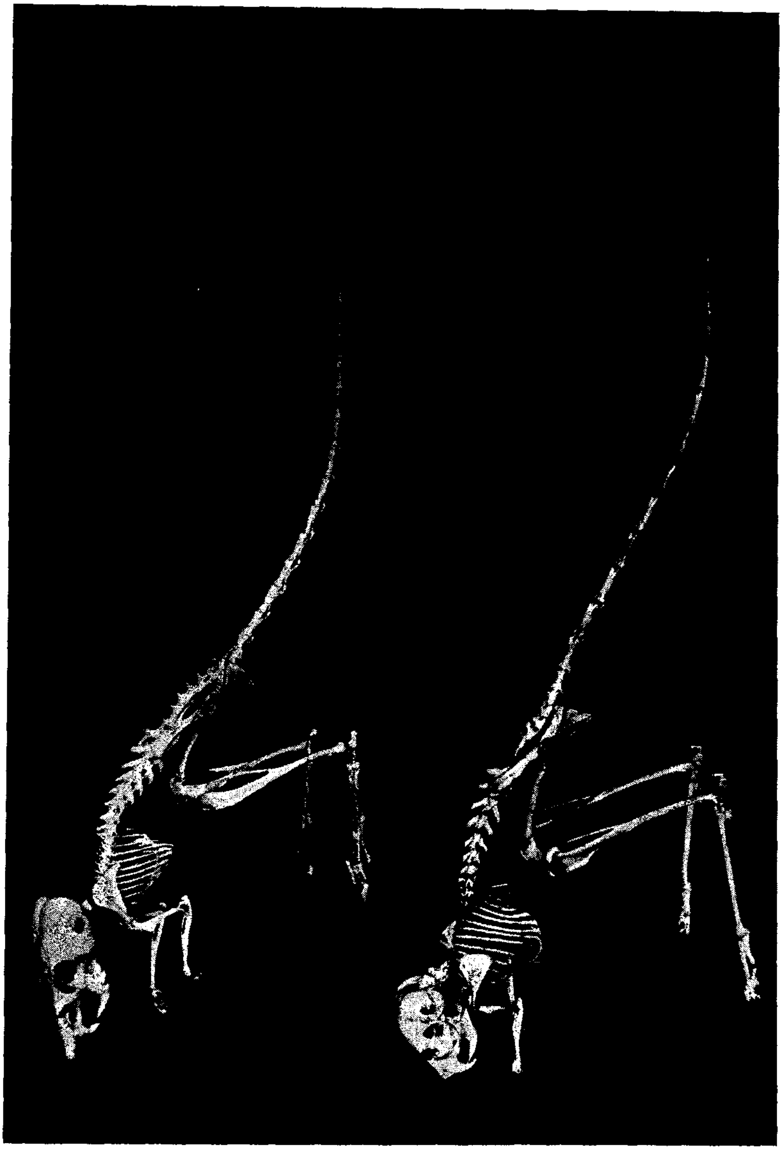
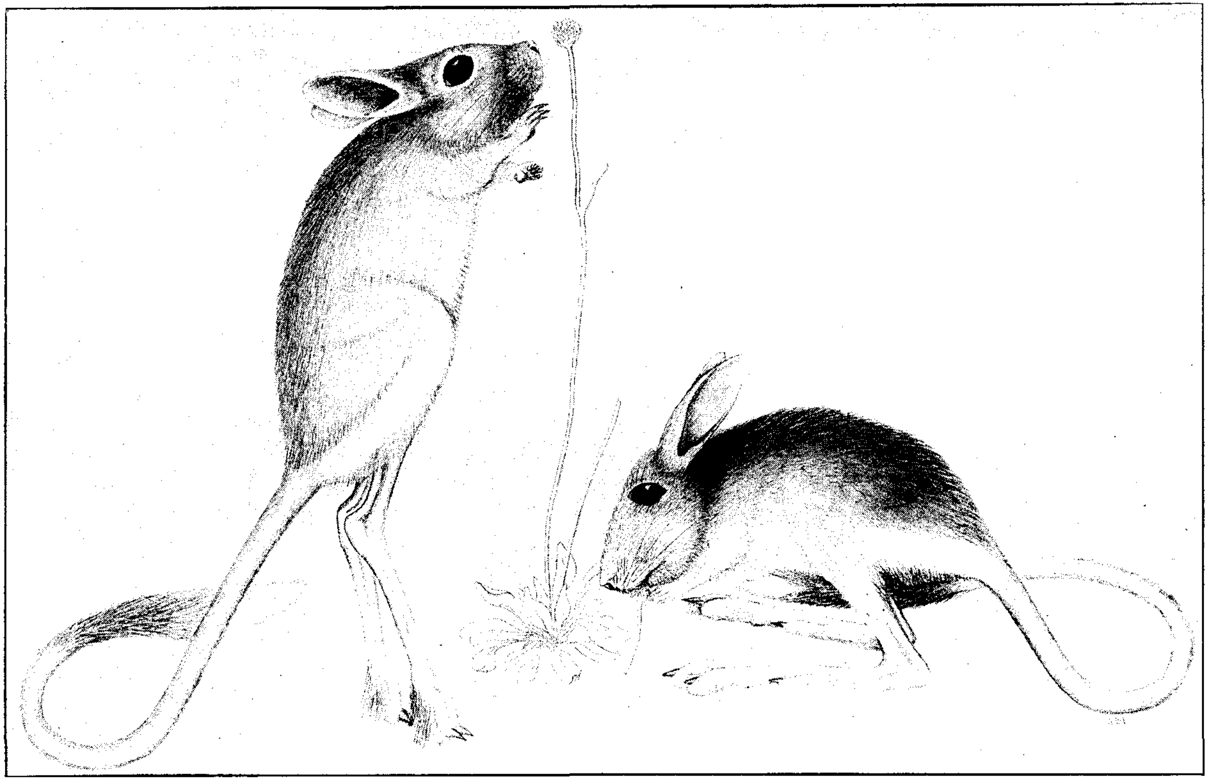


FIGURE 2. Mounted skeletons of kangaroo rat (Dipodomys deserti) and Egyptian jerboa (Scirtopoda orientalis).



FRONTISPIECE. The Egyptian jerboa (Scirtopoda, left) and the Chinese five-toed jerboa (Allactaga, right).

It is well known, of course, that mammals derive sufficient moisture from the metabolism of their food for their physiologic needs save, in the majority of instances, those connected with excretion (Babcock, 1912). But it is extremely doubtful that such metabolic water can be manufactured in more copious amount by the desert dweller than by its relative of more humid clime. In other words it seems certain that the products of metabolism of a given rate are the same regardless of the mammal concerned, and the physiology of digestion, quite thoroughly known, will not admit of any process whereby either more or less than a definite quantity of water may be derived by the metabolism of a given quantity of any particular food substance. Therefore the amount of water manufactured in the body of a kangaroo rat is neither more nor less than of a meadow mouse of equal size fed an identical diet, unless there might be a differential rate of the basic metabolism.

Water is given off from the body in three ways; in the form of water vapor in respired air; in the form of sweat through the sudoriparous glands of the skin; and in the form of urine, or to some extent, of course, in moist feces. It is only by modifying these three physiologic processes that a mammal can contrive to conserve the water that its body contains. The fact that at least some of these rodents will partake of water when it is offered them, or seem to remain indifferent to its lack when it is withheld for months at a time, suggests that their excretion of water varies within fairly wide limits, dependent upon the moisture content of the body.

But little or nothing is known regarding the functioning of the sensory equipment of the kangaroo rat. Its infraorbital tactile sense, through the vibrissae, is certainly keen, but probably to no greater degree than in many rodents. Its large eyes denote good nocturnal or crepuscular vision, but whether this sense is depended upon to a greater degree than that of smell is a question. But one may safely say that the acoustic sense is extremely highly specialized in this rodent. Unfortunately we have no inkling of the precise, or even approximate, quality of hearing involved, for the acoustic apparatus differs from that of man in a number of features and thus it may involve factors of which we have no conception. The only opinion that I have had on the subject is that the kangaroo rat places great dependency upon resonance or vibrations through the ground—or in other words that its enormous, hollow bullae act as sounding boards—and perhaps less upon air borne sound waves than the majority of mammals. Thus an individual may quietly be followed, in the light of an electric torch, within three or four feet as it forages here and there. It will remain indifferent to conversation or other noises if of not too sudden a nature, even when these are rather loud. But the slight tap of a pencil upon the ground will send the subject out of the picture in a single bound. It is thus inferred that the genus is particularly alert for the detection of the light footfalls of its enemies, which must often approach unseen behind the shelter of the scattered desert brush.

In progressing at a deliberate pace the kangaroo rat employs the creeping gait of the majority of rodents, with the tail dragging behind. This is a walk, the fore feet being always employed; but during the frequent pauses the fore feet are elevated and the body is then supported solely by the hind legs, unless the posture involves the placement of the nose close to the ground. When exploring new surroundings, after being liberated in a large box or a room, *Dipodomys* appears to be more prone to stretch upward on its hind legs while steadied by its tail, for the purpose of testing and smelling the walls above its head, or rather to indulge in this action more frequently, than are the majority of rodents. This is doubtless for the reason that the shorter body and more powerful musculature of the posterior limbs facilitates this action, and it is thus more naturally indulged in.

Only rarely have I observed this animal to progress at slow speed by any gait other than a walk, and this has consisted of a slow hop, involving the placing of the fore feet upon the ground followed by the foreward movement of both hind feet in unison. When moving faster than a slow walk the animal hops, bird-like or kangaroo-like with tail held stiffly to the rear, the distance between foot prints varying with the speed. When this measures some two or three feet the actions are graceful, the tail then acting as an effective equilibrator and all of the movements of the animal appearing to be in perfect coördination. When panic increases the length of the leap to the neighborhood of six or eight feet the speed of the animal would seem to exceed its control. At least it has been my experience that it is then likely to make a rather poor, unbalanced landing, and that before full recovery has taken place a powerful kick by the hind feet will often send the animal hurtling through the air once more in a more or less lopsided fashion. If in an especial hurry it may thus fail to plan its course with proper foresight and occasionally will land amid the dense twigs of a small shrub in anything but a tactical position.

Perhaps, however, this tendency to leap anew before a perfect balance has been recovered after a hop has a certain advantage. For one thing it lends to the swift progression of this animal a certain erratic quality that must be exceedingly confusing to an enemy. This is further enhanced, at the will of the individual, by the fact that the tail is a very effective rudder. Its proper use will correct, in the midst of a trajectory, any error of balance caused by a faulty "take-off," and it must enable the animal, when closely pressed, to execute the marvelous dodges for which it is famous, and which are the despair of a pursuer. When chased by a dog (or other enemy) and hard pressed, the kangaroo rat will perform a series of right-angled zig-zags involving such abrupt changes of direction that the dog is reduced to complete helplessness. In fact it is likely that no mammal can catch this rodent in fair chase, but only by stealth.

The long tail of *Dipodomys* is subject to many vicissitudes and individuals having this member considerably shortened are frequently secured. If the loss be considerable in amount the animal is much handicapped in its long leaps and will frequently turn completely over in midair.

Whether the speed of these rodents in a linear sense would compare favorably with that of a rabbit, for instance, is questionable. I regard their long hind limbs as not remarkably serviceable in this regard, but rather that they have been developed for giving considerable speed with phenomenal dodging ability, as well as rather high cruising speed for the purpose of covering a considerable amount of territory in search of food where this is often scarce. The distance that it is possible for them to cover at a single bound remains to be determined. I have endeavored to ascertain this but have been unable to do so to my satisfaction. I think it doubtful if it is possible for them to cover more than about 12 feet, with a lesser distance for the smaller species.

There here intrudes the question of whether the gait of the kangaroo rat is digitigrade or plantigrade. At slow speed the latter is the case, although it is prone to stretch upon its toes to investigate an object above its head. At cruising speed it is digitigrade to a considerable extent; but so are many mammals that are classed as strictly plantigrade. Even man is digitigrade when running rapidly and the same may be said of *Dipodomys*. The final impulse for a leap is furnished by the toes, and the animal may even land on the toes, without the heel touching the ground.

The characteristic sleeping posture of *Dipodomys* is, like most rodents, resting on the heels with the head and nose tucked beneath the belly, at times with the forehead actually resting upon the ground, but occasionally a sleeping captive will be detected sprawled upon its side. When at perfect rest or dozing the weight of the animal will be rested squarely on the heels, but when on the alert and ready for action, but still motionless, the center of gravity is shifted forward over the metatarsi (fig. 3).

HABITS OF THE JERBOAS.

But few accounts of the habits of the jerboas are available and these are anything but satisfactory. For the following notes on *Scirtopoda orientalis* I am indebted to the late John Hornung, the collector of my specimens. He related to me that near Alexandria the species is common throughout stretches of aeolean sand but does not occur upon heavier soil. Thereabouts there is so little vegetation, and this is so scattered, that it is difficult to understand how the animals subsist, even though the length of their legs fit them well for covering long distances. Their burrow systems are quite extensive but very close to the surface; hence one may insert his hand in a burrow and with his arm "plow" along until the inhabitant is reached. Arab boys capture numbers in this way, and while endeavoring to dash past the hands of their pursuers, the animals often dart up the capacious sleeves of the native costume, from which they are easily extracted unharmed.

Clark and Sowerby (1912), have stated that the Chinese 3-toed jerboa *Dipus sowerbyi* is a purely sand-loving animal, inhabiting the dunes which exist in and on the borders of the Ordos Desert. Its burrows are almost impossible to find and it is a question whether it fills up the mouths of these purposely or whether this is accomplished naturally by the loose sand. At the time of their visit it was too late in the year to investigate this point as the jerboas were hibernating. From the tracks it was judged that they can leap for a distance of 8 feet. When asleep they lie upon their sides for the better accommodation of the long hind legs. When proceeding slowly they hop upon all four feet, but when progressing at some speed the hind limbs are employed exclusively. The present writer is of the opinion that the above figure of 8 feet refers to the average distance covered by each leap when the animal is progressing at reasonable speed, but that in sudden panic a considerably greater distance can undoubtedly be covered.

Blanford (1888), in writing of a genus of a much lesser degree of saltatory specialization, has stated that *Gerbillus* can cover a distance of 4 or 5 yards at a single bound.

Very little is known regarding the habits of the Chinese *Allactaga*. Sowerby (Clark and Sowerby, 1912) stated that it is not so essentially a sand-inhabiting animal as is *Dipus sowerbyi*. Captive animals sleep upon the side or back and if disturbed kick and bite savagely. When not on the alert they have a queer habit of drooping the terminal half of the ears.

From the meager references to the character of the habitats of the more saltatorial form of the *Dipodidae* it seems safe to state that their distributional preferences are almost identical to those of *Dipodomys*, save that they appear to be more strictly confined to a steppe or desert type of country, seeming to shun brushy surroundings such as at times harbor the kangaroo rat in numbers. This is to be expected in connection with the greater saltatorial specialization of the Asiatic animals.

One of the most interesting points in regard to the habits of the jerboas is concerned with the degree to which they are digitigrade. It seems to me clear beyond question that during slow progression all four feet are employed, and this contention appears to be substantiated by photographs. Even so, the extreme length of the hind limbs must prove a definite handicap during the creeping tactics employed by all rodents in searching for small fallen seeds. John Hornung has assured me that in captivity *Scirtopoda* is even more prone than is *Dipodomys* to stretch upward upon its toes, while using its tail as a third member of a tripod, for the purpose of investigating everything within reach, and in the wild this ability must be put to practical use in aiding the animal to secure seed pods that would otherwise prove unattainable. At considerable speed the jerboas are, beyond question, digitigrade, just as are kangaroo rats, but undoubtedly to a more pronounced extent, utilizing the longer pes for imparting an augmented thrust to the long legs.

EXTERNAL CHARACTERS.

Measurements and Weights.

Except in the case of foot length, reasonably satisfactory external measurements can be given only for those species listed below, for skins of these are available and measurements were made from the fresh specimens by the collectors. The dimensions, in millimeters, of two specimens of each are accordingly given, and those of the large *Dipodomys deserti* are also included.

	Head and Body	Tail	Pes
Dipodomys spectabilis....	136 and 140	190 and 210	50
Dipodomys deserti.......	136 and 145	214 and 225	55 and 60
Allactaga m. longior.....	134 and 138	209 and 205	69.5 and 71
Dipus sowerbyi.........	116 and 120	160 and 176	62 and 66

It was found that because of the difficulty in correctly allowing for the curve of the vertebral column in the hardened alcoholic preparations the relative proportions of the spirit specimens of *Allactaga* were so at variance with those of the study skins that I am loath to place any dependence upon the head and body measurement of *Scirtopoda*, taken from alcoholic specimens only. In the latter animal, however, the body appears to be relatively a bit longer, and the tail slightly shorter, than in *Allactaga*, while the foot is about the same.

A freshly caught specimen of *Dipodomys spectabilis* weighed 124 grams, and four years later when removed from spirits and allowed to drain for a few minutes, 135 grams. It had thus absorbed moisture to the extent of about 9 per cent of its original weight. On this basis a specimen of *Scirtopoda* before me, weighing 132 grams, must have tipped the scale when freshly caught at about 121 grams; and an individual of *Allactaga*, 106 grams. It is thus seen that the specimens dissected were as nearly similar in size as one could reasonably wish.

Head.

In considering the external characters of the three genera dissected account will also be taken, where it seems desirable, of the features of other saltatorial mammals, but only briefly, because many sorts have not been available for examination, and some of those seen have been in an unsatisfactory condition for proper study.

In saltatorial rodents the external ear conforms to two general patterns. At one extreme there is the phenomenal pinna of *Euchoreutes*, comparable in some respects to that of the desert hares, and at the other the small type of pinna of the sort encountered in kangaroo rats and mice and some jerboas. It is the rule that a large pinna accompanies an auditory bulla that is not unduly inflated, while a small pinna upon a mammal highly adapted for saltatorial progression indicates an enormous bulla. But there are exceptions to this rule, at least to the extent that two 3-toed jerboas with approximately equal development of the bulla may have the pinna either almost as short as in *Dipodomys* (i. e. *Dipus sowerbyi*) or considerably longer,

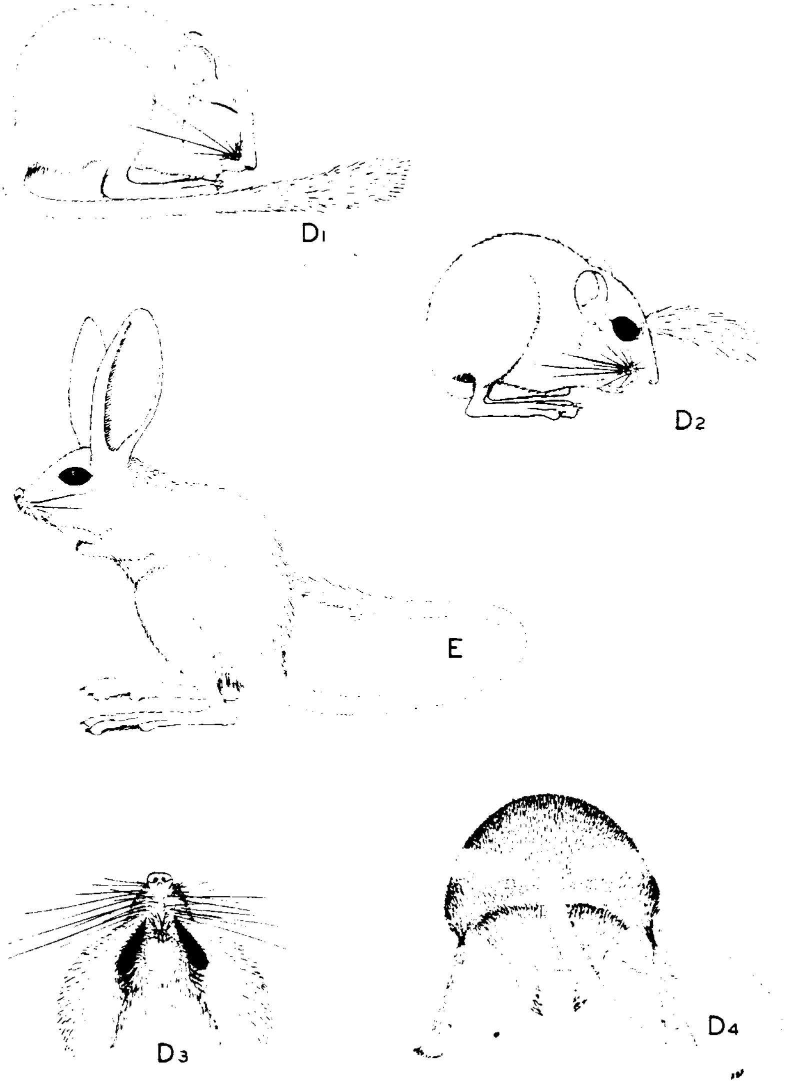


FIGURE 3. Postures and proportions of Dipodomys (D) and of the Asiatic Euchoreutes (E). D_1, lightly dozing, with center of gravity over heel; D_2, alert, with center of gravity over base of toes; D_3, openings of external cheek pouches, the latter represented as filled; D_4, typical digging posture. Shown without regard to uniform reduction.

as in *Scirtopoda*. But I know of no case in which an animal has both a long pinna and phenomenally inflated bulla.

Grinnell (1922) has called attention to the fact that within the genus *Dipodomys* "Consideration of habitats in connection with size of ear brings out unmistakably the correlation of small pinna with open type of habitat, and, in the other direction, of large pinna with a chaparral type of habitat. The same tendencies are to be observed in *Peromyscus* and *Perognathus*." The same author has further stated that "An interesting circumstance, however, with *Dipodomys* at least, is that, in many of the small-eared forms, the relative inflation of the mastoid and tympanic portions of the skull is far greater than in the large-eared forms. Increased effectiveness of the internal parts of the aural apparatus would here appear to be in some measure compensatory to the reduction of the external parts. There are apparent outstanding exceptions to the above rule of compensation, however."

In *Dipodomys spectabilis* the length of ear (in an alcoholic specimen) measured from the crown was 10, and from the notch 17 mm., with a width of 10 mm. The pinna was blunt and evenly rounded and the anterior border was folded back so as almost to hide the base, which was not tubular. The antihelix (Pocock's supratragus) was well marked and the antitragus was distinct, although not much developed. The tragus was absent as a true prominence. In this species the ear was covered with fine hair but there were no marked fringes to the border. No color pattern was apparent, although Grinnell (1922) has described and figured considerable variation in the auricular color plan in the different species of the genus, this being for the most part a basic pattern of darker distal and paler proximal parts, varying in intensity of tone according to the shade of the body coat. This general style of pinna is characteristic of the Heteromyidae.

Illustrations clearly show that the ears of the smaller of the two species of Egyptian jerboa (*Dipus jaculus*) are of the short type, apparently rounded much as in *Dipodomys*, and this also appears to be the case in the little Asiatic *Salpingotus*, with its enormous bullae. Perhaps the pinna of *Dipus sowerbyi* and its close Asiatic relatives should also be classed as short, for it measures 13 mm. from the crown and 17 from the notch; but it is considerably more pointed than in *Dipodomys*.

Pocock (1922) has stated that in *Scirtopoda* the ear, when laid forward, overlapped the eyes by a little. In my alcoholic specimens the ear reached half way from the eye to the nose, measuring 27 mm. from the crown and 30 from the notch, with a width of 13 mm. It was tubular at base and the antihelix was poorly defined except when the ear was laid back, when the cartilage became creased at this point. In fact this may always be regarded merely as a cartilaginous hinge. There was really no tragus defined, at least in its normal situation, although an inner fold near the base of the anterior border may well be homologous with this prominence. Although the ear was so much longer than in *Dipodomys* the distal border was hardly more pointed. The pinna was scantily covered with exceedingly fine, short hairs, as is usual in mammals, and in addition the anterior border was well fringed, these hairs being longer and more plentiful toward the base; and the base of the posterior border was also rather densely haired.

Pocock's illustration indicates that the ear of the four-toed African *Scarturus* is relatively longer and more pointed than in *Scirtopoda orientalis*, but not to the extent encountered in *Allactaga*. Pocock stated that in the species *indica* of the latter genus the ear reached about to the end of the muzzle, but in the alcoholic specimens of *A. mongolica longior* available the distance so reached was slightly more than half way from the eye to the nose. From the crown it measured 32 mm. and from the notch 34, with a width of 13 mm. It tapered more toward the tip than in *Scirtopoda* but was also tubular at base, with the anterior border more folded. The fold upon this border that may represent the tragus was smaller and hidden within the conch, and the antitragus, while small, was better defined than in *Scirtopoda*. There were two antihelical folds and the hairs were distributed more plentifully than in the 3-toed animal.

The external ear of *Euchoreutes* is relatively more developed than in any other living mammal, with the exception of some bats, and reaches well beyond the nose when laid forward. It is really astounding in its proportions (see fig. 3), but only a dried skin in poor condition is at hand and little or nothing can be told regarding its detail, save that the border fringe is very well developed.

The ear of *Pedetes* is also of the long type and is highly specialized (fig. 4). Pocock found that apparently at the will of the animal the ear can be folded lengthwise so that the anterior and posterior borders are in contact, while a slender process at the base, which seems to be a specialized tragus, fits between them. The antihelix or supratragal ridge extends lengthwise of the pinna so as to facilitate closure.

Sowerby found that when the Chinese *Allactaga* was not on the alert it had the odd habit of drooping the terminal half of the ears.

At least in the alcoholic specimens examined, and presumably in all saltatorial rodents that live in burrows amidst a desert habitat, it is apparent that when the ear is retracted the inner folds and prominences dove-tail into one another so as to effect practical closure, and thus exclude particles of fine sand. This is also facilitated by the marginal fringes.

In almost all rodents the eyes are so situated that the animal can see in every direction without movement of the head, but we know nothing whatever about the quality of vision concerned. In all certainty stereoscopic vision, of the sort characteristic of man, does not exist in rodents, but there is an effective binocular panoramic vision.

Size of eye is a criterion of an animal's habits, but the interpretation of this detail is not always easy. Thus an eye of moderate size may mean that an animal is diurnal with keen vision, or nocturnal and with rather mediocre sight. In rodents a difference of a couple of millimeters means little when tabulated, for it is almost impossible to compare this detail properly with body size, and so obtain uniform results. But after making observations I am of the opinion that all saltatorial mammals have rather large eyes, this possibly being mainly attributable to uniformly nocturnal habits, save to some extent in the case of the lagomorphs or hares. In species but slightly modified, and usually of small size, which must depend more upon caution than speed to escape from enemies, thus being more prone to forage near protective vegetation, increase in the size of the eye is not marked, while in the more specialized sorts, whose ability to leap and dodge enables them to range well into the open with reasonable safety, the eye is inclined to be larger.

In those animals dissected the distance between the canthi of the eye was 9 mm. in *Dipodomys* and *Scirtopoda* and 8 in *Allactaga*, while in a small wood rat (*Neotoma*) of comparable size this measurement was less than 7 mm. In all of these the center of the eyeball was about 60 per cent of the distance between the nose and the notch of the ear, save in *Scirtopoda* with its short muzzle, in which this percentage was 55.

In these saltatorial rodents there is a relative increase in the distance between the eyes. In a specimen of *Neotoma* this distance was but 43 per cent of the head length, while in *Dipodomys* and *Allactaga* this item rose to 61, and in *Scirtopoda* to as much as 76 per cent. It seems, however, that this eye width is more strongly determined by such considerations as are influenced by inflation of the audital bulla than by any purely visual requirement.

The most interesting ocular detail from the present viewpoint, however, is in the angle at which the eye is set. After having borne this question in mind for a number of years I have been led by numerous observations to believe that in mammals the eye is so situated that in the posture in which the animal most often carries its head, or let us say in the posture of the head during its most critical activities, a line drawn between the canthi will be to all intents horizontal with the ground. So far as I know this detail has received no previous consideration, but it is a development that would be entirely logical, enabling the greatest possible breadth of vision to be attained. It seems then that this is a valuable index to the angle at which the head is normally carried. Employing it thus it has been found that in a wood rat this line through the canthi forms an angle of 15 degrees

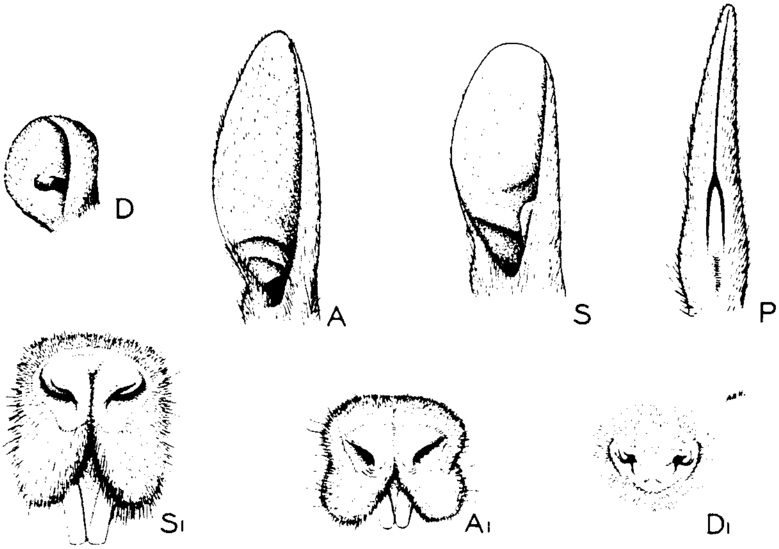


FIGURE 4. Right ears (× 1) and rhinarial region (× 2) of Dipodomys (D) Allactaga (A), and Scirtopoda (S). The ear of Pedetes (P), redrawn from Pocock, is shown in the closed position, supposedly peculiar to this genus.

with another line stretching from the tip of the nose to the notch of the ear. Although the latter is not the precise axis of the skull it is here so considered for convenience. Thus the wood rat can be judged to carry its head depressed to an extent of only 15 degrees from the horizontal. In *Dipodomys* this angle of deflection was found to be 32 degrees, in *Scirtopoda* 40, and in *Allactaga* 50 degrees. In the wood rat it is impossible to decide whether the need for widest vision, or in other words the determinant for canthi angle, is experienced while the animal is sneaking through dense vegetation on the watch for enemies, or while searching for seeds on the ground beneath. In the jerboas and kangaroo rats, however, it is apparent that greatest keenness of vision is not needed so much when the animal is in full flight, but rather when it is quietly going about its business, nose to ground, either for finding food or with a wary eye watching for enemies.

In *Dipodomys* some of the mystacial vibrissae are as long as 75 mm., and these are black, while the shorter ones, located farther forward, are white. There are also two or three superciliary vibrissae, but no others could be found. Over the face, however, and especially below the eyes, the guard hairs are very long, and these doubtless have some tactile function. In *Allactaga* the longest vibrissae were about equal to conditions in *Dipodomys*, but in *Scirtopoda* they were slightly longer (82 mm.). In both the vibrissae were entirely black at base, although a number had white tips, and in *Dipus sowerbyi* there was more white. The two or three most ventrocaudally situated were the longest, and also stouter than the rest. Two or three superciliaries were present and nearer the eye a number of black bristles, 6 or 8 mm. long, more plentiful upon the lower lid. These were more scanty in *Dipus sowerbyi*.

In *Dipodomys* the nose had no differentiated rhinarium and as the tip was approached the hairs of the rostrum became progressively shorter until upon the nose proper they were exceedingly short, and white. The borders of the nostrils were somewhat folded so that complete closure, while digging in loose sand, should be easy of accomplishment. There was no sharp philtrum or medial groove, although this was slightly indicated. In *Scirtopoda* there was a well differentiated rhinarium, abruptly hairless. The nostrils were of scroll shape, on the same general principle as, but of a different shape than, those of *Dipodomys*, and there was a deep philtrum which tended to branch above the nostrils. In *Allactaga* the scroll-like part of the nostrils was more deeply situated so that it was less apparent, the philtrum did not divide above the nostrils nor was the nasal part quite so deep, and there was a slight lateral depression between the nasal and labial parts of the muzzle. In the single study skin of *Euchoreutes* which I have examined the nose appears to have dried in a position that is relatively undistorted. In detail it is considerably different from the other jerboas discussed for there is a raised terminal border, which gives it the appearance of being the snout of a minute pig.

Although in *Dipodomys* there is no well marked labial philtrum, the upper lip is divided so as to form a slight inverted V, and there is therefore no maxillary oral vestibule. The bases of the mandibular incisors are ensheathed by a labial fold and here there is a vestibule a couple of millimeters in depth. Lingual cheek pads in the region of the diastema act so as to close the mouth completely posterior to the incisors as in many other rodents. The opening of the external cheek pouches is parallel to a line passing from the nose to the undershot chin and is about 20 mm. in length, varying in precise position and measurement according to the position in which the specimen was fixed by the preservative. The cheek pouches are about 35 mm. in depth but are capable of considerable stretching. The upper part of the opening is situated opposite the base of the maxillary incisors, and the lower angle far beneath the chin. The lateral lips extend uninterruptedly into the pouch, which is lined with sparse, short, white hairs. It appears from both superficial and myological examinations that these pouches have always been external in situation since their original inception, and not that they have developed from any sort of internal pouch as now found in squirrels, hamsters, etc.

In *Scirtopoda* there is a deep labial philtrum and the cleft character of the upper lip is much more marked than in *Dipodomys*. The mandibular vestibule is as deep as 7 mm. and closure of the mouth is effected more exclusively by the posterior part of the upper lips, or rather, the chief action of the lingual pads is more dorsal, so that after closure the orifice of the mouth may be compared to the letter T rather than to a short vertical line as in *Dipodomys*. In this animal the hairs of the upper lip bordering the incisors are very long. In *Allactaga* conditions are largely the same, but the maxillary incisors are more projecting and the lower ones longer, giving space for longer lingual cheek pads which appear to be less efficient in their valvular action.

Body.

Character of pelage as well as pattern was determined by observing alcoholic specimens immersed in a dish of water, as well as study skins.

Although a large part of the range of *Dipodomys spectabilis* is arid desert this species is but little paler than many non-desert rodents, and much darker than the excessively pale *Dipodomys deserti*, whose range immediately adjoins, and perhaps overlaps, that of *spectabilis*. But it is very much paler than many species inhabiting more humid districts. *D. spectabilis* is slightly darker than *Dipus sowerbyi* and *Scirtopoda orientalis* in its darker pelage. It is, however, considerably paler and less sooty than *Allactaga mongolica*. One of the specimens of *Scirtopoda* was in changing pelage, the shorter part, evidently comprising patches of the summer coat, was almost as pale as *Dipodomys deserti*, which is one of the palest rodents in existence, while the longer patches, evidently of the winter coat, were practically as dark as is *Allactaga* but more ochraceous. The character of the pelage of *Dipus sowerbyi*, *Allactaga mongolica*, and of the dorsum of *Scirtopoda orientalis* is not noteworthy, fluffing out in water to the same degree as in most rodents. The entire pelage of *Dipodomys* of all species, and of the thighs, lower shoulders and much of the underparts of *Scirtopoda*, however, is particularly lax, long, and appearing as though slightly matted by grease. That this is characteristic of the pelage regardless of its condition is certain, but when live *Dipodomys* are deprived of fine sand in which to dust and cleanse the fur the condition becomes aggravated by the products of the skin glands and the animals then present an unkept, bedraggled appearance.

Many species of leaping mammals, including the lagomorphs, have an unusual amount of ochraceous coloration in their pelage, this often being intensified over the sides to almost an apricot shade. But this is attributable to the fact that very many of the saltatorial species are inhabitants of desert areas, where paleness and brightness of pelage are a corollary of arid climatic environment, and their near relatives of more humid regions are darker and duller. In the pattern of the coat markings, however, saltatorial rodents are prone to exhibit a noteworthy degree of convergence. White underparts are too uniform among mammals in general to deserve more than mention, but there is a tendency, although this in not by any means invariably shown, for there to be whitish areas between the nose and eye, above the eye, and at the base of the ear. More remarkable is the tendency to develop a broad and abruptly white stripe extending from the base of the tail to the thigh. Apparently this is present in all kangaroo rats and all jerboas, although not in the kangaroo mice nor in mammals of minor saltatorial development. This seems not to be so strictly an independent white marking, but rather that the white of the underparts extend thus high upon the flank, while the darker coloration of the upper parts have secondarily invaded the thigh. In some specimens of *Allactaga*, but not in others, the white of the flank is bordered below by long black hairs (frontispiece). The result of this uniformity of color pattern, including that of the tail, in connection with the similarity in bodily conformation, results in the fact that in regard to general external features the peculiarities that have been developed by the kangaroo rats appear almost exactly to match those of the unrelated jerboas. Included in these features are apparent shortness of body and relatively large head, as well as the fact that following the development of the fore and hind limbs, the posterior part of the body is heavier than the anterior; but these details will be discussed more fully in connection with the osteology.

Attention should be called to the presence in *Dipodomys* only, among the Heteromyidae and other saltatorial mammals, of a small cutaneous gland situated mid-dorsally and slightly posterior to the shoulder blades. This is of about the same size as the ocular aperture, is hairless, granular or warty in texture, and stands slightly above the surface of the skin. It is probably a scent gland but extrudes no apparent exudation and the surrounding hairs are always dry. That a gland should have developed upon the back or sides is not surprising, but it is somewhat remarkable that glands in this precise situation should have been acquired by the unrelated ground squirrels of the genus *Callospermophilus* and at least some species of African hyraxes (Hyracoidea), of ungulate affinity. Presumably these glands serve to scent the top of the burrows through which the animals pass.

The mammillary formula of those species of *Dipodomys* examined for this feature is two pairs inguinal and one pair pectoral.

In saltatorial mammals the tail has the double function of equilibrator during speedy progression and as a third leg or prop while a bipedal posture is assumed. Hence it is directly dependent in considerable degree upon length of hind limb, or in other words, leaping ability. The tendency as specialization advances is undoubtedly for an increase in length of tail and acquisition of a terminal tuft or brush, which usually becomes black in color at some later stage, and ultimately may be expected to acquire an abruptly contrasting white tip.

In many of the pocket mice of the genus *Perognathus* the tail is actually shorter than the head and body, reflecting the tendency for generalized, non-leaping, non-climbing desert rodents to have shorter tails than their relatives of less arid regions. In no *Perognathus* is the tail much more than one fifth longer than the head and body, and this is no longer than in many of the spiny heteromyid mice of non-saltatorial propensities from Mexico and farther south. In the shorter-tailed pocket mice there is no terminal brush developed by the tail, nor is this present in the kangaroo mouse (*Microdipodops*) whose tail is but little longer than the head and body. In pocket mice with longer tails the terminal pencil is usually present and entirely of a color with the remainder of the dorsal aspect of the tail. In some species of the Old World genus *Gerbillus* the tail is also pencilled and in others not. In at least some species of the partly saltatorial genera *Taterillus*, *Dipodillus*, *Meriones* and *Rhombomys* the terminal tuft is darker than the rest of the tail, and this becomes virtually black in *Pedetes*, some species of *Tatera*, and some of the Allactaginae, while there is an abruptly white tip to the generally black pencil in most of the highly saltatorial jerboas. Apparently in the latter animals this white tip is acquired by the gradual spread of a terminal area in which each hair is completely white. In *Dipodomys*, however, I judge that the development of this detail is on a different plan.

In some of the paler races of *Dipodomys* the brush has neither black nor white areas, but the color is the same as the rest of the dorsal caudal surface, while in some of the darker, smaller sorts the tail is uniformly sooty above. From this a white tip seems to be formed, in those that exhibit this character, not by the progressive encroachment of an abruptly white area, but by the gradual extension distally of the white bases of individual hairs. This process, clearly illustrated in some forms, possibly may not have taken place in a few exceptions, such as in *D. spectabilis*, for here the white tip is as abrupt as in any jerboas. In its greatest development the caudal hairs begin to lengthen at the middle of the tail or even proximal to this point. In the Heteromyidae the dorsal hairs are the ones that show the first and greatest development, although those of the side are later involved also, while in the jerboas the lateral hairs are the ones chiefly concerned, giving to the tail a somewhat feathered effect. In *Dipodomys* the basic tail pattern is quadriform, the sides being white and the dorsal and ventral borders darker. In a form as pale as *D. deserti* the ventral stripe is very narrow and faint, having disappeared in a few specimens, while the proximal part of the dorsal stripe is of a very pale salt-and-pepper effect, but in darker species both of these stripes are sooty, and these may be broader than the lateral white ones which separate them. In the jerboas, however, the proximal part of the tail is bicolor, buffy above and white below. The latter may extend entirely to the tip, or else be practically eliminated where the black hairs of the pencil encroach ventrally.

Mention may here be made that frequently one finds in his traps specimens of *Dipodomys* whose tails have been shortened through accident. If the condition of the stump suggests that this took place at some considerable time in the past, there will likely be present at the tip an accessory tuft of hair that is more abruptly defined, of lesser extent, and composed of longer, more bristly hairs than was the original. This, clearly, is an effort on the part of a damaged tail to regenerate a detail that is of much importance in the animal's economy.

There is considerable variation in relative length of tail within the genus *Dipodomys* and at present this cannot be correlated with other features. Grinnell (1922) has stated that "the average ratio of tail to body in 33 California forms is 145 per cent. . . . The two extremes in the averages are 116 per cent and 167 per cent for *monoensis* and *californicus* respectively." But "In so far as the writer can see there is no significant correlation between either length or slenderness of tail with size of hind foot, with presence or absence of first claw on hind foot, with size of ear, or with geographic distribution." An attempt to determine from my alcoholic specimens the percentage of tail length to the length of head and body proved somewhat unsatisfactory because of the cramped position of the hardened bodies. But judging by the collectors' measurements on study skin labels the tail in these rodents is close to 150 per cent of the head and body length (*Dipodomys spectabilis*, 140–150; *D. deserti*, 155–157; *Allactaga*, 148–155; *Dipus sowerbyi*, 138–147; *Zapus*, 143). Judging by alcoholic specimens the tail of *Scirtopoda orientalis* is certainly no longer than that of *Allactaga m. longior*, and appears to be shorter.

From an examination of the animals studied in the present connection it would appear that there is no alteration in the diameter of the tails of these highly saltatorial mammals, for in this feature they are strictly comparable to rats of about equal bulk. Grinnell (1922), however, found that among the California forms of the genus there is variation in the thickness of the tail, to the extent that freshly caught specimens, of about the same size but representative of different groups, may be distinguished by this feature alone. "*Merriami* apparently exhibits the slender extreme of this genus (within the state), while the opposite obtains in *ingens*."

The tails of the lagomorphs, or hares and their allies, are not comparable with those of jerboas and kangaroo rats. Their tails are vestigeal and non-functional, in the little pikas as well as the big desert jack rabbits, and they were probably in this condition before the latter animals developed marked cursorial and leaping ability. A tail that has thus been reduced to a vestigeal condition appears utterly unable subsequently to attain to respectable proportions.

There are many factors which affect the conformation of the feet of rodents. In the majority the fore limbs have two critical functions, consisting of those concerned with feeding activities, and those connected with locomotion. In the genera in which the hind limbs have developed so that speedy progression is accomplished by saltation, the second of these functions has been abandoned, and the critical role of the fore limbs comprises their use as hands, with support of the anterior body relegated to a secondary position. The result, apparently, is a decrease in the robustness of these members. Whether they also become shorter depends upon feeding requirements but not, directly, upon any corollary change in the length of the hind limbs. In other words it seems quite certain that elongation of the hind limbs can not, per se, effect any compensational shortening of the fore limbs.

In *Dipodomys* the pollex is a mere rudiment with a short, very blunt nail. The other nails are long (5.5 mm. in the third) and taper very little. Digit 2 is slightly shorter than 4, which is practically as long as 3. The metacarpals appear normally to be held over-extended so as to form with the forearm an angle of about 45 degrees, while the nails are at the same time flexed to form an angle of 90 degrees with the metacarpals. This is so in the preserved *Scirtopoda* also, but in *Allactaga* this respective extension and flexion is much less pronounced. There are slight transverse creases upon the palmar aspect of the digits of all three animals, as is so often the case in rodents, and there are fine papillae upon that part of the distal palm that is not covered by the pads There is a very small but high tubercle just posterior to and fairly between digits 3 and 4, and another at the ulnar base of 5. There are two main palmar pads, large and high, but not quite as sharp as in the jerboas. The more radial one is partially double, and the other has a row of short hairs extending obliquely distally from the wrist (fig. 5, D1). As in other rodents similarly equipped these pads function in a manner comparable to the human thumb in aiding the digits to grasp objects which the animal desires to handle. There is no differentiated carpal vibrissae area, but the hairs over the wrist are quite long, moderately stiff, and must have some tactile ability.

In a fresh specimen of another species (*Dipodomys agilis*) the manus was found to be highly mobile, with possible pronation to about 75 degrees, and supination through the joint and the bones of the forearm to at least 150 degrees.

The nail of the pollex of *Scirtopoda* is heavier and longer than in *Dipodomys*, but still very blunt. The other nails are about of equal length (5.9 mm.) but more tapering. The palmer surface of the digit is finely haired, which is not the case to an apparent degree in *Dipodomys*, and the distal palm is without fine papillae. To all intents there is but one large and very high pad, partially divided by a crease and with the anterior border somewhat irregular. The carpal hairs are much as in *Dipodomys* but in addition there is a differentiated carpal vibrissae area near the wrist.

In the two above animals the nail of the pollex is slightly worn, but in *Allactaga* it is unworn and conical, so I judge that the growth may be slower and that it is even more rudimentary. The other nails are a bit shorter (4.3 mm.) and are tapering, and the palmer surface of the digits is marked by transverse ridges more faintly than in *Dipodomys*, and like the latter there are two small but high tubercles upon the distal palm similarly situated, and two large and high palmar pads as well. There is no differentiated vibrissae area upon the wrist.

In *Scarturus* there are said to be three tubercles, but only two in *Euchoreutes*, according to Pocock, who also found that in *Allactaga indica* there was but one. In most jerboas the nail of the pollex is blunt, but in *Euchoreutes* it is clawlike and as in the other digits, and this is also the case in *Pedetes*, the especially large palmar pads of which were considered by Pocock to have the ability of independent movement. The possibility of the latter feature is very doubtful because of the impracticability of the development of controlling musculature.

In reporting on the wood rat the foot length compared to that of head and body of the specimens whose measurements were given varied from 17.2 to 20.2 per cent according to species. For *Dipodomys* I have determined this percentage by taking average measurements as given by Grinnell (1922) for all (16) of the full species which he listed, and added to them those of *D. spectabilis* and *D. gravipes*. The average for the entire lot was 33.4 per cent, varying from 37.7 for the small *D. nitratoides* and the medium sized *D. agilis*, and 37.6 for the large *D. deserti*, down to 34.7 for *D. ingens* and 33.8 for *D. gravipes*, both large species. The foot of *D. spectabilis* is somewhat above the average, being about 36.5. In the American jumping mouse *Zapus* the percentage is about the same as the average for the kangaroo rat, or 33.

But the above figures do not tell the whole story, for *D. gravipes* has a foot that, although short, is particularly broad and heavy, while in other sorts the foot may be relatively narrow. Thus, in speaking of the California races, Grinnell (1922) has stated that the pes in "the *merriami* group shows an extreme of slenderness, while the *heermanni* and *agilis* group shows an extreme of heaviness (that is, the foot is relatively broad). There is a fairly clear correlation here with existence on bare, open ground sparsely grown to vegetation, as compared with more or less grassy or bushy territory. The slender-footed *merriami* group is most abundant as to individuals on the most open ground. If "broad-footed" groups are represented in the same general vicinity, they will be found to predominate on ground of the opposite characteristic."

For the jerboas whose skins are available the percentage of foot length to that of head and body is 41 for the little *Euchoreutes*, 52 for *Allactaga*, and 54 and 55 for *Dipus sowerbyi*, according to collector's measurements. Thus the relative size of hind foot compared with length of head and body averages for the wood rat, kangaroo rat, and jerboa, respectively, about 18.7, 33.4, and 50, or expressed still more roughly for convenience, as the numerals 4, 7 and 10 compare with one another.

As compared with the foot of *Neotoma* that of *Dipodomys* is both relatively and actually less broad, the proximal part of the sole is greatly elongated and the distal part (area of palmar pads) much reduced. Whereas in *Neotoma* the length of the third toe without its claw, measured on the longer, medial side, is one-fifth of the total foot length, in *Dipodomys* it is one-quarter, showing that the toes have increased in length even more rapidly than the rest of the foot. The relative proportions of the four lateral toes have suffered no alteration, however, save that digit 5 appears to be very slightly more reduced than in *Neotoma*. Some species of *Dipodomys* entirely lack an external hallux, some are variable in this respect, and others normally have this toe represented vestigeally, it being either entire or with the claw and terminal phalanx lacking. But this will be further discussed elsewhere.

The whole sole has the skin in a faintly scale-like pattern and the plantar aspect of the digits is transversely creased. The plantar pad area is not sharply marked but consists only of an irregular prominence of the sole near the base of the toes. The juncture of digits 3 and 4 is situated farther distally than those of the others. Hairiness of foot varies somewhat with the species, but in *D. spectabilis* hair occurs upon the toes save upon the terminal pad, and is longest upon the sides, here being fringe-like. The sole is also densely covered with short hairs which are longest near the base of the toes. Upon the dorsal surface of the foot the hairs are white, but sooty upon the plantar surface of this species, although white in *D. deserti*.

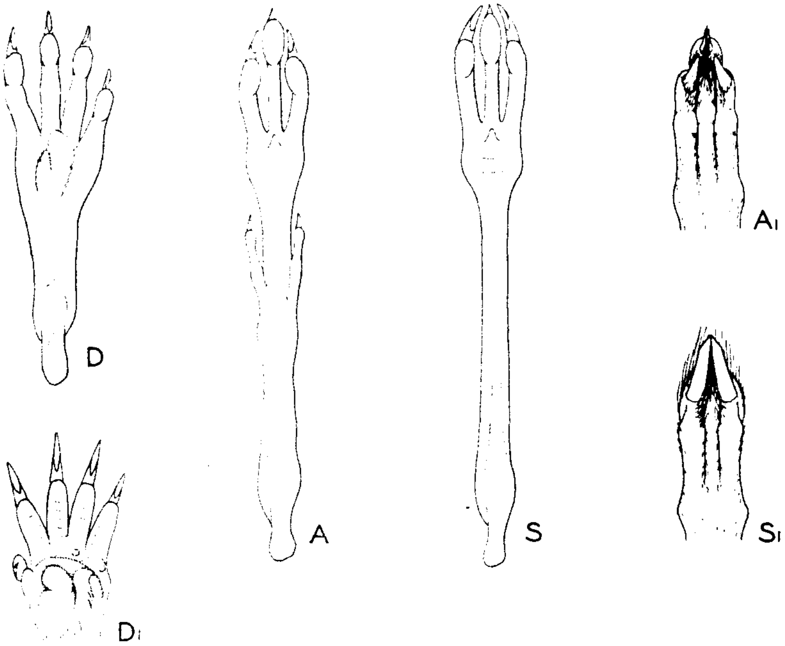


FIGURE 5. Plantar aspect (× 1) of the left pes of Dipodomys (D), Allactaga (A), and Scirtopoda (S). Palmar aspect (× 2) of the left fore foot of Dipodomys (D_1). Dorsum of the distal pes of Allactaga (A_1) and Scirtopoda (S_1), showing characteristic central inclination of nails.

In the jerboas the hind foot, as well as the shank, is greatly elongated. The middle toe without claw is about 23 per cent of the total foot length, so the toes have experienced approximately equal elongation with the foot as a whole. The number of digits is variable, but there are always three terminal digits,—the second, third, and fourth. These may be all that are externally present, as in *Dipus* and *Scirtopoda*, the first and fifth may be retained farther back on the pes, as *Allactaga* and *Euchoreutes*, or the first may have disappeared and the fifth retained in a reduced state, this condition being solely represented by the genus *Scarturus*.

In *Dipodomys* there is a tendency for the terminal part of the second and fourth digits to slope toward the third, as discussed more fully elsewhere, and in *Scirtopoda* this is carried to an extreme. In preserved specimens of the latter, as undoubtedly in life, the tip of the second claw touches that of the fourth, these being relatively on a line with the digital axis. The much shorter and smaller claw of the third digit, however, is almost at a right angle to its first and second phalanges, and with the other two claws, it therefore points upward between the other two. All the digits in this animal would therefore seem normally to be used fairly closely adpressed and as a single complex. It therefore seems indicated that the toes of the jerboas are not spread fanwise to function on soft sand like a snowshoe, but are held together to act as a single digital detail. In the preserved specimens they are maintained at a fair right angle with the metatarsus, irrespective of whether the latter be flexed or extended in regard to the shank, being held in this position by the tone of the flexor fibularis tendons.

The terminal digital pads are not broad but are very deep indeed and are definitely creased. With the toes adpressed the middle pad is somewhat in advance of the other two, and the middle digit is considerably the slenderest. In figure 5 the foot is illustrated as without hair, to show detail, but actually the sides of the toes are very densely fringed with long, sooty hairs (see frontispiece). These do not hide the terminal pads of digits 2 and 4, but they do cover that of 3. This long hair also covers the metatarsophalangeal expansion of the foot and becomes progressively more scanty and shorter proximally. Over most of the metatarsus the hairs are most dense along the depression, clearly visible, between the flexor tendons and the bone. The tarsal prominence and the heel are naked. There is but one plantar pad or tubercle, high and pointed and set well back from the base of digit 3.

Although the middle three digits of *Allactaga* superficially resemble those of *Scirtopoda* because of the elongation of the foot, basically they are somewhat different. The second and fourth nails also slope toward the third, but the latter is not elevated to a right angle with its phalanges, nor do the second and fourth nails approach one another to the point of contact, for the third digit interposes. The nails are also shorter than in *Scirtopoda*. The terminal pad of the third toe is very broad and flat, while those of the second and fourth, although equally as extensive in a vertical direction are rather narrow where they touch the ground, and fit back of, and to either side of, that of 3. The borders of the digits and of the foot are lightly fringed with rather long hairs, but not to anywhere near the extent as in *Scirtopoda*. The single plantar tubercle is larger than in that animal and situated farther forward. The first and fifth digits are very much more slender than the others and the first is slightly shorter than the fifth. Their positions upon the foot are better illustrated than described. The metatarsal area of the sole is naked, indicating, I should think, more frequent contact with the ground than in *Scirtopoda*.

Excessive hairness of the digital area appears to be characteristic of the 3-toed jerboas, but not of those having four or five digits. The foot of *Euchoreutes* is essentially like that of *Allactaga*, except that in the former the terminal digital pads are phenomenally developed, and this is also the case with the 4-toed *Scarturus* save that the hallux is lacking. The foot of *Pedetes* is on the general plan of that of *Dipodomys*, although somewhat more specialized.

In a fresh specimen of *Dipodomys* (*D. agilis*) examined soon after death and manipulated so as to overcome the effects of rigor mortis it was found that the greatest angle which the thigh could assume with respect to the vertebral axis was about 135 degrees, but it is doubtful whether so extreme a position (in a posterior direction) can be attained voluntarily. In normal static posture the thigh-vertebral angle approximates zero; or in other words, the femur is then virtually parallel with the body axis.

Maximum extension of the shank with respect to the thigh was about 135 degrees, so that it is not quite possible for the animal to hold these two segments in a straight line. In fact it is doubtful whether in practice this angle exceeds 120.

In shank-thigh flexion there was only a slight amount of tonal flexion of the pes—to about 120 degrees—which can probably easily be overcome by antagonistic muscular effort. In shank-thigh extension there was strong inhibition of foot extension, the shank-foot angle remaining at practically 180 degrees, and the latter was the greatest amount of flexion of which the foot is capable.

At the heel there was no discernible pronation or supination, but because of the long foot and the looseness of its ligaments movement of the toes was possible for about 75 degrees in either direction. The corresponding figures for *Neotoma* were 100 degrees in supination and 75 in pronation. When the leg was flexed there was gradual involuntary flexure of the toes, undoubtedly through the action of the deep flexor tendons, until at the maximum of this action the foot-toe angle was about 135 degrees.

Unfortunately there has been no opportunity to examine a fresh specimen of jerboa so as to study the limb action, and preserved specimens are entirely untrustworthy for this feature. On the whole the flexor-extensor action of the limb segments is probably not greatly different from that of *Dipodomys*, save that there appears to be a greater amount of automatic flexion of the toes of *Scirtopoda* when the pes is sharply extended with respect to the shank. In the more specialized jerboas, such as those dissected, there is probably also some reduction in the rotational action of the segments, especially of the pes, in which there should be possible very little supination and pronation.

MYOLOGY.

The material, representing saltatorial rodents, which was dissected in the present connection comprised a number of individuals of *Dipodomys spectabilis*, two of *Scirtopoda orientalis*, and two of *Allactaga mongolica longior*. Not all of the *Dipodomys* were completely dissected, only particular points of some of the specimens having been investigated. The condition of these was excellent, as was that of the *Allactaga*, save that both of the latter had been caught in traps and some of the muscles of the thorax quite severely damaged. In addition these and the specimens of *Scirtopoda* had become hardened in a flexed position so that some difficulty was experienced in investigating small muscles of the subcervical and other regions. This was especially pronounced in *Scirtopoda*, the muscles of which had been bleached and rendered especially tender by years in alchohol, so that they were very prone to tearing. While dissecting all of these certain details of the genus *Zapus* were also examined.

In connection with the myology it should again be mentioned that the affinity of the jerboas is more nearly with hystricomorphine, and of *Dipodomys* rather with the sciuromorphine rodents, and in consequence many of the differences are attributable to phylogenetic rather than adaptational considerations. This also applies to the wood rat, *Neotoma*, a myomorphine representative, details of which are mentioned from time to time.

A word of explanation anent the designation of muscle groups as flexors or extensors may here be in order as investigators, including myself, have frequently neglected properly to indicate the nomenclature which they have intended to follow. In the case of a typical quadruped it is certainly more convenient, from a functional viewpoint, to look upon the forward movement of the knee as flexion of the thigh, corresponding to backward movement of the elbow, and this action has been so treated in my previous reports. But this treatment leads to confusion when the phylogeny of the muscle groups is also being considered and is technically incorrect for the reason that all the dorsal muscles of the limbs, including triceps brachii and quadriceps femoris, are extensors of the limbs in the primitive tetrapod posture. This system of terminology is the one that will be followed in the present instance.

The innervation is given for all muscles of *Dipodomys*. No attempt was made, however, to determine the exact number of nerve components serving such segmental groups as the abdominal muscles and sacrospinalis. Furthermore it was found so exceedingly difficult to work advantageously within the reduced space between the shoulder and the greatly inflated bulla that the finer details of the cervicobrachial plexus are not offered. Because of the limited amount of jerboa material available the innervation within this group was sought only in the case of those muscles presenting some particular problem.

Perhaps no plan of arrangement of the muscle system will ever be entirely satisfactory from every viewpoint, and this certainly cannot be the case at present when so much remains to be done on the morphology of muscles. I have found the plan followed in reporting upon the anatomy of the wood rat to be convenient and reasonably satisfactory. Nevertheless it is somewhat deficient from the morphological aspect and accordingly I have herewith adopted a system of muscle arrangement that is essentially phylogenetic in character, based, for the most part, on tenets that are reasonably well known and established. Some compromises, however, have been necessary, as in separating, under extrinsic and intrinsic muscles of the shoulder, the latissimus dorsi and teres major, which are components of a common group. Such a system will not be found as convenient for the student who is not thoroughly versed in myology as one more purely topographical in character, for he may not see anything to be gained by dividing groups which he has been wont to consider as closely related, such as the dorsal scalene from the two other scaleni divisions, but this is the fault of nomenclature that is firmly entrenched—not of the phylogenetic arrangement employed.

The system herewith followed may be briefly presented as follows:

- Branchiomeric Musculature.
  - Trigeminal Field (N. V).
  - Facial Field (N. VII).
  - Glossopharyngeal Field (N. IX).
  - Vago-accessory Field (Nn. X, XI).
- Myotomic Musculature.
  - Axial.
    - Ocular.
    - Dorsal.
    - Ventral.
      - Infero-lateral.
      - Superior (Hypaxial).
  - Appendicular.

I. BRANCHIOMERIC MUSCULATURE.

A. Trigeminal Field (N. V).

The Mm. tensor tympani and tensor veli palatini were not investigated.

Masticatory Group.

M. masseter. In the two groups, represented by the jerboas on the one hand and *Dipodomys* on the other, the masseter muscle, with accompanying bony conformation of the skull, constitutes one of the best criteria for supergeneric groupings. Hence, as might be inferred, the masseter muscles of the two groups are basically different.

In both, as in all rodents, I believe, there is a typical masseter superficialis, distinct and largely triangular. In *Dipodomys* the apical half was tendinous and arose from a slight prominence at the premaxillary-maxillary suture between the infraorbital and incisive foramina. Insertion was along the caudal border of the expanded angular process of the mandible, not, however, extending quite to the dorsal tip of the process. In *Scirtopoda* the tendinous part was more restricted and it further differed in that origin was also fleshy, the whole being from the inferior border of the maxillary portion of the zygomatic arch. It thus probably included some of the masseter major components as found separable in *Dipodomys*. Insertion was upon both outer and inner borders of the angular process and dorsalward to include the accessory process between the angle and the condyle of the mandible. In *Allactaga* origin was even more extensive, to include half or more of the ventral border of the jugal.

In *Dipodomys* the masseter major was robust and arose from the whole rostral surface of the infraorbital lamina of the maxilla and as far rostrally as above the infraorbital foramen. It was inserted, in common with the deep division, upon a characteristic groove upon the mandible, below and parallel with the dental arch. Posteriorly its deep fibers merged with those of the deep division. In the jerboas the masseter major did not occur as a separable muscle.

In reporting on the wood rat I designated the caudal division of the masseter as the masseter zygomaticus, which I now discard as less desirable than the term masseter posterior. In *Dipodomys* this was very well developed and arose chiefly from the posterior part of the zygomatic process of the maxilla, but also from the jugal. Insertion was upon both lateral and medial surfaces of the superior angular process. In the jerboas origin was continuous with and inseparable from that of the superficial division. Insertion, however, was distinct, in *Scirtopoda* upon the dorsal part of the accessory angular process and in *Allactaga* upon the well defined prominence formed below the condyle by the projection of the incisor root; thence, in both animals, for a short distance anteriorly.

The masseter profundus in *Dipodomys* arose from the entire orbital surface of the maxilla and had a broadly tendinous insertion upon the mandible in company with the masseter major. In the jerboas the division was intricate and especially posteriorly its relationship with contiguous divisions was very intimate. It has evidently developed to subserve the same functions as the masseter major in myomorph rodents, which it has been able to do by extending anteriorly through the large infraorbital aperture of the maxilla, and it accordingly arose as shown (fig. 6), as well as from the orbital surface of the maxilla and adjacent jugal. Insertion was as in *Dipodomys* but less extensive.

M. temporalis. This muscle was very much reduced in all three animals. In *Dipodomys* it was double, the dorsal division arising from most of the squamosal, including its process directed toward the meatus, and inserting upon both aspects of the mandibular coronoid process. The deeper, more ventral division arose from the ventral aspect of the squamosal laterally adjacent to the glenoid cavity, and inserted posterior to the coronoid process. In the jerboas the muscle was single and even weaker, comparable to the dorsal division in *Dipodomys*.

M. pterygoideus externus. This was quite strong in *Dipodomys* but smaller than the internus. Origin was from the lateral surface of the parapterygoid plate with insertion upon the medial condylar process of the mandible. The muscle was smaller in the jerboas.

M. pterygoideus internus. In *Dipodomys* this was very large, origin being from the parapterygoid fossa as usual, and insertion upon the inner, or rather anterior, surface of the extensive angular plate of the mandible. In *Allactaga* it was almost as large but insertion was necessarily less extensive, following the conformation of the mandible. A part of the muscle ended over the entire angular vacuity, so it inserted directly upon the fascia covering this part of the masseter. In *Scirtopoda* it was considerably smaller.

INFERIOR ALVEOLAR GROUP.

M. digastricus anterior. In all specimens the anterior belly of the digastric was distinct from the posterior, but the plan was different in the two groups. In *Dipodomys* the anterior belly, well separated from the midline in its posterior part, arose from the divisional tendon and inserted upon the mandible near the symphysis menti. It was thus not essentially of the sciuromorphine type, as mentioned by Parsons (1894). On the other hand, that of the jerboas was, although in this feature the latter did not conform to what the latter author termed the hystricomorphine type. In the jerboas the digastric tendon formed a tendinous arch over the hyoid and was partly attached thereto. In *Scirtopoda* the anterior belly arose from half this tendinous arch and passed rostrally in contact with its antimere throughout its length. In *Allactaga*, however, a very odd condition obtained. The fibers that reached the mandible arose from the lateral part of the arch, converging toward the symphysis, while the fibers from the more medial part of the arch inserted upon a midline raphe, and the part of the digastric so inserting seemed to be separable from the remainder. That the latter has nothing to do with the mylohyoid is shown by the presence between the latter and the digastricus anterior of the transversus mandibularis.

M. transversus mandibularis. This was present in all, best developed in *Allactaga* and least so in *Dipodomys*.

M. mylohyoideus. In all three animals attachment was partly to the hyoid, as well as to the medial raphe. The latter part in *Dipodomys* extended but two-thirds the distance from the hyoid to the symphysis, half the distance in *Scirtopoda,* and one-quarter of the distance in *Allactaga,* in which the muscle was much reduced in area.

FACIAL FIELD (N. VII).

Superficial Group.

Although there has been some dissension with regard to the platysma, I believe that work on the facial field within recent years by Ruge, Huber et al shows conclusively that all of the superficial musculature that is served by the facial nerve has been derived from the primitive sphincter colli of the lower vertebrates. In mammals this has differentiated into three layers, comprising a sphincter colli superficialis, often entirely lacking in the less primitive sorts, a platysma layer, and a sphincter colli profundus. Remnants of the sphincter colli primitivus may occur over the neck. According to Huber the superficial sheet has not led to the formation of any specialized divisions, while the other two have done so, the derivatives often being of extreme complexity and secondarily having migrated over or under other slips in a very intricate manner. It is only by interpreting such slips by means of a generalized ground plan common to many unspecialized mammals that they may be satisfactorily homologized.

The facial musculature of *Scirtopoda* and *Allactaga* is much the same and of a rather simple pattern, so that they may be discussed together. That of *Dipodomys,* however, is so exceedingly intricate, because of the specialization necessary as the external cheek pouches developed, that it must be accorded separate discussion. In four specimens of the latter in which the superficial musculature was dissected there was considerable variation in the definition of individual slips, certain of them being thinner in one specimen than another, or with definable fibers extending farther. Furthermore, the pouch is so extensible in all directions that in one specimen a slip of its controlling musculature may be narrow and in another stretched much more broadly, this depending upon the position in which the pouch was fixed by the preservative, and whether it was empty or distended with food.

In terminology and plan of treatment for this group of muscles I follow Ernst Huber. The jerboas are discussed first, as in them the conditions are so much simpler than in heteromyids.

Scirtopoda and *Allactaga.*

M. sphincter colli superficialis. In *Scirtopoda* this sheet was represented by extremely tenuous fibers over a limited area below the ear (fig. 6), but it was not distinguished in *Allactaga.*

M. platysma. In *Scirtopoda* the platysma proper arose at the midline partly deep to the retro-auricular musculature and, expanding slightly between the superficial sphincter colli and the sterno-auricularis, divided into two slender but well defined slips, one of which extended to the angle of the mouth where it ended superficially to the orbicularis oris, and the other more ventrally to the chin. Its dorsal border was in contact with the auriculolabialis where careful dissection was necessary to separate the two. In *Allactaga* the plan was essentially the same, save that the dorsal border did not extend quite to the auriculolabialis.

In *Scirtopoda* the retro-auricular musculature, consisting of a postauriculo-occipitalis complex, occurred in two layers. The more superficial of these entirely covered the deeper and extended quite broadly from the midline to the base of the ear. The second layer was also broad, but thinner, with fibers running more strictly laterally to the base of the ear. In *Allactaga* there were also two layers. The superficial was about as in *Scirtopoda* but heavier, while the deeper, extending farther ventrally and therefore visible from beneath the posterior border of the superficial layer, extended to the cartilage of the ear in three tenuous slips.

In both jerboas the mandibulo-auricularis was narrow but robust, extending from the auricular cartilage to the posterior border of the mandible just ventral to the condyle.

M. sphincter colli profundus. All except the specialized parts of this sheet were lacking. Posteriorly in both genera there was a slender sterno-auricularis, extending from the ear cartilage, beneath the platysma, and quite to the sternum, attachment being about at the first sterneber rather than clearly to the manubrium. The more ventral part of this slip is of course homologous to the sterno-fascialis, where this occurs alone as in *Neotoma.* In both animals the auriculolabialis extended from the ear cartilage, below and in close relation with the orbicularis oculi, and deep to the superior branch of the platysma and the orbicularis oris. Above its origin was a reduced frontalis, a mere remnant in *Scirtopoda* but larger in *Allactaga* and extending as far as above the eye. There was also a very small interscutularis in both, extending but a short distance

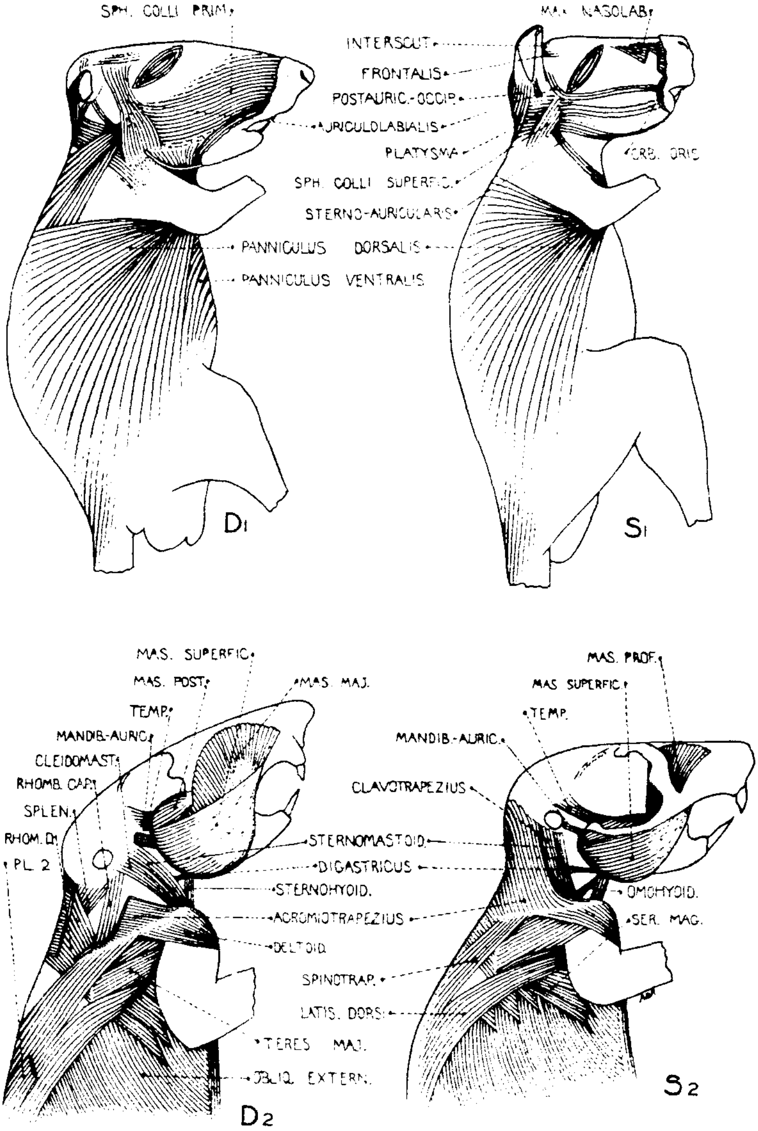

SPH. COLLI PRIM.
MAX. NASOLAB.
INTERSCUT.
FRONTALIS
POSTAURIC.-OCCIP.
AURICULOLABIALIS
PLATYSMA
ORB. ORIS
SPH. COLLI SUPERFIC.
STERNO-AURICULARIS
PANNICULUS DORSALIS
PANNICULUS VENTRALIS
D1
S1
MAS. SUPERFIC.
MAS. POST.
MAS. MAJ.
TEMP.
MANDIB.-AURIC.
CLEIDOMAST.
RHOMB. CAP.
SPLEN.
RHOM. D.
PL. 2
MAS. PROF.
MAS SUPERFIC.
TEMP.
MANDIB.-AURIC.
CLAVOTRAPEZIUS
STERNOMASTOID.
DIGASTRICUS
STERNOHYOID.
OMOHYOID.
ACROMIOTRAPEZIUS
SER. MAG.
DELTOID.
SPINOTRAP.
LATIS. DORS.
TERES MAJ.
OBLIQ. EXTERN.
D2
S2

FIGURE 6. Superficial cutaneous (facial and panniculus) musculature of Dipodomys (D_1) and Scirtopoda (S_1), above. Below, the superficial musculature of the shoulder and masticatory musculature of the same genera (D_2 and S_2).

from the ear. In both, also, the nasolabialis was about as usual in rodents and was simply triangular. Deep to this, but partially visible along its superior border, was the maxillonasolabialis. In *Scirtopoda* this took the form of a slender pars nasalis, and a short and equally slender pars labialis (fig. 6) extending beneath the more posterior of the vibrissae follicles, these being larger than the remainder of the follicles. In *Allactaga* the pars nasalis occurred in two divisions. The deeper was largely like the single slip in *Scirtopoda*, while the superficial arose in two slender parts, these then joining and sending a thread-like tendon which broadened over the dorsal part of the nose. The pars labialis was as in *Scirtopoda* save that it slanted more toward the nose than the lip.

The orbicularis oris is indicated in the drawings but all of its fibers were not followed with exactitude. The buccinatorius was as usual in rodents, being more extensive in *Allactaga* as an accompaniment of the relatively longer rostrum. It was a rather simple sheet in both, and its indicated ability to act as an oral sphincter was not considered to be more marked than in the wood rat.

Dipodomys.

There was no indication of any sphincter colli superficialis. The muscles present were exceedingly involved and curious, departing widely in their development from the fundamental arrangement, undoubtedly because of the requirements of the external cheek pouches. The control of the latter is intricate, for the muscles must act largely in groups.

M. sphincter colli primitivus. From this has developed the superficial sheet of muscle covering the pouch. It took origin from the midline chiefly deep to the postauriculo-occipitalis complex and passed deep to the two divisions of the platysma. A part, indicating its homology, then extended for a short distance over the shoulder before disappearing, but the main portion had secondarily invaded the side of the head to the mystacial pad. After passing deep to the platysma it expanded, as suggested in fig. 6, its relationship with the labial extension of the auriculolabialis being so intimate that the two could not be readily separated. Although it closely approached the orbicularis sacculi and the fibers were parallel, the latter was not derived from this sheet.

M. platysma. The platysma proper usually occurred dorsally over the shoulder in two slips. The more superficial arose from the fascia near the midline, in the same layer with, and almost joining, the panniculus. Above the posterior extremity of the pouch it disappeared beneath the ventral extension of the auriculolabialis. It there divided into three portions. The part (*a*) was the more superficial and extended to the dorsolateral part of the pouch as shown (fig. 7, D1), but stopped considerably short of the orbicularis sacculi. Entally a somewhat similar part (*b*) went deep to the pouch and inserted upon the middle half of its ental surface (fig. 7, D2). A third part (*c*) split off and in the form of a slender slip passed deep to the posterior termination of the pouch, emerged from its ventral border so that it could pass superficial to the caudal border of the gular sheet of the sphincter colli profundus (fig. 7, D1 and D4), and so extended to the chin.

The part of the platysma which is termed for convenience the deeper division arose from the fascia near the midline in a position which covered the anterior half of the latissimus dorsi origin—therefore deep to the panniculus. It emerged from the latter only to pass beneath the superficial division of the platysma, extending narrowly and partially deep to the dorsal border of the pouch and then expanding to insert upon the anterior part of its ental surface, medial to slip (*b*) of the superficial division. In another specimen the origins of platysma 1 and 2 were practically continuous and the second division did not separate until it dipped beneath the first, posterior to the auriculolabialis. In addition the first division covered the entire pouch.

The three divisions of platysma 1 are really not so surprising. It is clear that they separated from the primitive platysma sheet as the pouch developed, the latter obliging slip (*c*) to adopt an unusual expedient. The origin of platysma 2 has migrated rearward beneath the panniculus, but its path of development as it finally has passed beneath platysma 1 is less apparent.

In *Dipodomys* the postauriculo-occipitalis complex occurred in three layers (fig. 7, D3). The most superficial of these was a broad, thin, triangular sheet arising from the midline and inserting along the occiput to the craniomedial base of the ear. The second was an extremely tenuous slip, also from the midline and passing to the medial part of the bulla. The third extended to the base of the ear, where it inserted by two slips as shown. The mandibulo-auricularis arose from the auricular cartilage and inserted upon the medial aspect of the superior angular process of the mandible.

M. sphincter colli profundus. At the medial base of the ear there was a small remnant of the interscutularis that did not reach even half way to the midline; and rostrally, a better defined frontalis, extending somewhat over the eye.

In one specimen at least the auriculolabialis certainly was entirely superficial. The original part was well defined and there was nothing noteworthy regarding its anterior fibers, these extending to the snout. Posteriorly, however, the major portion of the muscle had secondarily migrated ventrally, thus covering both the platysma and sphincter colli primitivus, and disappearing over the lower border of the pouch. Over the rostrum there had also been a secondary expansion dorsally, so that fibers as shown (fig. 6, D1) now entirely hide the nasolabialis. This rostral portion is now, apparently, completely fused along its border with the forward extension of the sphincter colli primitivus. In at least one specimen dissected the condition of this muscle was definitely different. The ventral extension covering the sphincter colli primitivus was not so well defined, but there was another tenuous extension beneath the primitivus, and in less intimate relationship with that sheet than with the underlying platysma (division 1), which the connection had pulled dorsally so that the platysma extended superiorly practically to the base of the ear cartilage.

The gular sheet of the sphincter colli profundus also varied individually in its definition. In one specimen it did not extend as far laterocaudally as the pouch. Perhaps usually, however, it is more pronounced, arising from the midline at least as far posteriorly as about the second sterneber, and to a point somewhat forward of the ventral border of the external opening of the pouch, but not to the chin. Both posterior and anterior borders of this sheet were heavier, especially the latter, the fibers here passing around the ventral opening of the pouch (fig. 7, D1) to continue dorsorostrally as the orbiculoris sacculi, and so to the mystacial pad. Thus, although the orbiculoris oris is also a derivative of the sphincter colli profundus it is likely that the latter became differentiated far in advance of the orbiculoris sacculi, which was developed from the same layer as the pouch began to form. It is seen that the posterior border of the gular sphincter colli was pulled forward by the third slip of the superficial platysma (fig. 7, D4), and that its dorsal border was indicated (fig. 6, D1) as not quite reaching either the auriculolabialis or the sphincter colli primitivus. Effort was made in several specimens to ascertain whether fibers of one of these layers did not actually pass over or under the other, but they were too slightly differentiated at this point to determine the question.

Deep to all the other facial muscles upon the side of the head was the usual sterno-auricularis, extending slenderly from the base of the ear to about the first sterneber. The ventral portion was there-

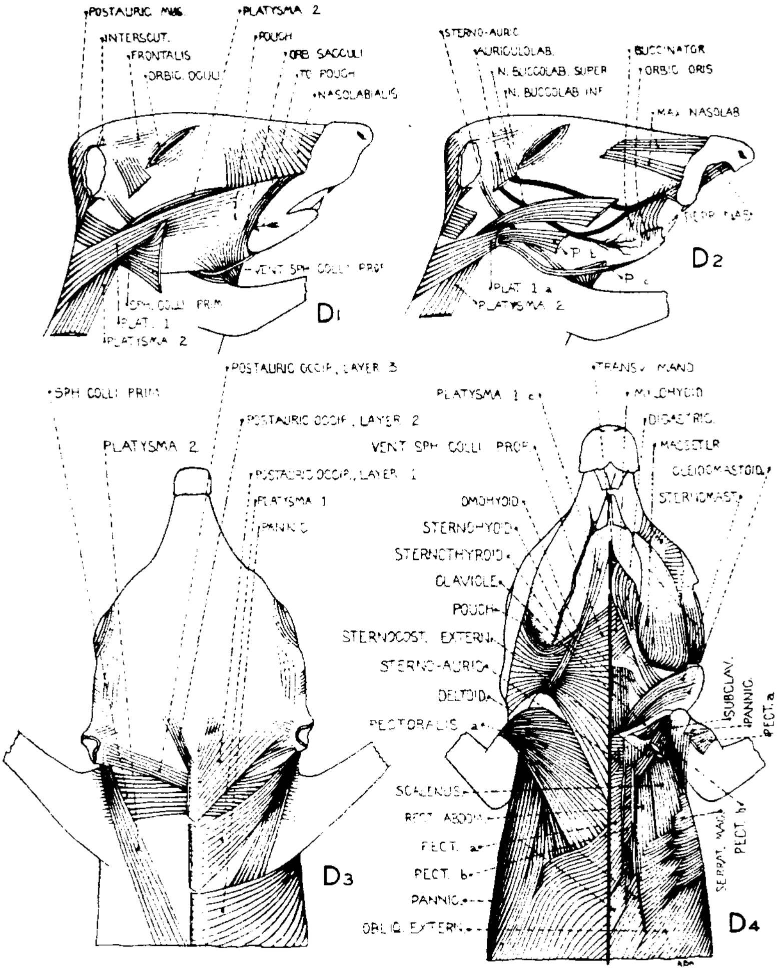

POSTAURIC MUS.
INTERSCUT.
FRONTALIS
ORBIC. OCULI
PLATYSMA 2
POUCH
ORB. SACCULI
TO POUCH
NASOLABIALIS
VENT. SPH. COLLI PROF.
SPH. COLLI PRIM.
PLAT. 1
PLATYSMA 2
D1
STERNO-AURIC.
AURICULOLAB.
N. BUCCOLAB. SUPER.
N. BUCCOLAB. INF.
BUCCINATOR
ORBIC. ORIS
MAX. NASOLAB.
DEPR. NAS.
P. b
P. c
PLAT. 1 a
PLATYSMA 2
D2
SPH. COLLI PRIM.
PLATYSMA 2
POSTAURIC OCCIP., LAYER 3
POSTAURIC OCCIP., LAYER 2
POSTAURIC OCCIP., LAYER 1
PLATYSMA 1
PANNIC.
D3
TRANSV. MAND.
MYLOHYOID
PLATYSMA 1 c
DIGASTRIC
MASSETER
VENT. SPH. COLLI PROF.
CLEIDOMASTOID
STERNOMAST.
OMOHYOID
STERNOHYOID
STERNOTHYROID
CLAVICLE
POUCH
STERNOCOST. EXTERN.
STERNO-AURIC.
DELTOID
PECTORALIS a
SUBCLAV.
PANNIC.
PECT. a
SCALENUS
RECT. ABDOM.
PECT. a
PECT. b
PANNIC.
OBLIQ. EXTERN.
SERRAT. MAG.
PECT. b
D4
ABH

FIGURE 7. The musculature of Dipodomys. Details from the lateral aspect of the middle (D_1) and deeper (D_2) layers of the facial muscles; and from the dorsal (D_3) aspect. Ventral (D_4) aspect of the cephalic and thoracic muscles, superficial layer to the left and deeper details to the right of the midline.

fore well beneath the gular sheet of the sphincter colli profundus, which is a very interesting occurrence. This slip is also a derivative of the sphincter colli profundus. Therefore it evidently separated from the latter in some ancestral form, the gular sheet then first contracting somewhat, but later extending to cover the ventral part of the sterno-auricularis as specialization of the pouch progressed.

The nasolabialis was not triangular as usual but the insertion, especially, was very extensive (fig. 7, D1), and it undoubtedly has developed for the purpose of aiding in control of the inner margin of the pouch. Beneath it were two small slips of the maxillonasolabialis. The pars nasalis extended to the nose, and the pars labialis, with more ventral origin, diverged to the upper lip.

The orbicularis oris was of the usual pattern and there was not discernible any specialization attributable to the pouch. The buccinatorius was, however, highly specialized. In the kangaroo rats the part of the mouth posterior to the incisors can evidently be much more tightly closed than in most rodents. Whereas in the wood rat there is an oral pad upon either side which the normal tone of the buccinators keeps in contact, thus closing the mouth posterior to the incisors, in *Dipodomys* the more caudal part of the buccinators themselves have differentiated, they being attached to the lower rostrum above and the mandible below, in such fashion that the pair can operate practically as a single, circular sphincter, thus closing the mouth very tightly. The anterior part of the buccinator, however, descends from the side of the rostrum as usual. From the sphincter colli profundus has also developed a small muscle extending dorsally from the anterior base of the upper incisors to the nose; this small muscle may be termed a depressor nasalis, but whether its relationship is closer to the orbicularis oris or buccinatorius is not clear.

Deep Group (of N. VII).

M. stapedius. In *Dipodomys* the muscular process of the stapes was especially poorly defined and the muscle appeared to be correspondingly smaller.

M. stylohyoideus. In *Dipodomys* origin was from the bulla medial to the styloglossus, and in the jerboas from the paroccipital process. Insertion was upon the hyoid as usual.

M. digastricus posterior. In *Dipodomys* origin was from the exoccipital process and fleshy fibers practically ceased at the tendinous division between the posterior and anterior bellies. The whole of this tendon did not pass to the latter, however, but a part split off medially to form with its antimere a poorly defined arch, with slight attachment to the hyoid. The latter detail was carried farther in the jerboas. In these the origin was from the paroccipital process and the whole of the tendinous division between the bellies formed, with its antimere, the tendinous arch, more firmly attached to the hyoid, and from which the fibers of the anterior belly took origin.

GLOSSOPHARYNGEAL FIELD (N. IX).

The part of the pharyngeal musculature served by this nerve was not dissected.

VAGO-ACCESSORY FIELD (Nn. X, XI).

The pharyngeal and laryngeal musculature innervated by this field was not dissected. In this group might properly be placed the sternomastoid, cleidomastoid and trapezius, for they are partially served by the ramus externus of the accessory nerve, but as they are also supplied by the cervical nerves they are grouped with the lateral set of the cervical, inferolateral, myotomic musculature.

II. MYOTOMIC MUSCULATURE.

1. AXIAL MUSCLES.

A. The muscles of the ocular division (Nn. III, IV and VI) were not investigated.

B. Intrinsic dorsal division.

The long back musculature might here be dismissed with a short description and the statement that it had the usual attachments. Discretion would dictate such a course, for when great specialization of this system is encountered it is extremely difficult to homologize the divisions with those of man, perhaps partly for the reason that certain portions are not homologous with anything in human anatomy but only analogous. Considering that there is a deal of uncertainty regarding the homology of the vertebral processes in mammals and that the erector spinae is now always a single muscle system with parts variously differentiated, this is only to be expected. Nevertheless an attempt will be made toward homologizing the extremely intricate divisions in saltatorial rodents, and these will be discussed at some length.

(*a*) Spino-transverso-occipital system.

In *Dipodomys* the *M. splenius* was extensive, the origin of the posterior part being deep to the serratus posterior superior, while insertion was over the lambdoidal area to a point caudoventral to the meatus. In *Scirtopoda* it was more extensive along the midline and this was still more pronounced in *Allactaga*. In *Zapus* it did not extend so far caudally as in the others.

(*b*) Sacro-transverso-transversalis system.

When one encounters two cervical divisions of the erector spinae located lateral to the trachelomastoid, or longissimus capitis, the inner should be considered as of longissimus affinity also, and the outer as a part of the iliocostal.

M. iliocostalis, partes cervicis, dorsi et lumborum. In *Dipodomys* this was greatly reduced in size, although the posterior part had excellent leverage. The pars lumborum arose mostly from the ilial spine, but a few fibers also came from the fascia mediodorsally. As a perfectly distinct, slender muscle the whole passed to an attachment of its fibers upon ribs 12, 11, 10 and 9. Fibers of the pars dorsi, indivisible from the pars lumborum, first arose from rib 11, with increment from each rib farther cranialward. The fiber bundles from each rib were attached to about the seventh rib next cranially, the whole ending in a pars cervicis, which inserted upon the transverse processes of the last four cervical vertebrae.

In *Scirtopoda* the iliocostal was even smaller. In the posterior lumbar region its fibers were inseparable from those of the longissimus, but definition was good behind the thorax. The lumbar fibers inserted upon the last five ribs and from these pars dorsi took origin. In the middle of the thorax the muscle narrowed so as almost to disappear, but became better defined anteriorly, and insertion was upon the transverse complex of the fused cervical vertebrae. In *Allactaga* it was determined that fibers of the iliocostal arose not from the ilium but clearly from the superficial aponeurosis of the longissimus, and that none extended farther to the rear than the middle of the lumbar region. This may really have been the case in *Scirtopoda* also, a point impossible of determination because of the bleached and fragile condition of the fibers. Insertion was as far forward as the third cervical.

The inner part of the sacro-transverso-transversalis system in reality consisted of two distinct divisions, a lateralis comprising that part of the usual longissimus that attaches to the costae, and a medialis that ordinarily has more medial attachments. These for convenience will accordingly be considered as two muscles.

M. longissimus lateralis, partes capitis, cervicis, dorsi, lumborum (et caudalis?). It is not at all improbable that a part of the extensor caudae lateralis as found is homologous with the longissimus lateralis, but this is not certain and for convenience it will be discussed with the medial division. In this connection it should be stated that the homology of the caudal muscles of long-tailed mammals has received so little attention that disposition of the caudal components in any phylogenetic scheme can be only tentative.

In *Dipodomys* pars lumborum of the lateral longissimus was extraordinarily robust. Its fibers arose partly from the medial surface of the ilium (but not from the sacrum) and from the dorsal fascia. From the ectal surface of the latter the more lateral fibers originated, and from the ental surface the deeper ones; and in the lumbar region all fibers curved laterocranially around the loin in a heavy, semicylindrical mass to insert upon the ventral surface of the anapophyses of the last two thoracic and first four lumbar vertebrae. As the curvature of the fibers was such that those caudally inserted no farther back than the fourth lumbar, well defined anapophyses were lacking upon the last three lumbars. All but the last two thoracics were also lacking in sharp anapophyses, and here the pars dorsi was accordingly of a different character. Its posterior, superficial fibers continued from the dorsal fascia but origin of the remainder was by small tendons, apparently from the dorsal metapophyses, with insertion upon the ribs considerably farther forward. The whole continued uninterruptedly into the pars cervicis, with tendinous attachment to the transverse processes of the last six cervicals. Pars capitis, constituting a trachelomastoid, was entirely separate. It seemed not to have attachment to any of the cervical vertebrae but arose from the first eight thoracics (metapophyses ?) and inserted upon the audital bullae caudoventrally to the meatus.

In *Scirtopoda* the pars lumborum was basically similar to, but of little more than half the width of, that of *Dipodomys*. The pars dorsi, however, was broader, and anterior attachment was upon the transverse cervical complex medial to the iliocostalis. In robustness this division in *Allactaga* was intermediate between conditions in the other two, with attachment to the last four cervicals. Accompanying the lesser size of this muscle in the jerboas the lumbar anapophyses were shorter, especially in *Allactaga*, than in *Dipodomys*.

Whereas in *Dipodomys* the cervical part of this division was definitely associated with pars dorsi and inseparable from it, in the jerboas the pars cervicis was rather associated with the trachelomastoid, constituting to all intents a shorter, cervical division of the latter. In *Scirtopoda* it arose from thoracics 4, 3 and 2, and inserted upon not only the transverse complex of the last six cervicals but upon the atlas as well. In *Allactaga* it arose from the first four thoracics as well as the posterior cervicals, and inserted upon the atlas. Pars capitis—the trachelomastoid—arose in *Scirtopoda* from thoracics 8, 7, 6, 5 and possibly 4, and inserted upon the caudal border of the meatus just deep to the sternomastoid, while in *Allactaga* conditions were the same save that insertion extended farther dorsally.

M. longissimus medialis should be treated as of the trunk and of the tail. Part of the caudal division, or extensor caudae lateralis, certainly belonged with the longissimus medialis, and perhaps all of it did. At any rate, as an accompaniment of the extreme specialization of the longissimus the fleshy part of the lateral extensor of the tail was perfectly divisible into three portions, which may be termed ectalis, intermedius and entalis (fig. 8, D2). In *Dipodomys* pars ectalis arose from the caudal aspect of the ilial aponeurosis and from the sacral transverse processes, and a single tendon extended from it upon the lateral side of the tail; so its functions are as much those of an abductor as an extensor. The pars intermedius had the same origin as the pars ectalis but farther forward, and the two were separated by a deep fascia. At the base of the tail this part was entirely deep to the ectalis but its three long, caudal tendons occurred upon both sides of that of the ectalis. It also sent shorter, deeper tendons to insert upon the metapophyses of the first few caudal vertebrae. Pars entalis was really nothing but a caudal continuation, by a single long tendon, of the middle division of the longissimus medialis, although shorter tendons from this muscle also inserted deep upon the basal caudal vertebrae. One gained the impression that upon contraction of this division of the longissimus farther forward the tail would be extended automatically, unless controlled by antagonistic action of the caudal flexors.

In the jerboas a largely similar condition prevailed, save that some of the fibers of the pars intermedius were in direct continuation of the longissimus lateralis, especially in *Scirtopoda*, and that in the latter the pars entalis sent four long tendons along the tail, while in *Allactaga* this was at first a robust, single tendon which later split into four branches.

The part of the longissimus medialis occurring upon the body proper of *Dipodomys* was also divisible into three parts, more distinct transversely than axially. These may be termed anterior, intermediate and posterior divisions. The anterior of these was really the most superficial (although all were mostly hidden by the longissimus lateralis) and its fibers took origin from the lateral side of the tendons of the semispinalis dorsi, which arose from the metapophyses of the first five lumbars. The fibers inserted upon the articular processes of the last four thoracics. Next, the posterior division arose from the aponeurosis attached to the superior border of the ilium. The fibers twisted around mostly deep to the intermediate division to insert upon the anapophyses of the first five lumbars and last two thoracics. Finally, the intermediate division arose under cover of the semispinalis lumborum from the sacrum and the tendons of the extensor caudae lateralis and also inserted, but dorsal to the posterior division, upon the anapophyses of the first five lumbars and possibly the last thoracic as well.

In the jerboas the medial longissimus was single and its fibers were distinct from those of the lateral division only in the superficial part of the muscle. It arose from the medial ilium and the dorsal aspect of the sacrum, and inserted upon the anapophyses of the last two thoracics and first four or five lumbars.

(*c*) Spino-spinalis system.

M. spinalis. In *Dipodomys* this system seemed to be entirely absent, although there may have been a few of its fibers present between the spines of the last thoracic and fourth sacral vertebrae. In *Scirtopoda* it was definitely present between these spines and in one specimen attachment was to the first caudal as well as the last sacral. In *Allactaga* what appeared to be a homologous slip occurred stretching forward from the last sacral spine, but it did not reach as far as the last lumbar, the fibers ceasing abruptly upon a medial aponeurosis.

(*d*) Transverso-spinalis system.

In many mammals, apparently, the relationship of the fibers constituting the semispinalis, multifidus and submultifidus (or rotatores) elements of this system is extremely close, so that no clear separation is practicable and one is uncertain of the proper term to use for some component that occurs. But after all it is largely an academic question whether the term semispinalis is employed, the fibers of which may pass over more than four vertebral spaces, or multifidus, which may pass over three. In my study of the wood rat the extensor caudae medialis appeared to continue directly forward into the lumbar region, and as this caudal muscle is conventionally believed to be of multifidus affinity I accordingly called the lumbar part by this name. In the rodents now under discussion, however, these two parts appear to be largely distinct. I am now reasonably certain that the lumbar part of this system in these mammals, and the wood rat as well, has fibers that are too long to be with propriety termed multifidus, while this objection does not hold for the caudal part. Suffice it to say, however, that the semispinalis lumborum, so termed, and the extensor caudae medialis in the rodents discussed, have both been derived from the transverso-spinalis system.

M. extensor caudae medialis took origin in all three genera from the first and subsequent caudal spines, insertion of each bundle then being upon the metapophyses of the third vertebrae next to the rear. The bundles rapidly became smaller and their insertions more tendinous.

M. semispinalis. The division which appeared to be representative of this group was composed of partes capitis, cervicis, dorsi et lumborum. In *Dipodomys* it was very broad and was largely superficial by virtue of the fact that the spinalis system was practically absent and the longissimus situated more laterally. Pars lumborum arose from the aponeurosis that extended from the ilium over the extensor caudae lateralis, and inserted upon the spines of all the lumbar and last three thoracic vertebrae, possibly with the exception of the last lumbar. Pars dorsi adjoined but was not strictly continuous with the last. Origin was chiefly by tendons from the metapophyses of the first four lumbars and last two thoracics. The fibers passed over several vertebral spaces and at the midline were tucked in to insert deep upon the short spines of the anterior thoracics, except that a number of the more lateral fibers, indivisible from the rest farther backward, separated and passed as a narrow slip superficial to the remainder in a medial direction (fig. 8, D2) to insert upon the ligamentum nuchae. There seems no possible course but to consider this as a part of the semispinalis. The pars cervicis, hidden by the pars capitis, was distinct and arose from the first six or seven thoracics and inserted upon the common spine of the fused second and third cervicals. The pars capitis was quite heavy but could not be separated into biventer cervicis and complexus. It arose from the first eleven thoracic vertebrae but apparently not from any cervical, and inserted upon the supra-occipital and just laterally upon the bulla.

In *Scirtopoda* the pars lumborum was smaller and insertion was upon the spines of the second to sixth lumbars. Origin of the pars dorsi did not begin so far to the rear as in *Dipodomys*, but the exact number of vertebrae involved was especially difficult to determine. Insertion was upon the thoracic spines, although there may have been no definite attachment to the first two; but fibers extended over these, nevertheless, to rather weak insertion upon the spinal complex of

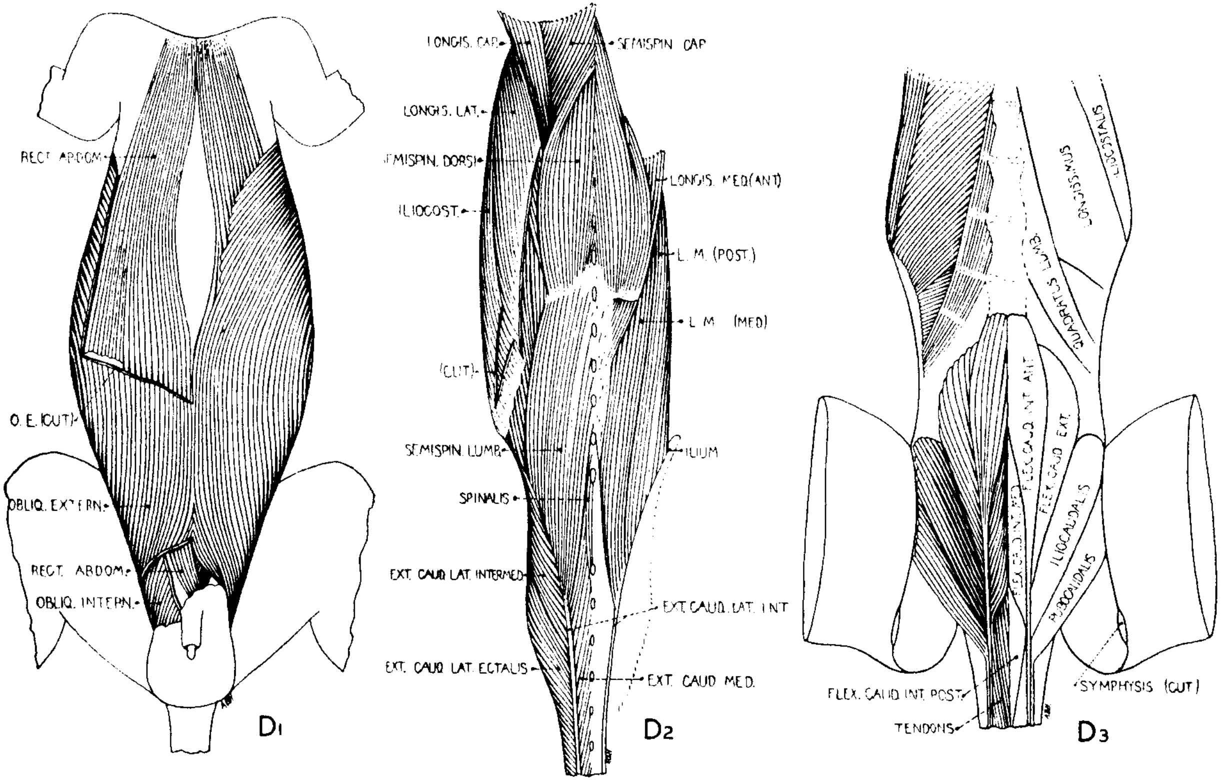

RECT. ABDOM.
O. E. (CUT)
OBLIQ. EXTERN.
RECT. ABDOM.
OBLIQ. INTERN.
D1
LONGIS. CAP.
LONGIS. LAT.
SEMISPIN. DORSI
ILIOCOST.
(CUT)
SEMISPIN. LUMB.
SPINALIS
EXT. CAUD. LAT. INTERMED.
EXT. CAUD. LAT. ECTALIS
SEMISPIN. CAP.
LONGIS. MED. (ANT.)
L. M. (POST.)
L. M. (MED.)
ILIUM
EXT. CAUD. LAT. INT.
EXT. CAUD. MED.
D2
LONGISSIMUS
ILIOCOSTALIS
QUADRATUS LUMB.
FLEX. CAUD. INT. ANT.
FLEX. CAUD. EXT.
FLEX. CAUD. INT. MED.
ILIOCAUDALIS
PUBOCAUDALIS
FLEX. CAUD. INT. POST.
TENDONS
SYMPHYSIS (CUT)
D3

FIGURE 8. The musculature of Dipodomys. (D$_1$) the abdominal musculature from ventral aspect. (D$_2$) The long back muscles, superficial aspect to the left and deeper to the right. (D$_3$) Ventral view of pelvo-sacro-caudal musculature.

the fused cervicals. Pars cervicis took origin from the first five, or possibly six, thoracics and inserted upon the cervical spinal complex. The lateral fibers of the thoracic part of the semispinalis were found to be separable and these, arising from all twelve thoracics, continued on to form a single pars capitis, inserting more broadly than in *Dipodomys* upon the more extensive supra-occipital.

In *Allactaga* pars lumborum was essentially as in *Scirtopoda,* but it is interesting to note that the thoracis mass showed a modified condition of that existing in *Dipodomys.* The lateral fibers occurred in a slip that was distinct throughout its length. The most superficial of these arose from the aponeurotic sheath of the longissimus, but the deeper fibers had origin from the original tendons of the medial longissimus, rather than from the processes direct, over about the last five thoracics, and passed superficially craniomedially over the rest of the muscle to insert upon the nuchal ligament. Because of the position of this nuchal slip, chiefly its original part, it appeared that its relationship in this animal, and therefore inferentially in *Dipodomys,* may really be with the semispinalis capitis rather than the pars dorsi. The remainder of the dorsal semispinalis, mostly deeper, arose from most if not all of the dorsal vertebrae, each bundle passing over several vertebral spaces to insert upon the dorsal spines, possibly to include the last one or two of the cervicals. Pars cervicis was essentially as in *Scirtopoda.* Origin of pars capitis began where origin of the outer slip of the pars dorsi ended and was from the first seven thoracics. The insertion was slightly more extensive upon the supra-occipital than in *Scirtopoda.*

Of the shorter elements of the transverso-spinal system of the trunk proper—multifidus and rotatores—there were, in *Dipodomys,* only the latter clearly defined in the lumbar region, while both occurred in the thorax. In the jerboas both seemed to be lacking in the lumbar region, while with certainty only the rotatores could be detected in the thorax.

(*c*) Intervertebral system.

Modified intervertebral group. Because of the peculiar occipital conditions in *Dipodomys* the conformation of the deep supracervical musculature was somewhat altered. The obliquus capitus superior extended from the lateral atlas to an extensive area over the caudomedial aspect of the audital bulla, the obliquus capitis inferior had its usual attachments to axial (plus third cervical) spine and lateral atlas, and probably because of the reduced occipital area the rectus capitis posterior was single, from the axial spine straight dorsally to the supra-occipital and in contact with its antimere. No distinct rectus lateralis could be demonstrated.

In the jerboas there were well developed major and minor divisions of the rectus capitis superior, and obliqui capitis superior and inferior. In *Scirtopoda* there was a very short rectus lateralis, which was better developed in *Allactaga* with its longer neck.

Of the remaining intervertebral muscles the interspinosi and intertransversarii elements were present in all, the former being especially strong in the lumbar region of the jerboas, possibly in compensation for the lack, or virtual lack, of multifidus and rotatores. It should here be mentioned that only the medial intertransversarii belong with the intrinsic dorsal musculature, the lateral ones having been derived from the ventral divisions of the myotomes.

(*f*) *M. scalenus posterior.* A perusal of anatomical text books shows with what uncertainty this division of the scaleni is regarded. Ordinarily it is stated, rather hesitatingly, to be served by twigs from the cervical plexus. Ernst Huber has, however, demonstrated to his entire satisfaction that it is innervated only by the posterior rami of the cervical nerves, wherefore it must be grouped with the dorsal, rather than the ventral, division of the axial musculature. Unfortunately, however, it is often the case that the scaleni of some of the lower mammals cannot be homologized, even topographically, exactly with the three divisions of man. Especially in such small, short-necked rodents as are under discussion it is virtually impossible to determine with thorough satisfaction whether or no the scalene complex is served also by branches of the rami posterior, and accordingly, whether or not the homologue of the human scalenus posterior be actually present. For this reason the components of the scaleni will all be discussed together (page 430).

Innervation of the intrinsic dorsal musculature is by short branches of the rami posteriores of the spinal nerves.

C. *Ventral division.*
  A′. Infero-lateral system.
    (a). Cervical region.
      a′. Lingual set (N. XII).

M. styloglossus. This muscle, superficial to the hyoglossus, wa very slender. In *Dipodomys* origin was from the bulla lateral, and in the jerboas dorsal, to the stylohyoid. Insertion was into the tongue as usual.

M. hyoglossus. Origin of this muscle, next deep to the geniohyoid, extended from the midline laterally upon the hyoid, the fibers diverging laterocranially to the tongue.

M. genioglossus. This was narrow and deep, with fibers parallel to and adjoining the midline. Origin was from the hyoid partly deep to that of the hyoglossus.

The intrinsic lingual musculature was not dissected.

Innervation of the lingual set is by the N. hypoglossus (XII).

b′. Medial set.

M. geniohyoideus. This muscle is immediately beneath the mylohyoid. It was parallel to and adjoined the midline, was rather narrow and stretched as usual from the hyoid to the mandible. Innervation is by a filament from the n. hypoglossus.

M. sternohyoideus. In all three rodents dissected origin of this muscle was not from the manubrium and the first sternal cartilage. On the contrary, presumably because of the shortness of the neck, origin was from the dorsal surface of the first sterneber and slightly from the second costal cartilage of *Dipodomys*, while in the jerboas it had spread far laterally upon the second cartilage. Insertion was as usual upon the hyoid.

M. omohyoideus. This was present in all three but absent in *Zapus*. In *Dipodomys* it was more nearly parallel to the sternohyoid than in the others, and had the usual attachments upon scapula and hyoid.

M. sternothyroideus. The origin of this muscle, fused with the sternohyoid, was from the dorsal surface of the first sterneber, the two separating at about the first rib. Insertion was upon the hyoid as usual.

Innervation of these three muscles is by branches from the ansa hypoglossi.

c′. Lateral set.

Mm. intertransversarii laterales. As already mentioned this part of the intertransversarii belongs with the ventral axial musculature. The bundles were well developed in the lumbar region.

M. scalenus. Although the dorsal scalene, as in man, belongs with the dorsal axial musculature, the whole complex will be described here, for the reason that it is not readily separable, as already explained. In most, if not all, rodents it seems safe to say that the strict homologue of the scalenus anterior of man is absent, for no part of this complex is situated ventral to the brachial plexus and subclavian artery. In *Dipodomys* there was but a single scalene, entirely indivisible anteriorly. Origin was by slips from ribs 2 to 5, those from 4 to 5 being deep to the two slips of the serratus magnus originating from ribs 3 and 4. Insertion was chiefly upon the atlas but there was slight attachment also to the transverse processes of the cervicals, chiefly 3, 4 and 5. The absence of a ventral division agrees with the situation in sciuromorph rodents as stated by Parsons.

In the jerboas there was distinctly a small, ventral scalenus, dorsal to the brachial plexus, which arose from the first rib and inserted upon the cervical transverse processes and upon the lateroventral process of the atlas, better developed than in *Dipodomys*. The larger dorsal division was more comparable to the scalenus of *Dipodomys*, but it arose from ribs 2, 3 and 4, and there was a diminutive deep slip arising from the extreme dorsal part of the first rib, the common insertion being mainly by tendon upon the lateroventral process of the atlas. This deep slip from the first rib might possibly be the homologue of the posterior scalene of man. The only innervation secured for the scaleni was by short branches from the cervicobrachial plexus.

The trapezius and sternocleidomastoid have usually been considered as partly of branchiomeric origin, because supplied by the accessory nerve as well as direct branches from the cervical plexus. Addens (1929) has claimed that the ramus externus of the accessory is a modified spinal nerve, but this remains to be proved. These three rodents well illustrate the real homogeneous character of these muscles.

M. sternomastoideus. This arose from the manubrium and possibly from a bit of the clavicle as well. Insertion in *Dipodomys* was upon the bulla below, and in the jerboas behind, the auditory meatus.

M. cleidomastoideus. In *Dipodomys* this muscle, arising from the clavicle immediately lateral to the sternomastoid origin, was about as broad as the latter but slightly deeper. In *Scirtopoda* the homologue of this slip might better be termed a calvotrapezius, but in *Allactaga* there had separated from the latter, toward the insertional end only, a cleidomastoid slip, arising from the clavicle continuously with the acromiotrapezius but soon separating from it in a slender slip and inserting deep to its lateral border behind the meatus. It would be difficult to find three mammals which illustrate to better advantage the theoretical origin of this muscle from the trapezius sheet.

M. trapezius. In *Dipodomys* origin of the spinotrapezius was fairly continuous with the latissimus dorsi, insertion being upon the dorsal two-fifths of the scapular spine. Origin of the acromiotrapezius extended along the midline as far as the occiput, with insertion along the entire spine to the tip of the acromion. In *Scirtopoda* the spinotrapezius was narrow, inserting upon the dorsal third of the spine. Origin of the acromiotrapezius extended along the midline from just dorsocranial of the spine to the occiput and then laterally upon the bulla dorsocaudal to the meatus. Insertion was as in *Dipodomys*. In addition there was in *Scirtopoda* a clavotrapezius, from the lateral quarter of the clavicle to a point behind the meatus, corresponding in all but position of origin to a normal cleidomastoid, and in reality it might be proper to term it so. In *Allactaga* the spinotrapezius was broad. At origin it was not quite continuous with the acromiotrapezius but the two joined toward insertion and attachment of the whole was to the ventral three-fifths of the spine. Origin of the acromiotrapezius extended laterally upon the skull to a point to the rear of the meatus, as in *Scirtopoda*, but at this point the muscle was fused with the clavotrapezius, the latter splitting off to an attachment upon the lateral clavicle. In *Zapus* the spinotrapezius was excessivley slender and long, having attachment to the dorsal half of the spine, and the acromiotrapezius did not extend laterally upon the skull.

(b). Thoracic region.

Mm. intercostales. There was little or nothing distinctive about the intercostals. The externals occurerd only between the bony ribs proper and not between the cartilages.

M. sternocostalis externus. The proper allocation of this small slip is problematical, for in small rodents I have found it impossible to determine its nerve supply. It appears, however, to have been derived from the intercostals. In *Dipodomys* it extended in a laterocranial direction from the fascia covering the anterior rectus abdominis to the angle of the first rib. In *Allactaga* it was essentially similar, but it was not detected in *Scirtopoda*. The innervation was not detected, but in a number of other mammals I have found it to be by an intercostal nerve, although some have claimed that it is served by the anterior thoracic.

M. serratus posterior superior. In *Dipodomys* this was narrow, occurring between ribs 5 to 9, origin being partly superficial to the splenius and mostly deep to the serratus magnus. In *Scirtopoda* it was smaller still, occurring between ribs 4 and 7, and in *Allactaga* even more tenuous.

M. serratus posterior inferior. In *Dipodomys* this was relatively heavy, but it was lighter in *Scirtopoda*, and was discernible with clarity in neither *Allactaga* nor *Zapus*. Innervation of both these muscles is by branches of the intercostal nerves.

(c) Abdominal region.

M. rectus abdominis. In *Dipodomys* this muscle was exceedingly broad, especially posterior to the thorax where a lateral slip ended (fig. 8). Origin was from the pelvic bones at the symphysis and there was slight decussation of the two halves, the right muscle here being U-shaped and inclosing the single slip of the left. The muscle was here quite narrow but rapidly broadened and its fibers became inextricably fused over the abdomen with the deeper fibers of the external oblique. Thoracic attachment was to the first rib only. In the jerboas there was no decussation at origin, as there is said often to be in octodonts but not in hystricomorphs. It was almost as broad as, although much less heavy than, in *Dipodomys*, and there was no very marked increase in width over the abdomen, nor was fusion of the fibers with those of the external oblique at all pronounced. Thoracic attachment was also to the first rib only.

All the abdominal muscles were very strong in *Dipodomys* but much weaker in the jerboas.

M. obliquus externus. In *Dipodomys* origin was from the dorsal fascia and by fleshy bundles at the termination of the serrati magnus (anterior) and posterior inferior. Anteriorly insertion was upon the sheath of the rectus, but over the abdomen the midline was practically reached, with fibers running almost sagitally and with the deep ones fused with those of the rectus. Caudally the lateral fibers in a heavier bundle passed lateral to the testis (in the male) and inserted, with the remainder of the fibers, upon the crural arch. Conditions in the jerboas were evidently largely similar, save that there was not such intimate contact with the rectus; but the abdomen of these jerboas was in bad condition and satisfactory dissection could not be accomplished.

M. obliquus internus. In *Dipodomys* origin was from the dorsal fascia slightly more dorsally than the external oblique, and from the crural arch and border of the pubis. Insertion was upon the last three ribs and then upon the transversalis on a line extending from the cartilage of the tenth rib to within about 25 mm. of the symphysis, and thence to the midline. Until well to the rear the ventral part of the muscle was entirely deep to the rectus but near the symphysis a part split off and passed superficial to the latter muscle to continue as the *M. cremaster*, covering the anterior half of the testis. In *Scirtopoda* this sheet was very thin and tenuous, the fibers hardly reaching the thorax, even dorsally. In *Allactaga* the condition was too poor for dissection.

M. transversus abdominis. Origin in *Dipodomys* was from the dorsal fascia slightly ventral to the internal oblique and from the inner border of the thorax, to the midline deep to the rectus abdominis.

Innervation of the abdominal muscles is by branches of the intercostal, and probably the first lumbar, nerves, not followed in the present instance.

B′. Superior (hypaxial) system.

(*a*) Cervical region.

M. rectus capitis anterior. As usual this extended from the atlas to the basioccipital and was much smaller in *Scirtopoda* than in the other two genera. No separate rectus capitis lateralis was detected.

M. longus capitis. Origin in *Dipodomys* appeared to be not from the vertebrae but exclusively from the expanded, dorsal portion of the first rib. In *Allactaga* origin seemed to be equally exclusively from the specialized ventral process of the sixth cervical. In *Scirtopoda* the latter process is virtually undeveloped and it was impossible to determine from what precise spot in its immediate vicinity this fragile slip arose. Insertion in all was upon the anterior basi-occipital, this being more restricted in *Scirtopoda*.

M. longus colli. The fibers of this muscle which in *Dipodomys* originated from the ventral process of the sixth cervical inserted upon the medial tubercle of the atlas. The remainder of the muscle arose from as far posteriorly as the fourth or fifth thoracic and inserted upon the cervical vertebrae. This was also the case in *Allactaga*, save that origin did not extend so far caudally upon the thoracic vertebrae; but in *Scirtopoda* the ventral process of the sixth cervical was so lightly developed that marked origin from this point was not apparent. Innervation of both of these divisions is by short branches of the cervical nerves, not sought in the present instance.

(*b*) Thoracic region—none.

(*c*) Lumbar region.

M. quadratus lumborum. This muscle occurred between the lumbar diapophyses as usual. In *Dipodomys* posterior attachment was chiefly to a process of the first sacral transverse process, well developed for this purpose, and to a lesser extent to the part of the ilium adjoining, while in the jerboas most of this connection was with the ilium. In *Dipodomys* diapophyses are virtually lacking upon the first two lumbars and reduced upon the third, while in the jerboas this condition occurs one vertebra farther forward. Accordingly the quadratus here passed over two and three vertebral spaces respectively to attachment upon the last rib. Innervation is by short branches of the lumbar nerves.

(*d*) Sacro-caudal region.

M. iliocaudalis. This muscle, homologous to a part of the levator ani of man, arose in *Dipodomys* from the medioventral border of the innominate ventrally and slightly cranially of the acetabulum. The fibers passed rearward into 6 tendons which extended upon the lateroventral surface of the tail. Conditions were essentially similar in the jerboas.

M. pubocaudalis. This, corresponding to the lower part of the levator ani in man, arose along the border of the pubis forward of the symphysis and caudally adjoining the iliocaudalis. Insertion was upon the tips of the transverse processes of the first four caudal vertebrae.

M. sacrospinosus, undoubtedly representing the combined m. coccygeus and sacrospinous ligament in man, was a short muscle easily overlooked in *Dipodomys* and *Allactaga*, passing from the spine of the ischium (just forward of the obturator notch) to the transverse complex of the sacrum. In *Scirtopoda* it seemed to be represented by the ligament only.

Innervation of these three muscles is apparently by short branches of the pudendal nerve.

Mm. flexores caudae. In the sacral region the caudal flexors were very intricate and they may best be described by treating them as occurring in the usual two divisions, an externus and an internus, and by separating the latter into partes antica, media, et postica. Pars antica was the most superficial of these, arising from the centra of the last lumbar and first sacral vertebrae. Its fibers developed 2 tendons which were the most dorsal of the lower caudal tendons. Pars media was partly deep to the last and arose from the centra of all five sacrals, developing tendons upon its lateral belly. Pars postica was considerably smaller, its fibers arising from the centra of the first two, or possibly three, caudals. The lateral fibers appeared to insert directly upon the third and fourth chevrons, while a tendon from the medial fibers, adjoining the midventral line of the tail, extended far toward the tip. The flexor externus arose from the ventral surface of all the sacral and first three caudal transverse processes, developing for the most part tendons upon the medial belly, although the posterior fibers inserted directly upon the tendons of the abductor caudae internus. Most of the tendons of all these flexor divisions finally inserted upon the chevron bones.

In the jerboas the fleshy part of these muscles was considerably less robust and more simple. The flexor caudae externus was largely as in *Dipodomys* but in *Scirtopoda* the flexor internus was uniform and indivisible, save as the parallel fibers could easily be forced apart. In *Allactaga*, however, the externus was divisible into two parts, these being anterior and posterior.

The perineal musculature of the sacrocaudal region comprised the following:

M. ischiocavernosus. In *Dipodomys* origin was from almost all of the caudal border of the ischium and forward as far as the symphysis, while insertion was upon the anterior base of the corpus cavernosum penis. Stronger connection with the pelvis was afforded by an elastic ligament from the vicinity of the symphysis to the anterior base of the penis, and another similar ligament posteriorly from the ischial border to the base. In this animal the crura of the corpora cavernosa penis were reduced and not apparent externally as lateral tumefactions. In *Scirtopoda*, however, these corpora were clearly indicated and quite swollen. Evidently as an accompaniment insertion of the ischiocavernosus was more apparently into, rather than on, the penis. Origin was as extensive as in *Dipodomys* but was shifted more dorsalward. Toward insertion the muscle passed beneath a superficial binding ligament that converged from the superficial surface of the crus to the posterior border of the symphysis. In *Allactaga* the corpora cavernosa penis were hardly more marked than in *Dipodomys* but the details of the muscle were more nearly like those of *Scirtopoda*.

M. bulbocavernosus. This was relatively large in *Dipodomys*. Some of its fibers were attached to the posterior ligament described under the ischiocavernosus, and the rest to the surface of the crus; but in the jerboas there was no attachment to the crus save indirectly by a ligament situated mediodorsal to the symphysis.

M. sphincter ani externus was as usual, surrounding the anus.

Innervation of the above three muscles is by branches of the pudendal nerve.

2. APPENDICULAR MUSCULATURE.

A. Pectoral limb.

a. EXTRINSIC SYSTEM.

Dorsal group.

M. latissimus dorsi. Morphologically this muscle should be placed with the teres major but for convenience the two are here separated on a topographic basis. In *Dipodomys* it was quite well developed and broad. Origin was from the dorsal fascia fairly continuous with the spinotrapezius, while insertion was upon the ventral part of the teres major tendon. In *Allactaga* origin was the same, and insertion as in *Scirtopoda*. In the latter animal origin was also continuous with the spinotrapezius but the muscle was considerably narrower. Fusion with the teres major tendon was not complete so that insertion may more properly be said to have been on the humerus. In *Zapus* the muscle was exceedingly slender. Innervation is by the thoracodorsal nerve.

M. rhomboideus. In *Dipodomys* this occurred in two parts. The origin of pars dorsi extended upon the midline anteriorly to the occiput, with insertion upon the entire vertebral border of the scapula. Pars capitis took origin narrowly from the mastoid region posterior to the meatus, between the splenius and the cleidomastoid, with insertion upon the anterior two-thirds of the vertebral border of the scapula. In *Scirtopoda* there was but a single rhomboid. Origin extended along the midline to the occiput and thence laterally to a point behind the meatus. Insertion was for a few millimeters along the dorsal portion of the spine and thence along the scapular border to the glenovertebral angle. Conditions in *Allactaga* were more comparable to those in *Dipodomys*, for there was a similar pars dorsi, and a pars capitis with scapular attachment confined to the vertebral border opposite the scapular spine. In *Zapus* there was also a pars dorsi and a pars capitis. Innervation is by the dorsal scapular nerve.

M. serratus magnus. In *Dipodomys* origin of the posterior part was by five slips, from ribs two to seven, the posterior four of which interdigitated with four slips of the obliquus externus, while the anterior two arose from deep slips of the scalenus. The main cervical part (depressor scapulae) was not separable save by its bundles of origin, which were apparently from the transverse processes of all of the last six cervicals. The atlantoscapularis superior might with equal propriety be considered as that part of the serratus which arose from the atlas. Insertion of the whole complex was upon the entire vertebral border of the scapula. In *Scirtopoda* origin did not extend posterior to the fifth rib and insertion continued for a short distance ventrally around the glenovertebral angle. In *Allactaga* origin included the sixth rib, but the thorax was torn so that some other details were impossible of investigation. In *Zapus* origin included the sixth rib. It will be seen that my observations on *Scirtopoda* do not agree with those of Parsons for the same animal (his *Dipus aegyptius*) for he stated that origin of the serratus extended as far as the eighth rib.

M. atlantoscapularis superior. This was present in all three animals but, as previously stated, it might just as well be considered as a part of the serratus magnus, for it is merely the separable part of that muscle that arises from the atlas.

M. atlantoscapularis inferior. This, too, is a derivative of that complex of which the serratus magnus is a unit, but the insertion has migrated distally. Origin was from the atlas as usual and insertion upon the border of the acromiotrapezius and the adjacent part of the clavicle. This was apparently the case in *Scirtopoda* as well, while in *Allactaga* the clavicle was not involved, and accordingly the muscle was situated a bit more dorsally.

Innervation of the above three muscles is by the long thoracic nerve (to most of the serratus magnus) and direct branches from the cervicobrachial plexus.

Ventral group.

M. pectoralis. In *Dipodomys* the pectorals were divisible into four parts at origin and two parts at insertion, and these may be designated as superficial and deep. The former arose in a broad sheet (*a*) from the midline between the manubrial tip and about the last sterneber, while the part (*a′*) was narrow and partly deep to the portion (*a*) over the xiphoid. Toward insertion it passed entirely deep to (*a*) and fused with it, the common insertion occupying a restricted area upon the proximal part of the deltoid process of the humerus. The portion (*b*) of the deep pectoral, situated the more anteriorly, was deep to the posterior half of the superficial (*a*), while the deep (*b′*) was, in origin, entirely to the rear of the superficial (*a′*), arising not strictly from the midline but with fibers decussating with those of the obliquus externus and rectus abdominis behind the xiphoid. Toward insertion (*b′*) passed deep to (*b*) and the single attachment was by a slender tendon upon the humeral head.

In the jerboas both superficial and deep pectorals were single, the former about as extensive as superficial (*a*) in *Dipodomys*, and the latter arising deep to the middle half of the superficial division. In *Zapus* the superficial pectoral was double, as in *Dipodomys*, but the deep pectoral was single, like that of the jerboas except for being very narrow.

In all three animals the scapuloclavicularis was absent, the only part of the sternoscapularis represented being the following:

M. subclavius. In *Dipodomys* this was relatively well developed. It arose from the first costal cartilage and passed dorsal to the clavicle to an attachment that was chiefly tendinous upon the lateral part of that bone. In *Scirtopoda* origin was farther lateral upon the costal cartilage, and still farther in *Allactaga*.

M. panniculus carnosus. There is no justification for considering this sheet as anything but a derivative of the primitive pectoral complex, even where there now occur segregated patches (over the hip of *Pedetes*, et cetera), for innervation of the entire sheet is invariably by the anterior thoracic nerve, the same as for the pectorals. In *Dipodomys* the main part of the panniculus was extensive. Origin of the dorsal part was from the vicinity of the midline, from a point above the axilla to and onto the base of the tail. In the latter situation the sheet was continuous over the midline and was heavier, as I have found to be the usual case in long-tailed rodents. Anterior to the tail base the fibers almost disappeared but became better defined once more over the dorsal part of the abdomen. Insertion of all of this dorsal sheet was upon the proximal part of the deltoid crest near the head of the femur and adjoining insertion of the superficial pectoral. Near insertion, however, the ventral border of the sheet separated to become tucked in deep to the remainder (fig. 6). There was also a ventral panniculus sheet in this animal arising from the midline almost as far forward as the gular sphincter colli profundus and posteriorly to just beyond the level of the groin. Its anterior fibers passed caudodorsally deep to the ventral fibers of the dorsal panniculus, where they soon diasppeared over the side, but farther to the rear the fibers were parallel and the two sheets became continuous.

In the jerboas only the dorsal sheet of the panniculus was present and this was very similar to that of *Dipodomys*, also being heavier over the base of the tail.

Innervation of the components of this ventral group is by branches of the anterior thoracic nerve.

b. INTRINSIC SYSTEM.

Muscles of the shoulder.

Dorsal group.

M. supraspinatus. In all three animals origin was from the entire supraspinous fossa with insertion upon the greater tuberocity of the humerus.

M. infraspinatus. Origin was from that part of the infraspinous space not occupied by the teres major. Insertion was upon the lateral part of the greater tuberosity. In the jerboas the origin was broader relative to the scapular width, therefore actually about the same width in *Scirtopoda* as in *Dipodomys*, and narrower in *Allactaga*. Innervation of these two muscles is by the suprascapular nerve.

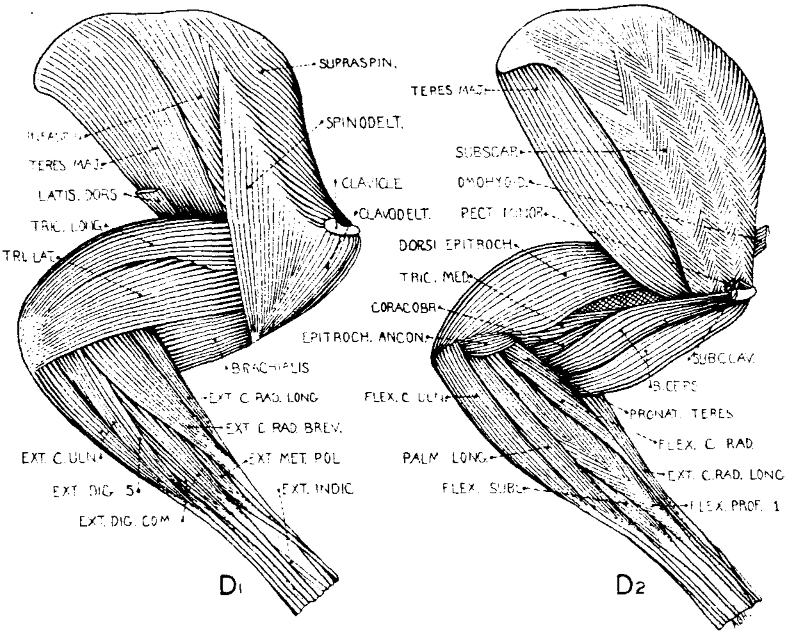

SUPRASPIN.
SPINODELT.
INFRASPIN.
TERES MAJ.
LATIS. DORS
TRIC. LONG.
TRI. LAT.
CLAVICLE
CLAVODELT.
BRACHIALIS
EXT. C. RAD. LONG
EXT. C. RAD. BREV.
EXT. C. ULN.
EXT. MET. POL
EXT. DIG. 5
EXT. INDIC
EXT. DIG. COM
D1
TERES MAJ.
SUBSCAP.
OMOHYOID.
PECT. MINOR
DORSI EPITROCH.
TRIC. MED.
CORACOBR.
EPITROCH. ANCON.
FLEX. C. ULN.
SUBCLAV.
BICEPS
PRONAT. TERES
FLEX. C. RAD.
PALM. LONG.
EXT. C. RAD. LONG
FLEX. SUBL.
FLEX. PROF. 1
D2

FIGURE 9. Musculature of the anterior limb of Dipodomys: lateral (D_1) and medial (D_2) views.

M. subscapularis. This muscle was highly multipinnated, with origin from the entire subscapular surface and insertion upon the lesser tuberosity of the humerus.

M. teres major. Morphologically this muscle belongs both with the subscapularis and latissimus dorsi, but the latter is placed with the extrinsic musculature of the arm for convenience. In *Dipodomys* it was very broad and as an accompaniment of the pronouncedly falcate shape of the glenovertebral angle of the scapula it overlapped the infraspinatus to a considerable extent. Origin was thus from most of this falcate angle and insertion was along the proximal part of the medial crest of the humerus. In *Scirtopoda* the muscle was hardly one-third as wide as in *Dipodomys*, the origin being correspondingly reduced with the smaller insertion situated more proximally. These features were even more pronounced in *Allactaga.*

Parsons (1894) has said that in *Scirtopoda* insertion of the teres major was into the tendon of the latissimus dorsi, but I found the opposite to be the case. He also said that in the Dipodidae insertion is posterior to that of the latissimus as is man, but this was certainly not the situation in my specimens. Innervation of the subscapularis and teres major is by the subscapular branches of the brachial plexus.

M. deltoideus. In *Dipodomys* the spinal, acromial and clavicular parts of the deltoid comprised a continuous muscle, to all intents homogeneous, but with care they could be separated. The spinal part arose from two-thirds of the spine, and the clavicular from practically the entire clavicle. Common insertion was largely tendinous and rather restricted upon the deltoid crest of the humerus. Conditions in the jerboas were similar save that in *Allactaga* the spinal origin was less extensive and its insertion more individual, passing deep to the acromial part.

M. teres minor. In *Dipodomys* this was partially fused with the infraspinatus. It arose from the distal two-thirds of the glenovertebral border of the scapula and inserted upon the greater tuberosity caudal to the infraspinatus. This was the case in *Allactaga* and *Zapus* also, but in *Scirtopoda* the muscle was easily separable from the infraspinatus. Origin, however, was only from the distal third of the glenovertebral border, and insertion in both jerboas was mostly deep to the infraspinatus. Innervation of both deltoid and teres minor is by the axillary nerve.

Muscles of the brachium.

Dorsal group.

The triceps complex occurred in four divisions as follows:

M. dorso-epitrochlearis, in *Dipodomys,* was distinct throughout its length, arising chiefly from the teres major tendon but to some extent from that of the of the latissimus as well. It extended broadly over the inner arm to an insertion by fasciculi upon the olecranon. *Scirtopoda* showed to excellent advantage how this muscle may appear by fission from, or disappear by fusion with, as the case may be, the medial surface of the triceps longus. Its superficial aponeurosis was double, one sheet coming from the scapula and the other from the teres major and latissimus tendons. Both shortly fused and passed deep within the substance of the muscle. In other words muscle fibers gradually developed upon the medial surface. This was a condition fairly intermediate between *Dipodomys* and *Allactaga*, in the latter of which there was no connection at all with the teres major or latissimus.

M. triceps longus. In *Dipodomys* this was relatively large. Origin was partly by tendon from the axillary border of the humeral head and partly fleshy, not from the bone but directly from an aponeurosis that passed from the acromion to the deep surface of the teres major and then around the axillary border of the scapula, this part of the origin being more clearly visible from the lateral than the medial side. Insertion was upon the posterior part of the olecranon. In the jerboas the entire origin was broadly tendinous, from the distal quarter of the scapular border.

M. triceps lateralis. In *Dipodomys* origin was by tendon from the lateral part of the humeral head just deep to the insertion of the teres minor, and insertion was upon the ulnar border of the olecranon. In *Scirtopoda* origin was from a trifle farther to the rear on the humeral head, and this was more pronounced in *Allactaga*.

M. triceps medialis. Origin was from the medial crest of the humerus distal to the teres major insertion and along the caudal border of the shaft, while insertion was upon the lateral olecranon.

No true triceps anconeus, either medial or lateral, was present as a separate division.

Innervation of the triceps group is by the radial nerve.

Ventral group.

M. biceps brachii. In *Dipodomys* the long head arose as usual from the bicipital process upon the glenovertebral border of the scapula, and the short head from the coracoid tip. The tendons remained separate throughout their length, the two partially twisting so that the one belonging to the long head inserted upon the medial ulna with the brachialis, and the other upon the usual area of bicipital attachment of the radius. In the jerboas there was only one head, as Parsons has stated. This was the long head, even smaller than the corresponding division in *Dipodomys*. Insertion was upon the ulna with the brachialis. In *Zapus* there was also a single head.

M. coracobrachialis. In *Dipodomys*, as well as *Zapus*, a pars profunda, or breve, of the coracobrachialis occurred, arising from the coracoid process and inserting upon the neck of the humerus above the latissimus tendon; but this could not be found in the jerboas, although Parsons (1894) stated that he encountered it in *Scirtopoda*. The pars media was represented in all, arising from the coracoid as usual. In *Dipodomys* it was very slender and inserted upon the humerus well below the latissimus tendon. In *Scirtopoda* it was very well marked and inserted from a point somewhat below the latissimus quite to the epicondyle, and similarly in *Allactaga*, save that insertion

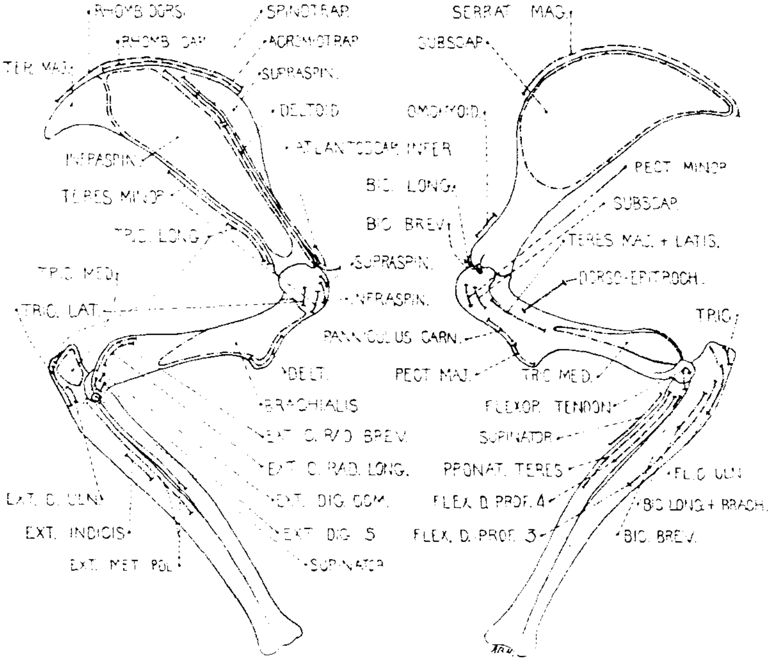

RHOMB. DORS.
RHOMB. CAP.
TER. MAJ.
SPINOTRAP.
ACROMIOTRAP.
SUPRASPIN.
DELTOID
INFRASPIN.
ATLANTOSCAP. INFER.
TERES MINOR
TRIC. LONG.
TRIC. MED.
TRIC. LAT.
SUPRASPIN.
INFRASPIN.
DELT.
BRACHIALIS
EXT. C. RAD. BREV.
EXT. C. RAD. LONG.
EXT. C. ULN.
EXT. DIG. COM.
EXT. INDICIS
EXT. DIG. 5
EXT. MET. POL.
SUPINATOR
SERRAT. MAG.
SUBSCAP.
OMOHYOID
BIC. LONG.
BIC. BREV.
PECT. MINOR
SUBSCAP.
TERES MAJ. + LATIS.
DORSO-EPITROCH.
TRIC.
PANNICULUS CARN.
PECT. MAJ.
TRIC. MED.
FLEXOR TENDON
SUPINATOR
PRONAT. TERES
FL. C. ULN.
FLEX. D. PROF. 4
BIC. LONG. + BRACH.
FLEX. D. PROF. 3
BIC. BREV.

FIGURE 10. The long bones of the anterior limb of Dipodomys showing areas, semidiagramatically, of muscle attachment: lateral (left) and medial views.

began immediately adjoining the latissimus. A relatively large brevis was present in *Zapus*.

M. brachialis. In all three animals this muscle was completely divisible into two parts. The more lateral arose from the surgical neck of the humerus lateral to the deltoid crest, and the medial, more upon the medial aspect of this crest and directly adjoining it. Insertion was upon the ridge on the medial ulna just below the sigmoid cavity, in company with the long head of the biceps.

Innervation of the brachial flexors, comprising the ventral group, is by the musculocutaneous nerve.

Muscles of the antibrachium.

Dorsal group.

M. brachioradialis (supinator longus). Parsons has stated that this muscle is present in most sciuromorphs, but it was absent in *Dipodomys*, as it was in the wood rat. In *Scirtopoda* it was slender and, as Parsons mentioned, insertion was not upon the radius but upon the medial process at the base of the pollex. In *Allactaga*, however, insertion was upon a process upon the medial side of the distal radius. The muscle was present in *Zapus*.

M. extensor carpi radialis longus. In *Dipodomys* this, the most radial muscle upon the lateral forearm, took origin from the lateral epicondyle partly deep to the brevis and inserted upon the medial base of metacarpus 2. In the jerboas it was smaller and origin was not partly deep to that of the brevis, while insertion was upon the middle of the medial side of metacarpus 2.

M. extensor carpi radialis brevis. In all three animals origin was from the lateral epicondyle distal to, and in *Dipodomys* partly superficial to, origin of the longus. Insertion was upon the lateral side of metacarpus 2.

M. extensor digitorum communis. Origin was narrowly from the lateral epicondyle adjacent to the extensor brevis, and tendons extended to digits 2, 3 and 4. In addition there was a partially separate slip which arose ulnarward from the communis aponeurosis and passed with the other tendons to insert upon the medial side of digit 5. In the jerboas insertion was upon the four lateral digits.

M. extensor digiti quinti. This was slender and arose from the lateral epicondyle and ulnarward of the extensor communis. Insertion was upon the lateral side of digit 5, and in *Allactaga* there was also a tendon to digit 4, the latter running deep to the communis tendon.

M. extensor carpi ulnaris. In *Dipodomys* origin was by two slips, one from the posterior border of the lateral epicondyle and the other from the lateral base of the olecranon. These soon fused and insertion was upon the process at the base of metacarpus 5, as was the case in the jerboas as well.

M. supinator. In *Dipodomys,* origin of this muscle, situated radialward of the extensor metacarpi pollicis, was largely by tendon from the capsule of the joint, but there had developed at this point a well defined sesamoid bone, located directly lateral to the radial head. Insertion was upon the medial aspect of one-third of the dorsal border of the radius. In the jerboas there was no sesamoid, but in *Scirtopoda* there was a well defined lateral process upon the radial head, and in *Allactaga* origin was more definitely from the humerus than in the other two.

M. extensor metacarpi pollicis (abductor pollicis longus). Origin was by two heads, both fleshy, from the proximal third of the lateroventral shaft of the radius and from two-fifths of the laterodorsal ulna below the joint. These soon joined and the common tendon slanted over the radial border superficial to the two extensor carpi radialis tendons to insert, in *Dipodomys* and *Scirtopoda,* upon the falciform bone of the carpus, rather than upon metacarpus 1. In *Allactaga* the chief attachment was also to the falciform but a tendinous band passed to metacarpus 1 as well.

M. extensor indicis. This was quite slender and situated just deep to the extensor digiti quinti. Origin was from the lateral border of the ulna and in *Dipodomys* this was bound down by a stout ligament which extended from the epicondyle for a considerable distance to an attachment upon the lateral ridge of the ulna. The tendon, passing deep to that of the extensor communis, was inserted mainly upon the dorsum of digit 2, but a slender branch also passed superficial to the extensor carpi radialis longus tendon to an insertion upon digit 1. In *Scirtopoda* the tendon was even smaller and the branch to digit 1 was stouter than that to 2. In *Allactaga* the ligamentous band over origin was not present and insertion was upon the lateral side of digit 2 only.

Innervation of all the muscles of the dorsal group of the antibrachium is by the radial nerve.

M. pronator teres. Origin was uniformly from the medial epicondyle and insertion upon the proximal third of the radial border of the radius.

M. flexor carpi radialis. Origin was from the common epicondylar tendon with insertion upon metacarpus 2. In *Scirtopoda* the tendon of insertion was exceedingly slender.

M. palmaris longus. Origin was from the medial epicondyle between the flexores carpi radialis and ulnaris, with insertion into the palmar fascia.

M. flexor digitorum sublimis. Origin was from the epicondyle and apparently the joint capsule, and in *Dipodomys* insertion was upon digits 1 to 4. In the jerboas only digits 2 to 4 were involved.

M. pronator quadratus. In *Dipodomys* there were only a few interosseous fibers representing this muscle, and in the jerboas definition was but slightly better.

Innervation of the pronators, palmaris longus, flexores carpi radialis and digitorum sublimis is by the median nerve.

M. flexor digitorum profundus. In *Dipodomys* and *Scirtopoda* origin was by four heads (1) from the medial epicondyle by means of the aponeurosis common to it and the flexor carpi radialis, (2) from the sigmoid border, (3) a robust head from the medial ulna, and (4) a weak one from the adjoining part of the radius. They joined to form a very heavy tendon, branches of which extended to all five digits. In *Allactaga*, however, no fourth head was detected. Innervation is by both median and ulnar nerves.

M. flexor carpi ulnaris. Origin was from the ventral part of the olecranon and the ulnar border of the ulna, while insertion was upon the pisiform. Innervation is by the ulnar nerve.

M. epitrochleo-anconeus. Whether a triceps anconeus, innervated by the radial nerve, ever occurs upon the medial elbow of rodents as it does in some other mammals (some primates), I have not determined to my own satisfaction. It is only recently that I have concerned myself with this detail, and although authors have almost always, either by direct statement or implication, placed this slip with the triceps group, with radial innervation, in all rodents (including *Dipodomys* and *Allactaga*) in which I have sought the innervation this has been by the ulnar nerve, allocating the muscle with the two muscles of the forearm so served. In all three saltators it was well developed, with a well rounded contour that rendered it impossible to mistake it for a distal continuation of the coracobrachialis, an error easily made in some other mammals. Origin was from the entepicondyle and insertion upon the olecranon.

No dissection of the muscles of the manus proper was attempted in the present connection. It was felt that with such small specimens there would be too large an element of doubt regarding many of the details and accordingly it was decided to make no attempt to describe them.

B. Pelvic limb.

It is to the posterior extremity to which we turn with greatest interest in any consideration of saltatorial mammals, and this feature will be discussed at some length. In this connection it will be advisable to examine the lumbar plexus (fig. 11). Unfortunately it was found that in the specimens of *Scirtopoda* available the proximal nerves of this plexus were so fragile and so prone to separate into their component fibers that the plan could not be satisfactorily determined.

Dipodomys. N. femoralis was from L. III and IV only. It pierced the psoas major before it appeared, sending twigs to the psoas, iliacus, and then a number to the pectineus and quadriceps femoris before continuing down the medial thigh as the saphenus nerve. The n. obturatorius was derived from L. III, IV and V. The n. ischiadicus was made up of branches from L. III to VII inclusive, which, for a rodent, appears to be an unusual number of elements. It would, perhaps, be preferable to designate by name the group of nerve twigs from the ischiadicus which serves the hamstring muscles, as these were rather sharply differentiated, but this is not attempted at present. The ischiadicus does not separate to form the tibialis and peronoeus communis until well over the hip. The n. pudeneus was weak and came only from L. VII.

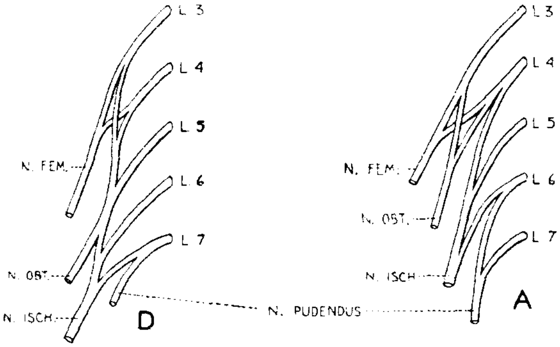


FIGURE 11. The lumbo-sacral plexus of Dipodomys (D) and Allactaga (A).

Allactaga. In the jerboas the femoral nerve was derived from L. III and IV as well, but instead of piercing the psoas major it emerged between that muscle and the quadratus lumborum. The n. obturatorius was also derived from these roots and had no connection with V. The n. ischiadicus came from L. IV, V and VI only, but the n. pudendus received a minute branch from L. VI, as well as the larger one from VII.

(*a*) EXTRINSIC SYSTEM.

Dorsal group.

Although the psoas complex is frequently included in the hypaxial system because topographically it is so situated and this course is often convenient, in reality it is a derivative of the extremity musculature and has secondarily invaded this region.

M. psoas minor. The anterior portion of this muscle was so tenuous that it was difficult to decide just where origin began. It may be said, however, that in *Dipodomys* it was from the first three lumbars, while in the jerboas it was more extensive, being from possibly the last thoracic as well as the first four lumbars. A broad, thin tendon developed upon the ental belly and upon this the fibers ended rather abruptly. Insertion was upon the psoas process of the innominate. Innervation is by short branches from the lumbar plexus.

M. psoas major. This muscle was very well developed, although not remarkably so. In *Dipodomys* and *Scirtopoda* origin was from the first five lumbars, while in *Allactaga* the first of these seemed not to be involved. The muscle was not quite so robust in *Scirtopoda* as in the other two animals. Insertion was upon the lesser trochanter of the femur in company with the iliacus. Innervation is by short branches from both the individual trunks of the lumbar plexus and the femoral nerve.

M. iliacus. In *Dipodomys* origin was from practically the whole of the inferior "gluteal" fossa of the ilium as far caudally as just below the rectus process. At insertion fusion with the psoas magnus took place and the whole had attachment to the anterior expanse of the lesser trochanter. In *Scirtopoda* origin was more extensive anteriorly, reaching the crest of the ilium, but the muscle as a whole was less robust. In *Allactaga* it was smaller still, and origin was from the inferior border only of the ilium. Innervation is by the femoral nerve.

(*b*) INTRINSIC SYSTEM.

Muscles of the hip and thigh.

Dorsal group.

M. vastus lateralis (or externus). In *Dipodomys* this muscle was greatly developed as is usual in rodents. It arose from the distal border of the greater trochanter, with insertion upon the patella, and at origin it did not completely cover the vastus medialis. In the jerboas, however, it did cover the latter muscle and the origin was located a trifle more posteriorly. Origin merely adjoined that of the gluteus minimus while in *Dipodomys* the tendon of origin of the latter was superficial to the vastus lateralis at origin.

M. vastus medialis. While this muscle in *Dipodomys* was no more massive than usual it was more extensive at origin. This was not only from the anterior surface of the lesser trochanter and the shaft adjoining, but passed dorsal to the head, onto the greater trochanter and to its caudal border, thus reappearing from beneath the caudal part of the vastus lateralis origin. Fibers of this posterior part inserted upon an aponeurosis that covered most of the vastus femoris, although the main insertion was medially upon the patella. In the jerboas this division was smaller and with origin less extensive.

M. vastus femoris (or intermedius). This arose over the anterior shaft both medially and laterally, and insertion may be said to have been upon the capsule of the knee joint, although fibers stopped short of the joint proper, and the muscle must accordingly be considered as a trifle degenerate. At origin the relationship of the more proximal fibers was extremely close with those of the lateralis, so that it was difficult to determine where origin of one left off and the other began. In *Scirtopoda* fibers extended more definitely upon the capsule. In *Allactaga* origin did not extend so far proximal and insertion was more definitely tendinous upon the capsule.

M. rectus femoris. Origin was from the rectus femoris process of the ilium, practically absent in *Allactaga*, and insertion was as usual upon the proximal patella.

Innervation of the vasti and rectus femoris, comprising the quadriceps femoris, is by the femoral nerve.

Usually, in the case of rodents, it is stated that there is a sartorius and a tensor fasciae latae, or when there is no separation, that the two have fused. In discussing the wood rat I pointed out that this mass could not be divided and as there was no innervation by the femoral nerve, the sartorius element must be considered as absent.

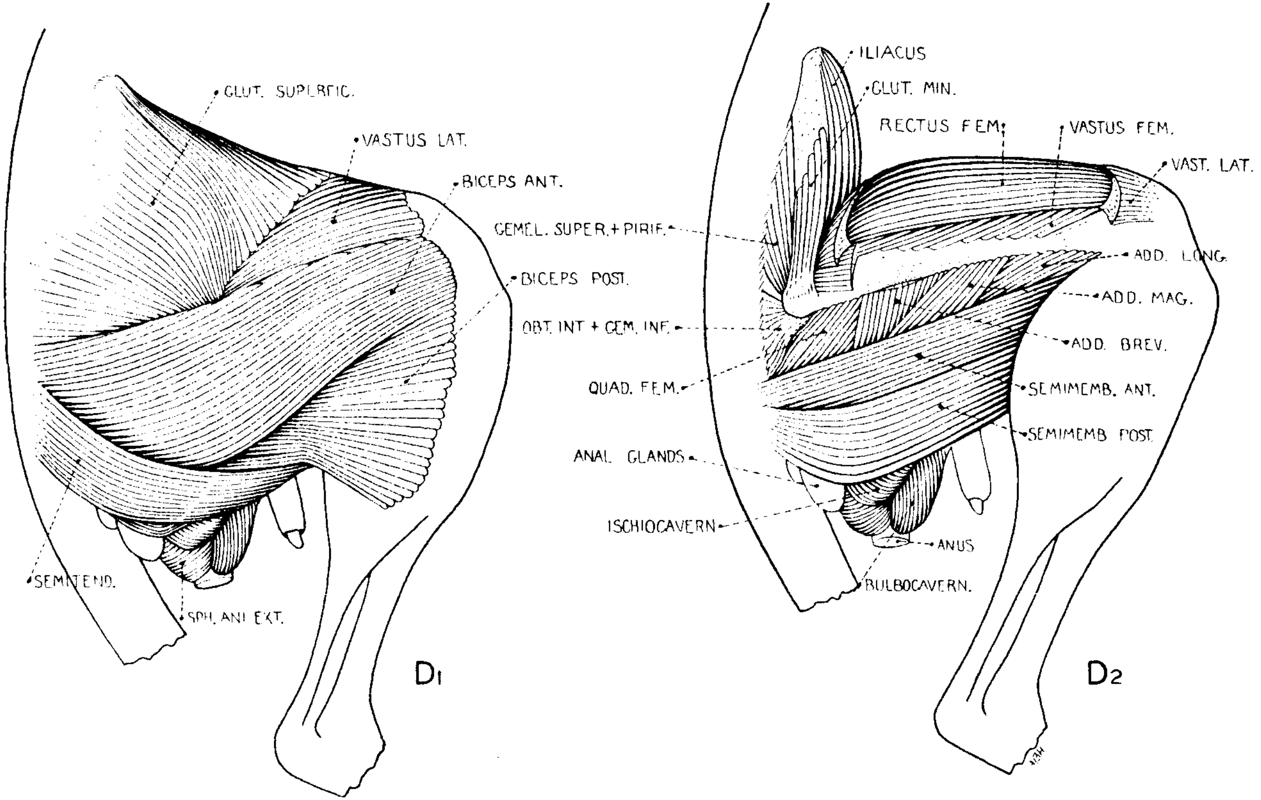

GLUT. SUPERFIC.
VASTUS LAT.
BICEPS ANT.
GEMEL. SUPER.+PIRIF.
BICEPS POST.
OBT. INT + GEM. INF.
QUAD. FEM.
ANAL GLANDS
ISCHIOCAVERN.
SEMITEND.
SPH. ANI EXT.
D1
ILIACUS
GLUT. MIN.
RECTUS FEM.
VASTUS FEM.
VAST. LAT.
ADD. LONG.
ADD. MAG.
ADD. BREV.
SEMIMEMB. ANT.
SEMIMEMB. POST.
ANUS
BULBOCAVERN.
D2

FIGURE 12. Muscles of the right thigh of Dipodomys: superficial (D_1) and deeper (D_2) (with gluteus maximus and medius removed) layers.

Innervation was exclusively by the superior gluteal nerve and hence the tensor fasciae latae was the only element present. A similar condition was encountered in the three saltatorial rodents dissected. In three specimens of jerboas and several of *Dipodomys* in which this detail was examined I could discover no service by the femoral nerve and I shall continue calling this sheet as follows:

M. gluteus superficialis. In *Dipodomys* origin was from the dorsal aponeurosis and from the iliac crest, while insertion was toughly upon the proximal part of the lateral femoral crest and the fascia covering the anterior thigh, the muscle fibers not reaching to the knee. Although the appearance was exactly what one would expect to result from fusion of a tensor latae with a sartorius, the evidence, I think, is entirely conclusive that the former element was the only one present, this superficial division of the gluteus mass having been specialized in an interesting manner to assume at least in part the function of a nonexistent sartorius. Attention may also be called to the statement by Parsons (1894) that the sartorius which he found in the rodents he dissected was served by the superior gluteal nerve, showing that this was *not* the element present.

In *Scirtopoda* the superficial gluteus was smaller, and whereas in *Dipodomys* the posterior border was entirely deep to the biceps, in the former it was superficial proximally, only the more distal portion twisting inward to insert, not upon the lateral crest of the femur, which the jerboas lack, but upon the deep aponeurosis between the vasti and the muscles posteriorly situated, and also upon the superficial aponeurosis over the front of the thigh as usual. The anterior part was partially divisible into a slip with a singular resemblance to a sartorius, arising from the inferior iliac border adjoining the crest, thence spreading well over the medial thigh; but absence of femoral innervation and the following of its nerve supply to junction with the ischiadicus proved this resemblance to be only superficial.

In *Allactaga* the lateral area of the muscle was even smaller than in *Scirtopoda* and no part of it was situated superficial to the biceps femoris border. The partially divisible section resembling a sartorius also extended more ventrocaudally upon the ilium, although the belly was less extensive over the medial thigh. The innervation was the same.

The partial division of this sheet in the jerboas is very interesting and illustrative, perhaps, of a rather successful attempt to differentiate a portion of the gluteal mass to take over the needed function fulfilled by a sartorius were one present.

At origin the main gluteal complex was largely inseparable, although the divisions were entirely distinct toward insertion.

M. gluteus medius. This was by far the most massive part of the gluteal complex. In *Dipodomys* origin was slightly from the fascia investing the iliocostalis, but mostly from the anterior part of the superior gluteal fossa of the ilium, and for a short distance ventrally along the crest. Insertion was robustly upon the greater trochanter superior to the vastus lateralis. In the jerboas origin was somewhat less extensive.

M. gluteus minimus. In *Dipodomys* origin was from practically the entire superior gluteal fossa of the ilium and caudally almost to the acetabulum, while insertion was upon the cranial border of the greater trochanter. In *Scirtopoda* origin was somewhat less extensive ventrally, allowing space for the scansorius. In *Allactaga* this division was fused with the gemellus superior and piriformis.

M. scansorius (or gluteus quartus). This was present in *Scirtopoda* only, with origin from the ventral part of the inferior gluteal fossa of the ilium with insertion upon the greater trochanter adjacent to the gluteus minimus.

Innervation of the above gluteal components is by the superior gluteal nerve.

M. gluteus maximus. In *Dipodomys* origin was mostly from the dorsal fascia above the ilium. It passed posterior to the femoral head as usual and insertion was upon the posterior border of the greater trochanter and distally to the base of the lateral crest or third trochanter. In the jerboas this muscle was smaller and insertion was more toward the base of the greater trochanter. Innervation of the gluteus maximus is by the inferior gluteal nerve.

M. piriformis. In all three animals this muscle had experienced at least partial fusion with the gemellus superior. In *Dipodomys* the two were virtually inseparable and the piriformis could be distinguished only by the fact that its fibers arose from the transverse processes of the third and fourth sacral vertebrae. Insertion was by tendon upon the greater trochanter. This was also the case in *Allactaga*, while in *Scirtopoda* separation of all but the insertional tendon was possible. The piriformis is innervated by short branches of the sacral nerves, not detected in the present instance.

M. biceps femoris anticus. This corresponded to the short femoral head in man, for it was served by the n. peroneus communis. It was very robust and in *Dipodomys* was simple, arising from the dorsal aponeurosis over the ischium and inserting upon the patellar

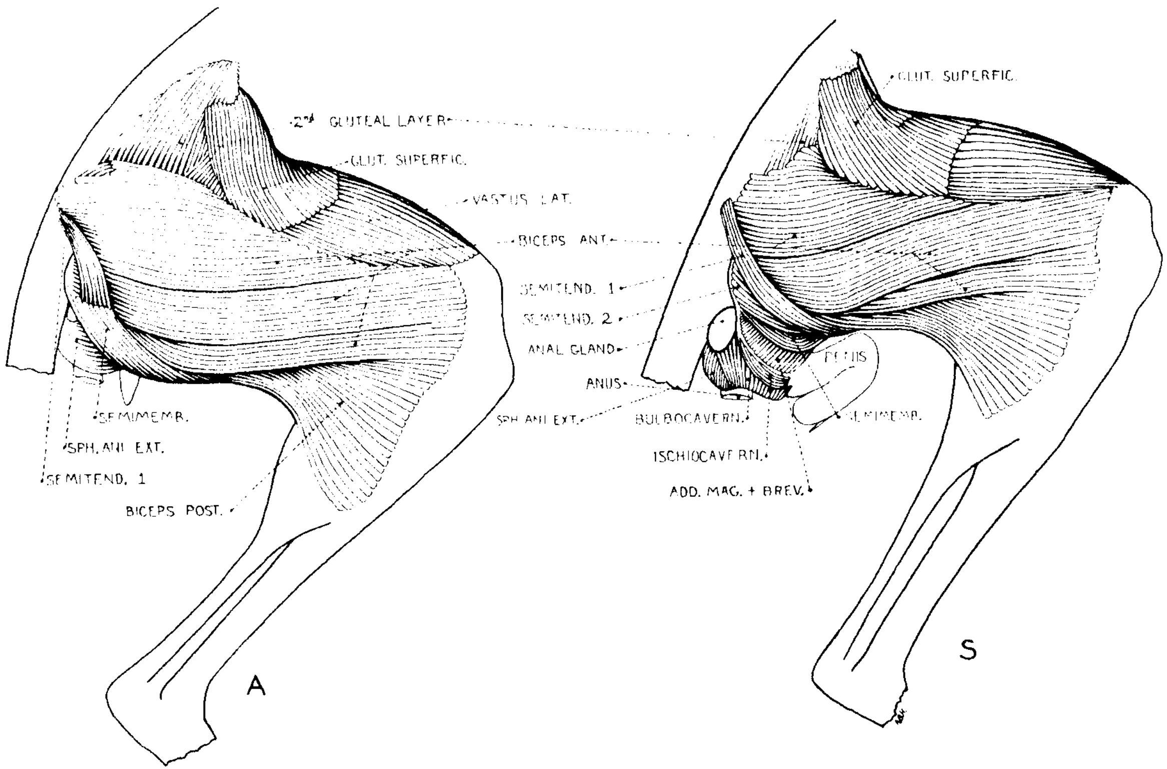

2nd GLUTEAL LAYER
GLUT. SUPERFIC.
VASTUS LAT.
BICEPS ANT.
SEMITEND. 1
SEMITEND. 2
ANAL GLAND
ANUS
SPH. ANI EXT.
BULBOCAVERN.
ISCHIOCAVERN.
ADD. MAG. + BREV.
PENIS
SEMIMEMB.
GLUT. SUPERFIC.
SEMIMEMB.
SPH. ANI EXT.
SEMITEND. 1
BICEPS POST.
A
S

FIGURE 13. Superficial muscles of the region of the right thigh of Allactaga (A) and Scirtopoda (S).

ligament. In all these rodents the two main heads of the biceps were considered to be distinct throughout their length, although actually their insertional parts were fairly continuous. In the jerboas this part was not only very broad but was divisible (with less ease in *Scirtopoda* than in *Allactaga*) into three portions. The first of these was composed of those fibers which arose from the dorsal fascia and from the aponeurosis between the dorsum and ischium, and inserted upon the deep aponeurosis adjacent to the vasti. The second part comprised those fibers which arose by tendon from the tuber ischii and inserted over the patella; and the third, those fibers farther posterior, arising from the aponeurosis common to this and a part of the biceps posticus, with insertion largely over the patellar ligament. In *Scirtopoda* the first and second of these parts were not surely separable from the posticus. In *Allactaga* the situation was rather peculiar in that only a part of the second division of the biceps anticus came into contact superficially with the semimembranosus, where these bridged over the semitendinosus, and at this point there was definite fascicular fusion of a number of their fibers. Innervation is by a twig diverging from the ischiadicus well above the point where it separates to form the peroneus and tibialis, but its affinity is doubtless with the former.

M. tenuissimus. The phylogeny of this muscle is still undecided but as many anatomists believe that it has been derived from the short biceps it is here placed for the present. It was encountered in *Dipodomys* only, in the long, extremely tenuous form which it invariably assumes. It arose from the deeper dorsal fascia deep to the anterior biceps origin, with fascial insertion upon the medial calf. Innervation is by a long, exceedingly tenuous filament diverging from the ischiadicus well beneath the gluteal mass.

Ventral group.

M. gemellus superior. As previously stated this muscle was almost completely fused with the piriformis. In *Dipodomys* it could be distinguished from the latter only by the fact that it arose from the middle of the superior iliac border as far caudally as the distinctive process there situated, as contrasted to the piriformis origin from the transverse processes. Insertion was tendinous upon the greater trochanter. In *Scirtopoda* origin extended almost to the iliac crest and separation of all but the insertional part was possible. In *Allactaga* fusion with the piriformis, as well as with the gluteus minimus, was more marked.

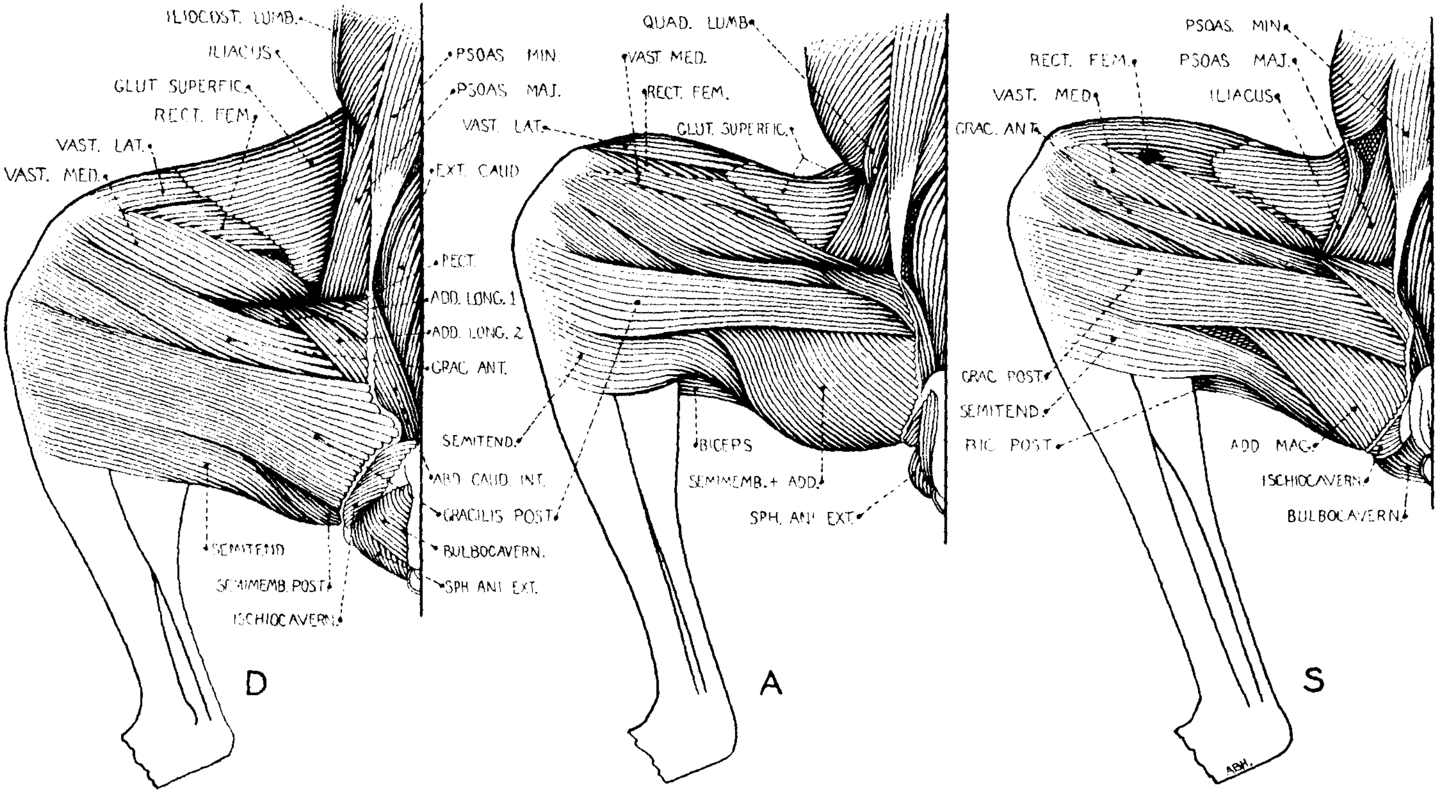

ILIOCOST. LUMB.
ILIACUS
GLUT. SUPERFIC.
RECT. FEM.
VAST. LAT.
VAST. MED.
SEMITEND.
SEMIMEMB. POST.
ISCHIOCAVERN.
D
PSOAS MIN.
PSOAS MAJ.
VAST. LAT.
EXT. CAUD.
PECT.
ADD. LONG. 1
ADD. LONG. 2
GRAC. ANT.
SEMITEND.
ABD. CAUD. INT.
GRACILIS POST.
BULBOCAVERN.
SPH. ANI EXT.
QUAD. LUMB.
VAST. MED.
RECT. FEM.
GLUT. SUPERFIC.
BICEPS
SEMIMEMB. + ADD.
SPH. ANI EXT.
A
PSOAS. MIN.
RECT. FEM.
PSOAS MAJ.
VAST. MED.
ILIACUS
GRAC. ANT.
GRAC. POST.
SEMITEND.
BIC. POST.
ADD. MAG.
ISCHIOCAVERN.
BULBOCAVERN.
S
ABH.

FIGURE 14. Superficial muscles of the region of the right thigh of Dipodomys (D), Allactaga (A), and Scirtopoda (S), from the medioventral aspect.

M. gemellus inferior. The relationship of this muscle with the quadratus femoris and tendon of the obturator internus was very close. In *Dipodomys* the main part of origin was from the ischial border *anterior* to the notch for the internal obturator tendon, but the muscle had extended slightly to the rear of this point as well, thus covering the obturator tendon and therefore with its fibers originating from the deep sacral fascia. Insertion was upon the obturator tendon. In the jerboas origin was as usual from posterior to the regular position of the obturator tendon (absent in these animals), with insertion more definitely into the obturator fossa of the femur.

M. obturator internus. In *Dipodomys* this muscle was double, just as Parsons (1898) reported in *Pedetes*. The proper, more posterior part was less extensive than usual, covering only the caudal part of the obturator foramen. The accessory part, entirely separate, occupied the well defined fossa situated medial to the acetabulum, and sent a separate tendon over the obturator notch, which was especially well defined in this genus. No muscle fibers could be observed from the lateral side and the two tendons, joining, inserted into the obturator fossa. In the jerboas this muscle was very decadent and the part passing over the pelvic brim had so nearly disappeared that no tendon could be detected.

M. quadratus femoris. In *Dipodomys* this presented the appearance of being in two portions, although in reality these were indivisible, by virtue of the fact that part of the origin was tendinous from the cranial part of the lateral tuber ischii ridge, while insertion was fleshy upon the proximal base of the third trochanter. In the remainder of the muscle these conditions were reversed, for origin was fleshy and extensive, from almost all of the ischium posterior to the obturator foramen, while insertion was tendinous upon the lesser trochanter and its proximal ridge. In the jerboas this condition did not obtain and the muscle was simple in appearance: nor was it so robust as in *Dipodomys*. Origin covered about the same area but insertion was only upon the tip of the lesser trochanter.

The gemelli, internal obturator and quadratus femoris are all served by short twigs of the ischiadicus.

M. semitendinosus. This muscle arose by two heads. In *Dipodomys* origin of the fascial head was from the dorsum, and of the ischial, from the tuber. They shortly fused and there then occurred a completely transverse aponeurotic inscription. From this the main portion of the muscle continued to the medial calf, but there was

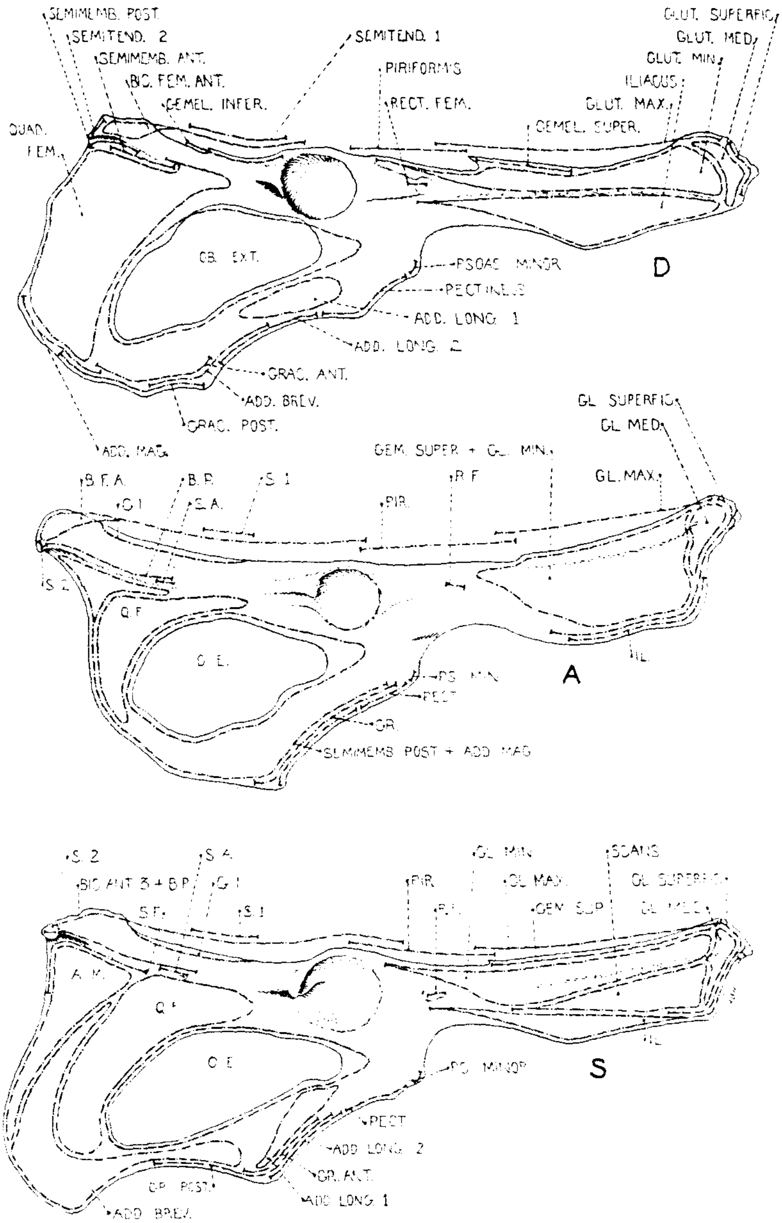

SEMIMEMB. POST.
SEMITEND. 2
SEMIMEMB. ANT.
BIC. FEM. ANT.
GEMEL. INFER.
SEMITEND. 1
PIRIFORMIS
RECT. FEM.
GEMEL. SUPER.
GLUT. SUPERFIC.
GLUT. MED.
GLUT. MIN.
ILIACUS
GLUT. MAX.
QUAD. FEM.
OB. EXT.
PSOAS MINOR
PECTINEUS
ADD. LONG. 1
ADD. LONG. 2
GRAC. ANT.
ADD. BREV.
GRAC. POST.
ADD. MAG.
D
GL. SUPERFIC.
GL. MED.
GEM. SUPER. + GL. MIN.
GL. MAX.
B. F. A.
B. P.
S. 1
R. F.
G. I
S. A.
PIR.
S. 2
Q. F.
O. E.
IL.
PS. MIN.
PECT.
GR.
SEMIMEMB. POST. + ADD. MAG.
A
S. 2
S. A.
GL. MIN.
SCANS.
BIC. ANT. 3 + B. P.
G. I
PIR.
GL. MAX.
GL. SUPERFIC.
S. F.
S. 1
R. F.
GEM. SUP.
GL. MED.
A. M.
Q. F.
O. E.
IL.
PS. MINOR
S
PECT.
ADD. LONG. 2
GR. ANT.
GR. POST.
ADD. LONG. 1
ADD. BREV.

FIGURE 15. Right innominate bone of Dipodomys (D), Allactaga (A), and Scirtopoda (S) showing, semidiagramatically, the areas of muscle attachment.

a small slip which diverged to fuse with the posterior biceps and pass to the lateral thigh. It is interesting to note that a precisely similar slip was encountered in the sea otter (*Enhydra*). In the jerboas there was no transverse aponeurotic inscription. Save for being narrow conditions were otherwise essentially similar in *Scirtopoda*. In *Allactaga*, however, as the superficial division crossed the ischial border it disappeared from sight between the semimembranosus and the biceps, reappearing once more from between these muscles upon the middle of the thigh, where the superficial head joined the deeper head. The origin of the latter was not only completely hidden by the other division, but even after removal of the latter practically none of the deeper division could be seen, because situated mostly deep between the semimembranosus and the biceps.

M. semimembranosus. In *Dipodomys* this also occurred in two divisions. Pars postica, the larger (about as in *Neotoma*), arose from the ischium along the lateral ridge extending from the tuberosity. Upon the medial aspect of the limb it passed deep to the gracilis and inserted by fascia upon the distal part of the patellar ligament. Pars antica arose slightly below (deep to) and anterior to the postica origin. Although origin was by a single aponeurosis there were really two divisions of this part, the larger and more medial inserting fleshily under cover of the anterior gracilis upon the medial epicondyle of the femur, and the second slip upon the base of the femoral shaft adjacent to, but more toward, its lateral side.

In *Scirtopoda* both divisions were small but distinct. Origin of the more posterior was from the ventral ridge of the tuber ischii and ventrally slightly along the caudal border of the bone. Origin of the anterior division was by a slender tendon from the cranial part of the tuber ridge. It then divided into two small slips, the more lateral inserting upon the posterior part of the extreme distal femoral shaft and the other more medially. In *Allactaga* a peculiar condition was encountered. It was possible easily to differentiate a single anterior division, thin but with very coarse fibers extending from the ischium to the femoral shaft, like the more superficial division of *Scirtopoda*, but here differentiation ceased. The fibers were so coarse and loose that perfect separation was possible at many points, but all were parallel and although it was possible to distinguish the different elements present, the posterior semimembranosus and all of the adductors proper had to all intents become one powerful muscle and must so be treated. Parsons mentioned the partial blending of the

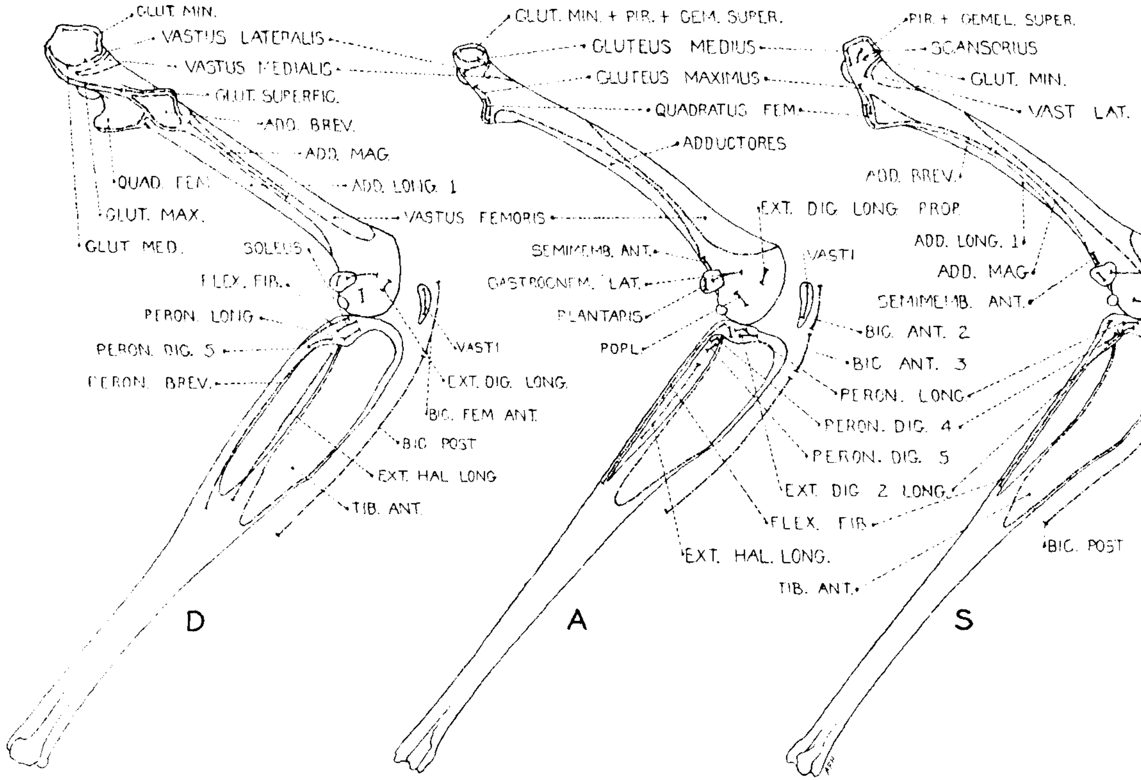

GLUT. MIN.
VASTUS LATERALIS
VASTUS MEDIALIS
GLUT. SUPERFIC.
ADD. BREV.
ADD. MAG.
ADD. LONG. 1
QUAD. FEM.
GLUT. MAX.
GLUT. MED.
SOLEUS
FLEX. FIB.
PERON. LONG.
PERON. DIG. 5
PERON. BREV.
VASTI
EXT. DIG. LONG.
BIC. FEM. ANT.
BIC. POST
EXT. HAL. LONG.
TIB. ANT.
D
GLUT. MIN. + PIR. + GEM. SUPER.
GLUTEUS MEDIUS
GLUTEUS MAXIMUS
QUADRATUS FEM.
ADDUCTORES
VASTUS FEMORIS
SEMIMEMB. ANT.
GASTROCNEM. LAT.
PLANTARIS
POPL.
A
PIR. + GEMEL. SUPER.
SCANSORIUS
GLUT. MIN.
VAST. LAT.
ADD. BREV.
EXT. DIG. LONG. PROP.
ADD. LONG. 1
ADD. MAG.
SEMIMEMB. ANT.
VASTI
BIC. ANT. 2
BIC. ANT. 3
PERON. LONG.
PERON. DIG. 4
PERON. DIG. 5
EXT. DIG. 2 LONG.
FLEX. FIB.
EXT. HAL. LONG.
TIB. ANT.
BIC. POST
S

FIGURE 16. Lateral aspect of the bones of the right thigh and shank of Dipodomys (D), Allactaga (A), and Scirtopoda (S) showing the areas of muscle attachment.

several elements in *Scirtopoda* but the tendency seems to culminate in *Allactaga*. The details will be presented under the adductors.

M. biceps femoris posticus. In *Dipodomys* this arose narrowly by aponeurosis from the tuber and spread to an insertion over the shank, as illustrated (fig. 12). It should be noted that in this division the origin was narrow and insertion broad, while in the anterior biceps the opposite condition prevailed. In the jerboas this posterior part was very narrow.

Innervation of the semitendinosus, semimembranosus, and biceps posticus is by twigs of the ischiadicus which separate high up on the trunk and pass deep to the origin of the anterior semimembranous before joining the muscles.

Mm. adductores femoris. The femoral adductors of rodents often prove to be very exasperating for the reason that there appears to be no certain manner in which they can be homologized with the accepted longus, brevis, and magnus divisions of man. *Dipodomys* was no exception to this rule, but if it were not for the short slip adjacent to the pectineus, in position and appearance from the superficial aspect entirely similar to the human adductor longus, matters would be simple, for it is possible to distinguish three other adductor divisions which conform essentially to human conditions. As one of these, deep to the questionable slip, is a perfectly good longus I am of the opinion that the former comprises an accessory muscle, but whether it be a derivative of the pectineus or of the true adductor longus I am not prepared to say. Perhaps this is not of great moment, for the part of the pectineus innervated by the obturatorius is nothing but a part of the adductor complex. At any rate I shall for the present retain for this questionable slip the term adductor longus, but with the qualitative term *accessorius*, while for the deeper and more extensive division which in other respects conforms in better degree to the human longus, I shall adopt the name adductor longus *primus*.

The adductor magnus was the most laterally situated. In *Dipodomys* this arose from the caudoventral part of the ischial border and it inserted upon the femoral shaft distal to the third trochanter for about three-fifths of the distance therefrom to the condyle. In *Scirtopoda* origin was from the entire caudal border of the ischium ventral to the tuber ridge, this being more dorsal than in *Dipodomys*. Insertion was upon the shaft over the distal half of the femur. This is a compound muscle, served by both the ischiadicus and obturatorius.

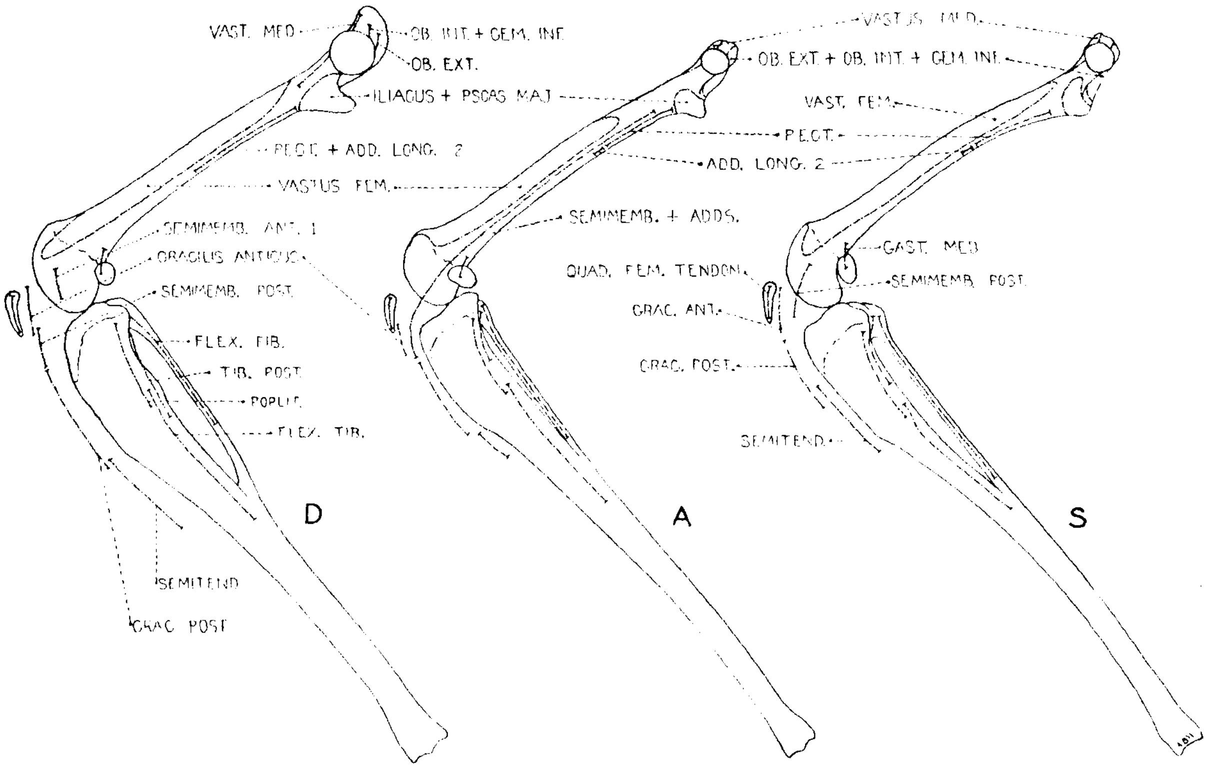

VAST. MED.
OB. INT. + GEM. INF.
OB. EXT.
ILIACUS + PSOAS MAJ.
PECT. + ADD. LONG. 2
VASTUS FEM.
SEMIMEMB. ANT. 1
GRACILIS ANTICUS
SEMIMEMB. POST.
FLEX. FIB.
TIB. POST.
POPLIT.
FLEX. TIB.
SEMITEND.
GRAC. POST.
D
VASTUS MED.
OB. EXT. + OB. INT. + GEM. INF.
VAST. FEM.
PECT.
ADD. LONG. 2
SEMIMEMB. + ADDS.
QUAD. FEM. TENDON
GRAC. ANT.
GRAC. POST.
SEMITEND.
A
GAST. MED.
SEMIMEMB. POST.
S

FIGURE 17. Medial aspect of the bones of the right thigh and shank of Dipodomys (D), Allactaga (A), and Scirtopoda (S) showing the areas of muscle attachment.

The adductor brevis was extremely robust in *Dipodomys,* although origin was thinly from the ventral ischial border between the adductores longus accessorius and magnus, with insertion robustly upon the caudal surface of the third trochanter. In *Scirtopoda* it was also massive, with origin from the ventral two-thirds of the caudal ischial border and almost as far cranially as the symphysis, and insertion upon the proximal half of the femur to and including the caudal surface of the lesser trochanter.

The adductor longus primus arose in *Dipodomys* along the pubic border from the adductor brevis to the pectineus and deep to the longus accessorius and a part of the gracilis anticus, while insertion was along almost the entire shaft of the femur. In *Scirtopoda* the gracilis origin had more thoroughly invaded the space between the two divisions of the longus, but origin was from a comparable area, and insertion was distal to the lesser trochanter for half the distance to the condyles.

The adductor longus accessorius resembled only from the superficial aspect the adductor longus of man, for it was very thin, arising along the pubic border caudal to the pectineus. In *Dipodomys* it ended in a slender tendon which joined the distal part of the pectineal tendon. In *Scirtopoda,* however, the tendon did not actually join that of the pectineus but inserted with it at a point about two-fifths of the distance distally upon the femoral shaft. This slip is served by a twig of the obturator nerve, labelling it indubitably as one of the adductors.

In *Allactaga,* as mentioned previously, the adductor complex and the posterior semimembranosus had experienced such a complete degree of fusion that it was impracticable to separate the component parts with any measure of satisfaction. The whole constituted a very robust muscle arising (the semimembranous part) from the ischium ventral to its tuber ridge and around the ventral border to the pubis at the anterior part of the symphysis. Here, however, origin of the adductor longus accessorius was distinct. Insertion of the semimembranous part was upon the medial epicondyle of the humerus, its fascia continuing onto the tibial head, while attachment of the adductor part continued therefrom proximally along the caudal border of the femoral shaft to and including the lesser trochanter.

M. gracilis. This was double in all three rodents. In *Dipodomys* the posterior division, which was extensive, originated along the caudoventral border of the ischium as far as the anterior part of the symphysis, with insertion along the shank over an area proximal to the semitendinosus, practically as far as the patellar ligament. Origin of the more anterior part was hidden by the adductor longus accessorius and the posterior part by the posterior gracilis. Insertion was upon the patellar ligament cranial to the semimembranosus posticus. Although in *Scirtopoda* the fascia of origin of the posterior gracilis extended as far caudally as in *Dipodomys*, muscle fibers did not begin until farther cranially. Also insertion differed in that it extended deep to a considerable portion of the visible adductor longus. Origin of the anterior division was entirely deep to the posterior, while insertion was the same as in *Dipodomys*. In *Allactaga* even a greater area of the posterior aponeurosis of origin was devoid of muscle fibers.

M. obturator externus. This arose from all of the margin of the obturator foramen except the extreme cranial portion. Insertion was into the trochanteric fossa of the femur.

Innervation of the adductor group, including gracilis and obturator externus, is by the obturator nerve. I hesitate to deny that the adductor magnus was served also by the ischiadicus, but I failed to detect this in the specimens examined.

Dorsoventral complex.

M. pectineus. This is believed to be a compound muscle, its components derived from a dorsal and a ventral element, because innervation is by the femoral, and in most cases by the obturator nerve as well, although it is stated that the latter service is often lacking in man. Twigs from both were demonstrated to occur in *Dipodomys*. Origin was as usual for a short distance along the ventral pelvic border immediately caudal to the psoas process, and insertion was along the proximal part of the femoral shaft.

Muscles of the crus.

Dorsal group.

M. peroneus longus. Origin in *Dipodomys* was from the fibular head in intimate connection with the collateral ligament. Its tendon passed behind the outer malleolus and thence under the peroneal retinacula and between metatarsus 5 and the cuboid, to insert upon the plantar process of metatarsus 1. In *Scirtopoda* this was the smaller of the two peronei, with origin partly fleshy and partly tendinous, the latter giving the appearance of being a direct continuation of a portion of the collateral ligament. The tendon continued to the sole through a groove formed by a well defined process of metatarsus 4 and the cuboid, and this was covered by the lateral tarsal sesamoid. Insertion then was upon the lateral side of the high plantar process of metatarsus 1. Except for the fact that in *Allactaga* no part of the origin appeared to be a continuation of the collateral ligament the situation was similar to that in *Scirtopoda*.

M. peroneus brevis. In *Dipodomys* origin was from the medial border of the fibula for its entire length and continuing onto the tibia at the junction of the two bones, while insertion was upon the base of metatarsus 5. The muscle was absent in the *Dipodidae*.

M. peroneus digiti quarti. In *Dipodomys* this arose from the caudolateral belly of the peroneus digiti quinti and its tendon passed beneath those of the latter muscle and extensor digitorum longus to the lateral side of digit 4, inserting upon the second phalanx. In *Scirtopoda* origin was from the cranial border of the fibular head and slightly from the interosseous membrane, and the tendon experienced a definite thickening just before reaching the digit. In *Allactaga* origin was hardly separable from the peroneus digiti quinti.

M. peroneus digiti quinti. In *Dipodomys* this may be considered to arise from the proximal fibular shaft, although relationship here was very intimate with the other peronei. Insertion was upon the lateral side of digit 5, at the terminal phalanx. In *Allactaga* conditions were much the same, but the muscle does not occur in *Scirtopoda*.

Innervation of the peroneal group is by the superficial peroneal nerve.

M. tibialis anterior. In *Dipodomys* this muscle was very robust. It arose from the entire tibial fossa, very deep, upon the lateral aspect of the bone. Its heavy tendon passed beneath the exceedingly stout transverse ligament of the ankle (Lig. transversus cruris) and then to the medial ankle, beneath another but smaller ligament in company with the tendon of the extensor hallucis, and to insertion upon the middle of metatarsus 1.

In *Scirtopoda* conditions were similar except that at the transverse ligament there was a deeper retaining groove and insertion was upon the distal end of metatarsus 1. The muscle was larger in *Allactaga*. At insertion the tendon divided into two, but one branch did not go to the internal tarsale as Parsons (1894) stated is the case in mammals with a hallux.

M. extensor digitorum longus. In *Dipodomys* origin was by tendon from the lateral epicondyle of the femur near the patellar ligament.

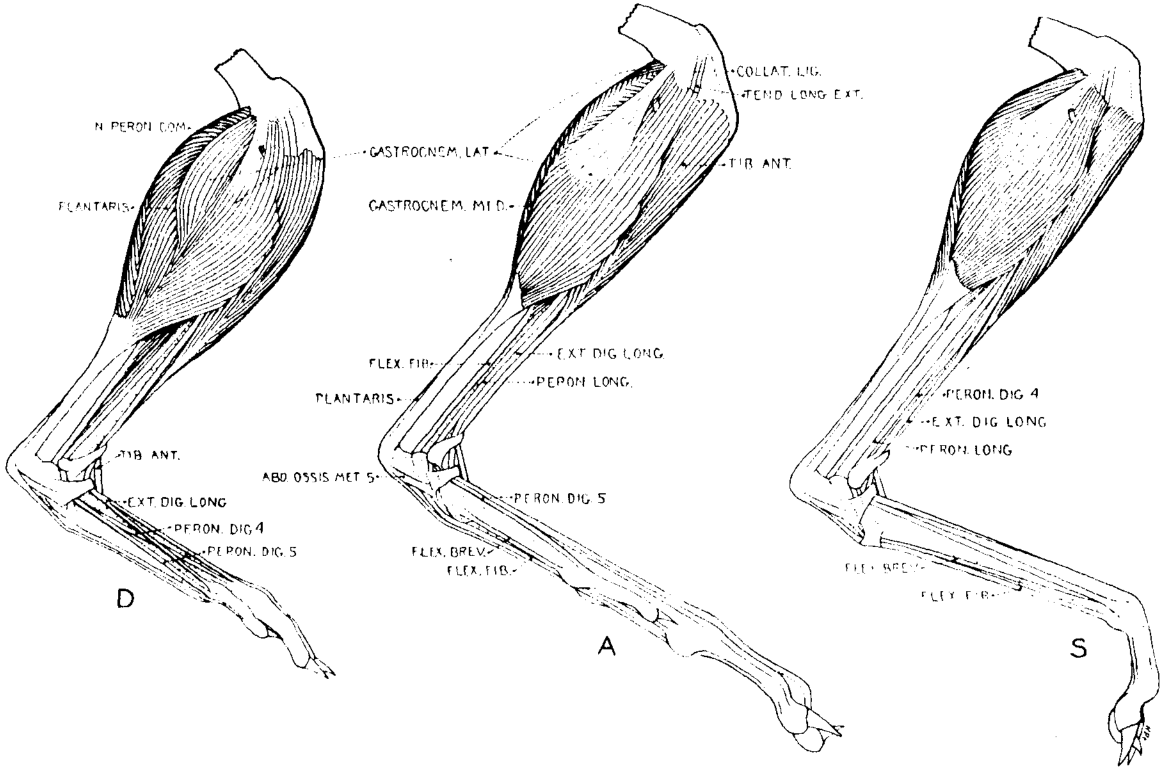

N. PERON. COM.
PLANTARIS
TIB. ANT.
EXT. DIG. LONG
PERON. DIG 4
PERON. DIG. 5
D
GASTROCNEM. LAT.
GASTROCNEM. MED.
FLEX. FIB.
PLANTARIS
ABD. OSSIS MET 5
COLLAT. LIG.
TEND. LONG. EXT.
TIB. ANT.
EXT. DIG. LONG.
PERON. LONG.
PERON. DIG. 5
FLEX. BREV.
FLEX. FIB.
A
PERON. DIG. 4
EXT. DIG. LONG
PERON. LONG
FLEX. BREV.
FLEX. FIB.
S

Figure 18. Lateral view of the superficial musculature of the right shank and foot of Dipodomys (D), Allactaga (A), and Scirtopoda (S).

The tendon extending from the muscle split almost immediately into four branches, passed beneath the transverse ligament in company with the tibialis anterior and extensor hallucis, and extended to the four toes, these branches being superficial to the other tendons and bound down by the fasciculi transversi which connect the extensor hallucis with the peroneus digiti quarti. The main tendons joined the toes upon the lateral side of digit 2, center of 3, and medial side of digits 4 and 5; but the tendon passing to digit 2 gave off a smaller, lateral branch, and that of digit 4 a medial branch, both of which were attached to the sides of digit 3.

Not only were the tendons of the extensor longus which passed to the three toes of *Scirtopoda* perfectly separable throughout their length, but this was also the case with the muscle. These divisions may be described as follows: The extensor digiti secundi longus arose from the cranial aspect of the fibula and was the most cranially situated of the three divisions. Its tendon passed with those of the other two and with that of the tibialis anterior beneath the transverse ligament. Near the toes the three tendons were confined by a thin, tendinous band or vincula, and the one to digit 2 then diverged to the medial side of the toe, but split into two parts at the metatarsal-phalangeal joint, the more medial of these inserting upon the second phalanx and the other upon the third. The extensor digiti tertii longus was situated next caudalward. It arose from the aponeurosis of the third extensor division near the fibular head. The fleshy part was weak and short, and of the crural part fully two-thirds consisted of tendon. Upon the foot the latter passed deep to that of the third division, and beneath the binding ligament near the toes was ensheathed by it. It emerged once more at the base of the third toe and inserted upon the terminal phalanx. The extensor digitorum longus proprius evidently constituted the main part of the long extensor trinity, for it arose by a stout tendon from the lateral epicondyle of the femur. Its tendon passed beneath a strong ligament that stretched from the fibular head to the tibia. At the base of the toes the tendon split, one branch going to the lateral side of digit 2, and the other to the medial side of digit 4, insertion being upon the terminal phalanx. All these tendons were located lateral to the middle of the tarsus, to a strikingly noticeable degree.

In *Allactaga* there were four divisions of the extensor longus muscle. The transverse ligament of the ankle was much more gristly than in *Scirtopoda* and the bony pulley was better developed. The ligamentum cruciatum was also considerably tougher. As the tendons passed along the dorsum of the foot they were situated even more pronouncedly to the lateral side. The extensor digiti secundi longus arose from the tibial head adjacent to the fibula and its slender tendon passed to the medial side of digit 2 and thence, without splitting, to the second phalanx. The extensor digiti tertii longus was as in *Scirtopoda*. The extensor digitorum longus proprius was from the femoral epicondyle, as in *Scirtopoda*, but at the base of the toes the tendon was broader and thinner. It there split into four branches, the two middle ones, to either side of digit 3, appeared to insert upon the second phalanx, while the two others, along the lateral side of digit 2 and the middle of digit 4 respectively, inserted upon the terminal phalanx. The extensor digiti quinti longus diverged from the tendon of origin of the proprius. It was an exceedingly tenuous slip with thread-like tendon to the second phalanx of digit 5.

M. extensor hallucis longus. I prefer to retain this name for the present muscle even though the kangaroo rat dissected has no hallux. It was an exceedingly slender slip arising in *Dipodomys* from the crural interosseous membrane. Its tendon passed beneath the transverse ligament and then diverged with, but cranial to, the tibialis anterior tendon to the medial side of the foot. It then passed beneath a binding ligament near the first metatarsal and extended to the medial side of digit 2, with attachment to the second phalanx. Over the middle of the pes the tendon was joined to that of the peroneus quarti by fasciculi transversi. In *Scirtopoda* the muscle was absent. In *Allactaga* it was even smaller than in *Dipodomys*. As the tendon passed over the ankle it was situated *posterior* to that of the tibialis anterior, and insertion was upon the second phalanx of the hallux.

Innervation of the tibialis anterior, extensores digitorum longus and hallucis longus, is by the deep peroneal nerve.

Ventral group.

Mm. gastrocnemii. In *Dipodomys* these were large and robust. The lateralis arose from the lateral epicondyle of the femur and the lateral sesamoid. At the center of its superficial belly the origin was tendinous, and this diverged to form a small aponeurotic sheet. The fibers posterior to this aponeurosis were not separable near origin but they were toward insertion, and attached to one side of a deeper aponeurosis while the remaining fibers inserted upon the other side of it. Some of these more posterior fibers also had, near their origin, close connection with the medialis, thus suggesting the condition, presumably more primitive, existing in the jerboas. The medialis was much the same as the lateralis save that origin extended for a short distance proximally along the shaft of the femur, and the superficial aponeurosis was more extensive. The deep aponeurosis of the two divisions was just as intricate as in most rodents and fairly continuous with the deep aponeurosis of the plantaris. Just proximal to the true tendon the two aponeurotic sheets fused inseparably and the calcaneal tendon passed to the lateral side of the heel. Just above the latter there occurred a thickening of the tendon for a distance of 3.5 mm. this being attributable to a tendency toward the formation of cartilage, which had not progressed as far as calcification.

In *Scirtopoda* this mass was relatively less robust, with the lateral aponeurosis more extensive and coming more sharply from the lateral sesamoid. The medialis was a trifle smaller than the lateralis and the aponeuroses of the two did not fuse until some 7 mm. distal to the beginning of the true tendon. In the jerboas a part of the lateralis, with fleshy origin proximal to the aponeurotic part, curved to the back of the calf and separated, finally to insert upon the deep aponeurosis of the medialis. This slip was pierced by the twigs of the tibial nerve which served the remainder of the lateralis. Such intimacy of relationship between the fibers of the two divisions is a reflection of conditions that are primitive in certain respects. This slip might in reality be called a gastrocnemius tertius (Frey, 1919). In *Scirtopoda* the stiffening of the distal calcaneal tendon was more pronounced than in *Dipodomys,* with a length of 7 mm., and cleaned skeletons show that calcification has occurred within this more rigid portion for a distance of some 3 mm. In *Allactaga* the lateralis was very slightly less bulky but with the superficial aponeurosis more extensive. Other conditions were entirely comparable with those in *Scirtopoda* except that calcification in the distal calcaneal tendon was farther advanced, with a length of 7 mm. Parsons (1898) has stated that in certain kangaroos and in *Pedetes* the lateral gastrocnemius arises from the outer side of the patella, and reached the conclusion that this is a saltatorial modification.

M. soleus. In *Dipodomys* the tendon of origin, from the fibular head, was relatively no longer than in the average rodent. Distally the tendon of insertion fused with the common gastrocnemial tendon at about the middle of the latter. In the jerboas the tendon of origin was considerably longer, while there was no tendon of insertion,

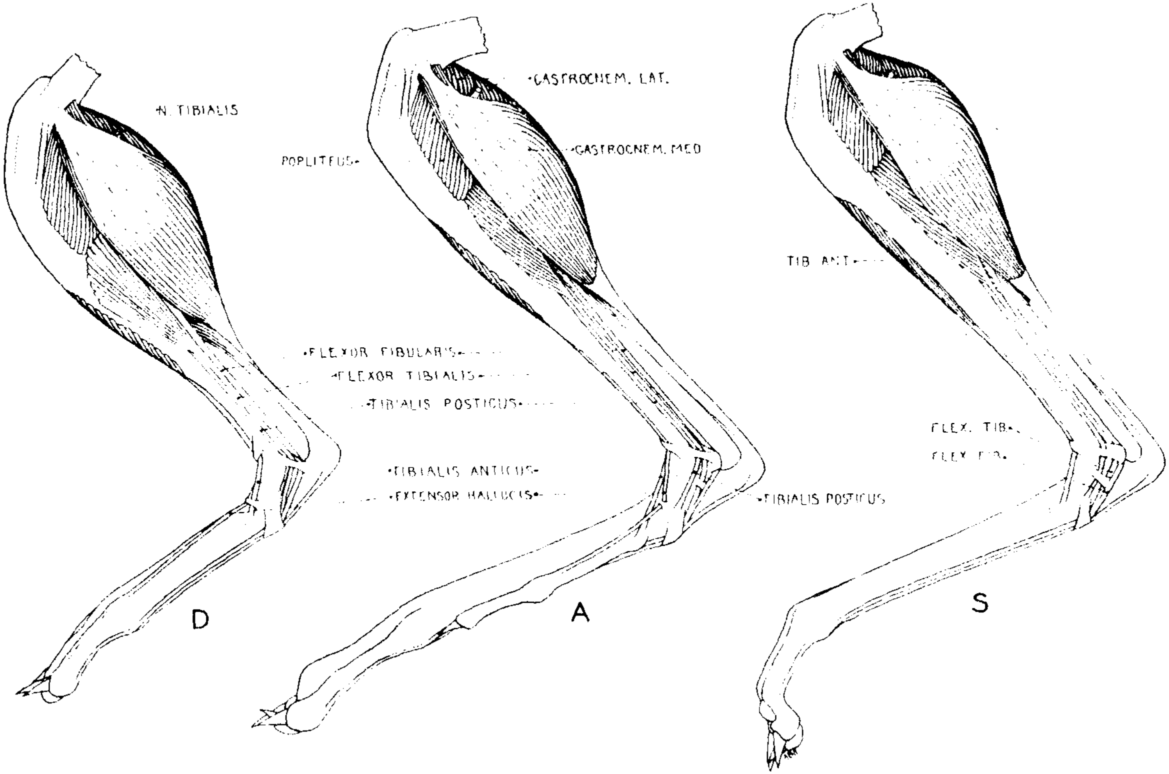

N. TIBIALIS
POPLITEUS
GASTROCNEM. LAT.
GASTROCNEM. MED
FLEXOR FIBULARIS
FLEXOR TIBIALIS
TIBIALIS POSTICUS
TIBIALIS ANTICUS
EXTENSOR HALLUCIS
TIB. ANT.
FLEX. TIB.
FLEX. FIB.
TIBIALIS POSTICUS
D
A
S

FIGURE 19. Medial view of the superficial musculature of the right shank and foot of Dipodomys (D), **Allactaga** (A), and Scirtopoda (S).

attachment of the muscle fibers being directly upon the deep aponeurosis of the lateral gastrocnemius.

M. plantaris. In *Dipodomys* tendon fibers at origin, from the lateral sesamoid, were hardly separable from those of the gastrocnemius lateralis, and its aponeurosis also was fairly continuous with that of the gastrocnemius. It was less than half the bulk of either division of the latter, however, and was thus entirely hidden. Its tendon passed medially and then around the gastrocnemial tendon, over the heel and to the sole beneath a binding ligament ventral to the tarsus. It then split into four true tendons, without intervening fasciculi transversi. Those to the three medial digits were robust, the one to the second appearing from the deep aspect of the common tendon, and each of the three was perforated at the base of the first phalanx to allow the emergence of the flexor fibularis tendon; and attachment was to the base of the second phalanx. The tendon to the fifth toe, however, was much more slender than the others and was not perforated, but merely passed to the lateral side of the digit.

In the jerboas the crural part of this muscle was much the same as in *Dipodomys.* Upon the sole the tendon did not divide into its three branches until it reached the base of the three terminal toes. Near this point it first broadened and the middle branch was partially deep to the other two. Each was perforated as in *Dipodomys.* The branches were much broader in *Allactaga* than *Scirtopoda,* however.

Dobson (1883) has stated that there is no flexor digitorum brevis in *Scirtopoda* (his *Dipus*), and he appears to have indicated the same condition for *Allactaga* and *Dipodomys.* True, in these saltatorial rodents this muscle does not appear as a separate unit. The perforation of the terminal plantaris tendons, however, indicates very clearly that this element is definitely present but that its muscle fibers have diappeared and the remainder become entirely fused with the plantaris.

M. popliteus. Origin was by tendon, in which there was a well formed sesamoid bone, from the lateral epicondyle of the femur beneath the extensor digitorum longus tendon. It diverged, binding down the origins of the long flexors, and inserted upon the border of the popliteal fossa, rather poorly defined, over the area indicated in the illustration (fig. 17). In the jerboas the muscle was a trifle smaller than in *Dipodomys,* with the sesamoid also smaller.

M. tibialis posterior. This was extraordinarily weak and the relationship of its fibers with those of the popliteus was close. Origin was from the fibular head, interosseous membrane, and posteromedial tibial border, partly deep to the popliteus. Its exceedingly slender tendon passed to the medial malleolus just cranial to the flexor tibialis tendon, with the latter through its groove and to the medial tarsal sesamoid. In the jerboas it was even more slender than in *Dipodomys*—nothing but a thread of a muscle.

M. flexor tibialis (flexor digitorum longus). In *Dipodomys* this was very slender, with tenuous origin from the medial tibial border mostly distal to the popliteus insertion. The tendon passed behind a groove on the medial malleolus and fused with the flexor fibularis tendon. In the jerboas conditions were similar save that fusion with the fibular flexor was not complete for some distance after the two met. I thought that in *Allactaga* the tendon fibers might possibly extend to the hallux but upon tearing the two apart I found that this was not the case.

M. flexor fibularis (flexor hallucis longus). This was the chief muscle concerned with the (largely) involuntary flexure of the toes accompanying flexure of the shank. In *Dipodomys* conditions were similar to those occurring in many rodents. The muscle was relatively very powerful and several times larger than the two other long flexors combined. Origin was from the fibular head medial to the soleus and distally along the shaft, from the interosseous membrane, and from the medial aspect of the posterior shaft of the tibia. The very broad tendon which was developed well proximally passed over the medial malleolus and so to the sole, deep to the plantaris. At about the middle of the sole it split into four very heavy tendons which, passing through the perforations of the plantaris tendons, were inserted upon the terminal phalanges. In the jerboas the situation was largely the same, but the muscle was less robust and with a smaller tendon. Like the plantaris the tendon to the three middle toes was single as far distally as the digital articulations. In *Allactaga*, however, a very slender branch diverged well proximally upon either side to join digits 1 and 5.

Innervation of the muscles of the ventral group of the crus is exclusively by the tibial nerve.

Muscles of the pes.

It is always possible, in the case of small rodents, that an investigator has overlooked muscle remnants represented by vestigeal fibers, but the following were indubitably present in the three genera dissected.

M. flexor digitorum brevis. This was represented by tendons in fusion with the plantaris, as described under the latter muscle.

M. abductor ossis metatarsi quinti. In *Dipodomys* this was chiefly tendinous and joined the calcaneum to the lateral process of metatarsal 5. In *Scirtopoda* it was more truly tendinous, to the vestige of the fifth metatarsal, while in *Allactaga* degeneration of the muscle had progressed even further and it occurred as little more than a ligamentous band.

Mm. lumbricales. In *Dipodomys* these were three in number, very tenuous, in the three intervals between the flexor fibularis tendons. *Allactaga* had two of them, well defined, but they had disappeared in *Scirtopoda.*

Mm. contrahentes digitorum. In *Dipodomys* there were two of these present. They arose by aponeurosis chiefly from the plantar process of the navicular. One was thread-like and extended to the inner side of digit 2, while the other was considerably larger and passed near insertion deep to the short flexor of digit 4, with attachment to both sides of the toe. The fact that insertion was deep (dorsal) to the short flexor introduced an element of doubt about the homology of this slip, but otherwise the two constitute perfectly good adductores digitorum secundi and quarti. Contrahentes were entirely lacking in the jerboas.

Mm. flexores breves (not flexor digitorum brevis). In *Dipodomys* there was a short flexor to each toe, with insertion upon both sides of the digit. The secundi arose mainly by a short tendinous slip from the navicular with an accessory fleshy origin apparently from the inner termination of the first metatarsal. In this connection it should be mentioned that it is extremely difficult, and often impossible, to be entirely positive of the precise tarsal bone to which these short muscles, in a rodent of such small size, are attached. The short flexor to digit 3 arose apparently from the cuboid, while those to digits 4 and 5 originated from the lateral sesamoid, situated plantarward from the base of metatarsus 5. In *Scirtopoda* the short flexors were represented by three stout tendons devoid of muscle fibers and arising from the lateral sesamoid of the tarsus. Origin of the one to digit 4 was the most ventral, and to the second, most dorsal. The one to digit 3 was of medium size and did not split until it reached the metatarsal-phalangeal sesamoid. The other two, however, both split at about one-third of the distance from the heel to the toe, sending a very robust branch to the inner, and a much weaker one to the outer, side of the respective digits. In *Allactaga* also muscle

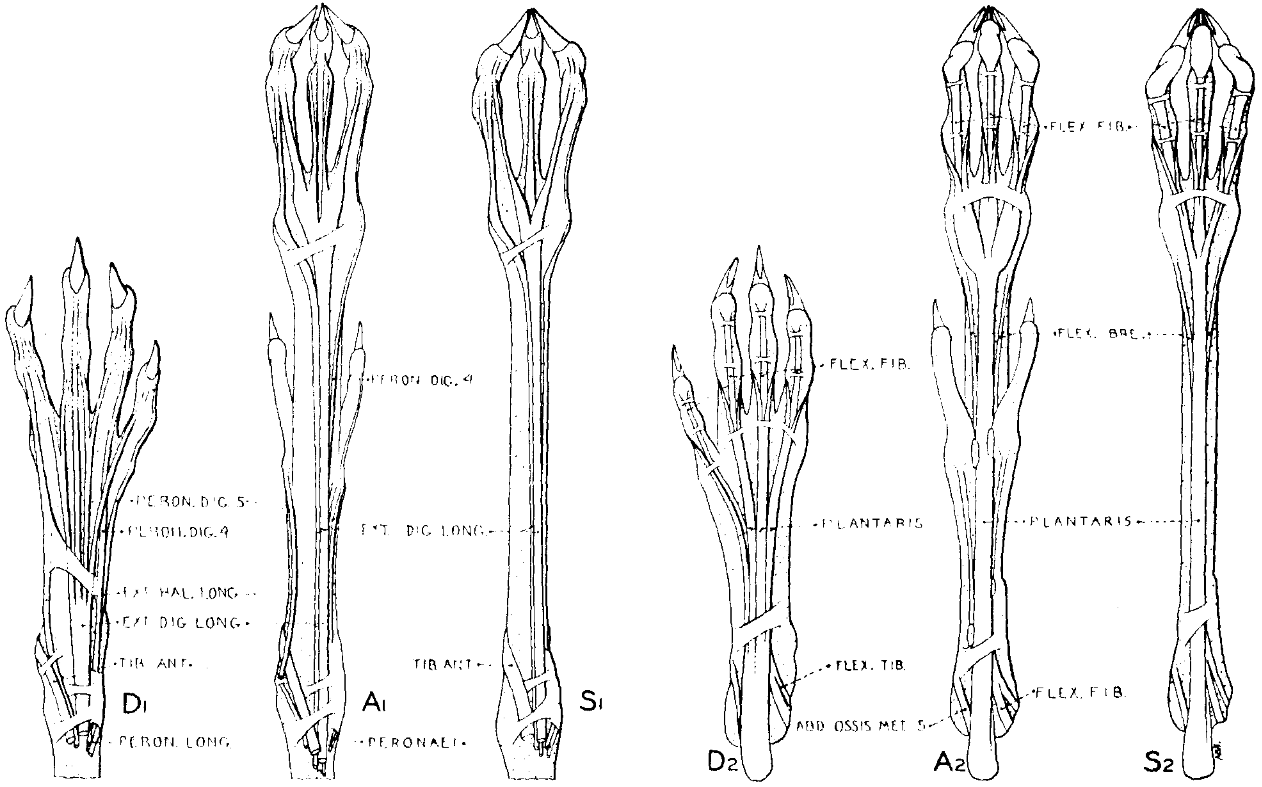

PERON. DIG. 5
PERON. DIG. 4
EXT. HAL. LONG.
EXT. DIG. LONG.
TIB. ANT.
D1
PERON. LONG.
PERON. DIG. 4
EXT. DIG. LONG.
TIB. ANT.
A1
PERONAEI
S1
FLEX. FIB.
PLANTARIS
FLEX. TIB.
ADD. OSSIS MET. 5
D2
FLEX. FIB.
FLEX. BRE.
PLANTARIS
FLEX. FIB.
A2
S2

FIGURE 20. Details of the tendons of the pes of Dipodomys (D), Allactaga, (A), and Scirtopoda (S): Dorsal aspect to the left (1), and ventral on right (2).

fibers had disappeared. The first and fifth were very slender and the other three, especially those to digits 2 and 4, very robust. Those to the first three digits arose from the much enlarged sesamoid at the base of metatarsus 1, and the other two from the slightly smaller sesamoid at the base of metatarsus 5, somewhat more posteriorly located.

Innervation of the short plantar muscles is by branches of the plantar nerves, from the n. tibialis.

OSTEOLOGY.

In the osteological portion of the present contribution the skull will be considered on a slightly different basis from the remainder of the skeleton, the former being accorded more lengthy treatment for the reason that few rodents in existence have a cranium more peculiarly and interestingly modified than *Dipodomys*.

The illustrations doubtless give a better idea of skull form than can a description, but nevertheless the general and chief characters of the *Dipodomys* skull in comparison with that of a rodent of typical murine form should be briefly presented. For the latter a small skull of *Neotoma*, with condylobasilar length about equal to the same measurement in *Dipodomys spectabilis*, will be selected.

SKULL.

The preorbital part of the skull of *Dipodomys* is longer than in *Neotoma* and the postorbital part correspondingly reduced. The shape is strongly triangular, with great width through both the mastoid and prezygomatic sections. The interorbital width has also increased, being more than twice as great as in the wood rat. The dominant character of the entire skull is the great size of the mastoid bullae.

In lateral aspect it is seen that the nasals of *Dipodomys* project especially far beyond the incisors, that the infraorbital foramen is located far forward upon the side of the rostrum instead of at the medial base of the maxillary root of the zygoma, and that the latter feature is much broader. Instead of the zygomatic width being the greatest transverse measurement of the skull, as in most rodents, this is surpassed both anterior and posterior to the orbit. Because of the crowding of the mastoids the architecture of the squamosal is greatly altered, and similarly the interparietal and occipital have been much restricted.

The incisive foramina are minute and the choanae much reduced in size, while the parapterygoid fossae are somewhat more capacious and each is perforated by a foramen of relatively large size. The cornua of the auditory part of the bullae meet in the posterior pterygoid region, and the inflated mastoid part have not only crowded extensively onto the dorsal surface of the skull, but overhang to the rear, projecting considerably posterior to the plane of the occipital.

In final analysis the modifications which the bones of the mammalian skeleton, other than the skull, have undergone are mostly, and indeed almost entirely, attributable to muscle stress. True, there have been mechanical stimuli of a variety of sorts. If there were rodents comparable in size to an elephant they would of necessity have a skeleton much different from that of a mouse, partly in order to support a great weight; and peculiar skeletal modifications often are correlated with special habits. But on the whole such differences must accompany, even though they may not be directly caused by, muscular specialization. This broad generality cannot be applied to the skull, however. In fact it is probable that other stimuli operating for cranial form are considerably stronger than muscle stress. There are the correlated factors of feeding habits and character of dental armature, sensory (including olfactory, ocular and auditory) requirements, and cerebral needs. All these must be nicely adjusted for proper interaction, as a properly formed scaffold for the respective organs, and so that the controlling musculature shall be well suited, in form and position, to operate with the requisite efficiency. When the vital cranial equipment is well balanced for a particular norm one encounters a skull suited to the needs of a generalized type varying with the requirements of particular groups of mammals—an "average" rodent skull, or that of a carnivore, or insectivore, or ungulate. If one of the factors of the vital cranial equipment depart widely from its norm for the particular type concerned, it will affect the entire skull to a greater or lesser degree, according to the quality and quantity of the alteration. Some of these changes are quite simple, affecting in appreciable extent only a single detail, while the surrounding bony topography is insignificantly altered. Others may be very complex, causing profound alteration in many features of the skull. In the latter category is the development of the audital bullae of *Dipodomys,* and the parallel feature in certain of the jerboas. This has had a very pronounced effect upon the skull and, fortunately, the successive steps by which the present conditions were attained may be followed with a considerable degree of assurance through other, more generalized forms of the Heteromyidae.

The above steps, leading to extreme hypertrophy of the bullae, are nicely illustrated by the following skulls, figured on page 479: *Liomys irroratus*, *Perognathus* (*Chaetodipus*) *baileyi*, *Perognathus* (*Perognathus*) *amplus*, *Dipodomys spectabilis*, and *Microdipodops pallidus*. With *Dipodomys* are also compared certain features of the species of jerboas which are available, and of the wood rat (*Neotoma*), the latter solely for the reason that I have previously chosen it as a representative, generalized rodent with which to compare other sorts. But the wood rat belongs to the family Cricetidae of the superfamily Muroidae, while *Dipodomys* is of the superfamily Sciuroidae. Hence it is fitting to use the skull of a squirrel (*Sciurus* (*Tamiasciurus*)) also in comparison.

Length of skull is a complex factor and can hardly be said to have anything to do with saltatorial habits. *Total* length of skull can be affected thereby, however, for in *Dipodomys* and other saltators with phenomenal bulla development the mastoid may project for a considerable distance posterior to the condyle. In *Dipodomys* this feature, as well as the long rostrum extending forward of the incisors, increases the total length of skull to the degree where it is about 55 per cent of the body length (the sum of the cervical, thoracic, lumbar and sacral vertebrae), which is very large indeed (*Scirtopoda* 46 per cent, *Allactaga* 48, *Pedetes* 34, *Lepus* 27, *Neotoma* 28). But if condylobasilar, instead of total, length be considered it is found that the percentage in *Dipodomys* is only about 27, and as great as 33 in *Neotoma*. In *Dipodomys* the items which increase the size of the head are relatively light, consisting of a slender muzzle and hollow bullae, so it is probable that its head is no heavier than in some mammals with heads appearing very much smaller.

Width of skull is definitely associated, to various extents, with a high degree of saltation. It is true that an animal may become a great leaper without having a very broad skull (*Allactaga*), but in those with large bullae this feature broadens the posterior part of the skull. Thus in *Dipodomys* the breadth is about 65 per cent of the condylobasilar length, and but 40 per cent in *Neotoma*. As such broadening takes place the eyes must also be placed farther apart, in order that the animal may, to a reasonable degree, watch for an enemy approaching from the rear, and this in turn is accompanied by some broadening of the details immediately anterior to the eyes.

The skull of *Dipodomys* and other rodents with comparable bullae is thus very broad posteriorly, which gives it a triangular appearance. Although it is not particularly deep, relative to length, this is so of the skulls of certain jerboas and their relatives, in which the bulla development is not phenomenal. This is the case with *Scirtopoda,* and especially *Pedetes.*

It should be noted that in all these rodents the temporal muscles are progressively weaker as saltatorial adaptation advances. Origin of the temporalis freely migrates from the parietal (chiefly) to the squamosal (either exclusively or practically so, as in *Pedetes*), and even to the mastoid part of the bulla, as in *Dipodomys.* In the latter case the temporal muscle is very weak indeed and has probably degenerated (in size) to as great an extent as in any rodent. Undoubtedly in compensation the various divisions of the masseter muscle in these rodents are very well developed indeed. It appears to be the case that in rodents with even moderately large bullae (all those dissected) origin of the styloglossus muscle has migrated from the inferior margin of the auditory meatus in a medial direction almost to the basioccipital border. In addition, the origin of the stylohyoid in *Dipodomys* seems to have migrated from the paroccipital process considerably farther forward. The attachment of these muscles to the bulla is very insecure, however, and it is difficult to define the precise points of origin. The insertion of the cleidomastoid of *Dipodomys* has shifted to a point below the meatus, and of the sternomastoid, definitely anterior to the meatus, which, of course, adds to the leverage of these muscles as compared with conditions in the usual rodent with narrower head, including the jerboas dissected.

Consideration of the position of the foramen magnum together with occipital tilt and the situation of the condyles shows clearly, I think, that the saltatorial rodents under consideration hold the head with the axis tilted downward to a greater degree with respect to the body axis than do the more generalized rodents. In other words the effect of the more erect carriage following the adoption of a partially bipedal mode of locomotion has been accompanied by an appreciable tilting of the head.

For the sake of convenience descriptions of the cranial foramina will be grouped together, preceding presentation of the other cranial elements.

Incisive foramen. This is minute in heteromyids, geomyids, and squirrels. It is enormous (about four times as long) in the wood rat, and intermediate in the jerboas. It is generally presumed that the development of the incisive foramina is correlated with Jacobson's organ, but nothing is known of the reasons for such a disparity in the size of these foramina.

Infraorbital foramen, for the passage of the infraorbital branch of the trigeminal (V) nerve. In murine rodents this is large and V-shaped, between the base of the rostrum and the zygomatic root. In true sciurine rodents it is minute and situated ventrally at the lower part of the zygomatic root. In the jerboas the huge infraorbital fenestration for the passage of a part of the masseter muscle, common to rodents of hystricomorph affinities, is generally termed the infraorbital foramen, but this is incorrect. The true foramen of this name is just as minute as in the squirrels and pierces the maxilla below and at the medial base of the larger fenestration. In all the Heteromyidae and Geomyidae, however, conditions are very different and apparently unique among the Rodentia, for the infraorbital foramen is situated not even adjacent to the zygomatic plate of the maxilla, but upon the side of the rostrum half way between the plate and the incisor. This is a detail which apparently is of great phylogenetic importance. In *Dipodomys* at least the maxillary branch of the trigeminus does not emerge into the orbit as it comes from the Gasserian ganglion as in most other (including sciurine) rodents. It may be seen, from the optic aspect, emerging from the foramen rotundum (practically confluent with the sphenoidal fissure in this animal) but it at once disappears into a canal at the level of the second molariform tooth, and the anterior opening of this canal forms the infraorbital foramen. Hence, in these rodents, the zygomatic plate of the maxilla is pierced by no foramen at all.

There is a foramen in certain rodents which apparently has never received a name. In the jerboas it is situated upon the side of the rostrum at a level with the maxillary root of the zygoma or a bit farther rostrally, somewhat similar but entirely within the orbit of the agouti (*Dasyprocta*), and at the dorsorostral part of the orbit in some heteromyids, but absent as a separate foramen in the wood rat. Dissection shows that it is for the passage of a cutaneous branch of the trigeminus (ramus frontalis) which serves the anterior region of the eye, and it may be known as the *intraorbital foramen*.

Optic foramen, for the passage of the optic nerve (II), is the foramen anterior to the sphenoidal fissure, as usual.

Sphenoidal fissure and *foramen rotundum*. These are separate in squirrels and the wood rat but combined in heteromyids and jerboas. The former, or anterior lacerated foramen, is for the passage of the oculomotor (III), trochlear (IV), abducent (VI), and first or ophthalmic division of the trigeminal (V) nerves. Through the foramen rotundum when it is present, or otherwise also through

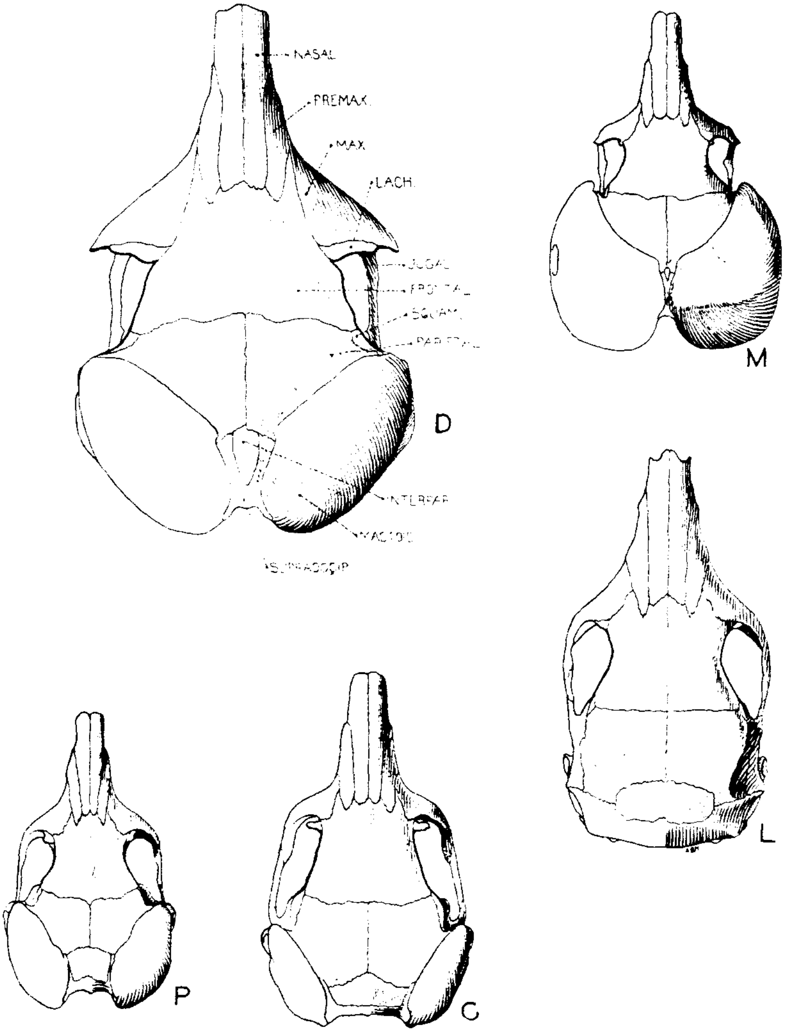

NASAL
PREMAX
MAX
LACH
JUGAL
FRONTAL
SQUAM
PARIETAL
INTERPAR
MASTOID
SUPRAOCCIP
D
M
L
P
C

Figure 21. Dorsal view (× 1½) of the crania of Heteromyidae: Dipodomys (D), Microdipodops (M), Liomys (L), and of the subgenera Perognathus (P) and Chaetopidus (C) of the genus Perognathus. The right half of the skulls is shaded, to show contour, and the left half left blank, to show sutures.

the sphenoidal fissure, passes the maxillary or second division of the trigeminus.

Foramen ovale, for the accommodation of the third or mandibular division of the trigeminus, is present in all examples and pierces the alisphenoid caudal to the foramen rotundum when this occurs.

Pterygoid foramen. Painstaking dissection is necessary before foramina may be homologized with certainty. For this reason it is unsafe to hazard a statement in regard to just what rodents may or may not have this foramen. It is very possible that in some families or genera it may be confused with an apparent foramen ovale. At any rate the well defined foramen pterygoideus, as represented in the Heteromyidae and Geomyidae by relatively large circular or subcircular vacuities situated between the mesa- and parapterygoid plates, seems clearly to be present in the squirrels, but in smaller form. Careful dissection in three specimens of *Dipodomys* showed that it is for the accommodation of a surprisingly large vein, emerging from between the external and internal pterygoid muscles and extending laterally to the base of the ear, where it joins with the facial and temporal veins to form the external jugular. I was unable to find a trace of the true internal jugular, although readily conceding that a minute one, in an animal of such size, not injected, may possibly have been overlooked. At any rate there is no doubt but that this vein through the pterygoid foramen is the main drain of blood from the brain, and that it has taken over the function of the more usual internal jugular vera. The true homology of what may be called this pterygoid vein is unknown. Certainly it seems to be entirely unrepresented in most mammals, including the cat, but in primitive Mammalia it may possibly occur as a small vessel and this, if of aid in draining the blood sinuses at the base of the brain (as inferior petrosal or cavernous sinuses) could easily, if the need arose, develop to assume the entire duty of a true internal jugular.

Middle lacerated foramen is variable in size but occurs as a rather large, irregular vacuity in *Dipodomys*, larger still in *Scirtopoda* and *Dipus*, and as an enormous, elongated one in *Allactaga*, between the alisphenoid and audital bulla. It seems that this is chiefly a vacuity. It is much larger than needful for the passage of small nerves and blood vessels even in *Dipodomys*, in which it was ascertained that the carotid artery does not enter the endocranium at this point, although it very probably does so in the jerboas, as in most other mammals.

Stylomastoid foramen, for the passage of the facial (VII) nerve. This is located immediately caudal to the auditory meatus, between the mastoid and audital bullae in most mammals, slightly ventral to the level of the meatus in the jerboas, and relatively far below it in heteromyids. In the more specialized of the latter it is useful in locating the junction of the petrous and tympanic portions of the temporal complex.

Posterior lacerated foramen. In the rodents examined this varied from an aggregation of several irregular foramina to a rather broad, slit-like vacuity between the basioccipital and the audital bulla. The latter is the condition in *Dipodomys*. In this genus it was found not only that there was no internal jugular, as already mentioned, but that the point of entry to the endocranium of the internal carotid had shifted, possibly as an accompaniment of bulla development, so that it entered through the rostral part of the posterior lacerated foramen. Emerging caudal to this artery were the glossopharyngeal (IX), vagus (X), and spinal accessory (XI) nerves. Presumably these conditions are the same in the other heteromyids but probably not in the jerboas, sufficient specimens of which were not available to determine all such points.

Stapedial foramen. This foramen, heretofore unnamed, is so termed for the reason that it accommodates the stapedial artery, which occurs in many mammals, but in man only until the third month of fetal life. In the species considered in the present connection it was absent only in *Liomys* and *Thomomys*, but its usual situation in nonheteromyids is farther caudalward near the paroccipital process. In rodents having this foramen it will be found that the internal branch of the carotid splits in the vicinity of the basioccipital into two arteries of approximately equal size. One of these becomes the true internal carotid and the other the stapedial artery, piercing the bulla near the basioccipital (fig. 26) and passing through the stapes.

Squamosal foramen (fig. 25). This is a foramen of generous size that pierces the squamosal of *Dipodomys* and is visible within the orbit from an obliquely anterior aspect. Dissection upon both sides of a spirit specimen, with this detail especially in view, showed that it is for the passage of a blood vessel. No nerve was here detected.

Condyloid foramen, for the passage of the hypoglossal (XII) nerve, is present in all skulls examined, but is especially small in heteromyids and jerboas. In *Allactaga* it is double. It occupies its usual situation ventrolateral to the condyle.

In addition to the foramen magnum and those mentioned, there are, of course, many small foramina in the skull for the passage of small blood vessels and nerve filaments. In certain forms some of these may ultimately prove to be of great importance but their proper treatment depends upon lengthy research.

Nasals. The shape of the nasals is influenced by the conformation of the snout proper and of the rostrum as a whole. In the Heteromyidae both of these features depart widely from cricetine and sciurine conditions. In heteromyids the snout is elongated and hence the nasals project well beyond the incisors, to as great an extent in *Dipodomys* as 4 mm., and are inclined to be tubular distally, each one in cross section describing about one third of a 4 mm. circle. The latter feature is most pronounced in *Microdipodops.* This character of projecting nasals is not shared by geomyids, with their short nose. Neither are the nasals definitely elongated in jerboas of the genera *Allactaga* and *Scirtopoda*, but there is some elongation encountered in *Salpingotus*, with its enormous bullae.

Premaxillae. The anteriormost portion of these bones in the Heteromyidae is prolonged into a distinct process upon each, which projects considerably beyond the vertical situation of the incisors to join the anterior thrust of the nasals. Thence they descend and join to form the U-shaped floor of the anterior nares, and thence descend still farther for a considerable distance to form a sharp, vertical, median ridge between the incisors. This is not the case in the wood rat, squirrel, or pocket gopher, but a somewhat similar situation prevails in the jerboas, especially *Scirtopoda*, in which the premaxillae ensheath the incisors more pronouncedly. In other respects these bones, in all the rodents here considered, conform to the same general plan. The ascending branches always extend posterior to the nasals, and the extent of their lateral areas conforms more strictly to the relative size of the skull than to relative length of rostrum. In other words the lateral area of the premaxilla is somewhat the same in an animal with long rostrum as in one having this feature much shorter. In the latter sort the maxilla is the detail which suffers reduction. The lateral area of the premaxilla is, however, affected in the Heteromyidae by the peculiar situation of the infraorbital foramen, for the premaxillo-maxillary suture always passes rostral to this foramen. The incisive foramina of the Heteromyidae are of the minute, sciurine type and they are bordered practically entirely by the premaxillae. In the jerboas, as in the wood rat, these foramina are much longer and the premaxillae and maxillae are about equally concerned. Toward the terminations of the ascending branches of the premaxillae of heteromyids there are no extremely small "nutrient" foramina such as are present in varying number in geomyids.

For the Geomyidae, Merriam (1895) has described an internal narial detail which he termed internal vertical plate of the maxilla. Its analogue is present in *Dipodomys* but in this animal the premaxillo-

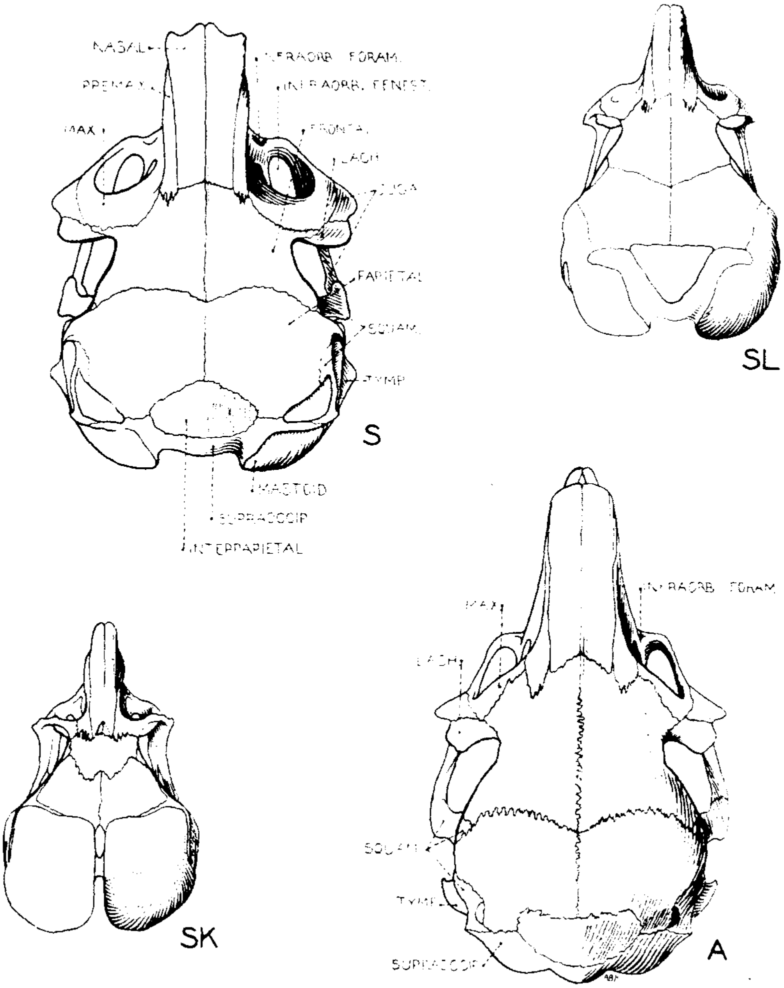

NASAL
PREMAX.
MAX.
INFRAORB. FORAM.
INFRAORB. FENEST.
FRONTAL
LACH.
JUGAL
PARIETAL
SQUAM.
TYMP.
S
MASTOID
SUPRAOCCIP.
INTERPARIETAL
SL
SK
INFRAORB. FORAM.
MAX.
LACH.
SQUAM.
TYMP.
SUPRAOCCIP.
A

FIGURE 22. Dorsal view (× 1½) of the crania of Dipodidae: Scirtopoda orientalis (S), Scirtopoda lichtensteini (SL, redrawn from Vinogradov), Salpingotus kozlovi (SK, redrawn from Vinogradov), and Allactaga (A).

maxillary suture is located farther forward in respect to the incisive foramina than in the pocket gopher. Presumably in correlation with this is the fact that in the kangaroo rat the internal vertical plate, in the form of a lamina arising lateral to the vomer, is formed by the premaxilla.

Maxillae. The maxillaries are largely affected not only by rostral features but by any peculiarities that may occur in either the anterior brain case or zygomatic arches. In the Heteromyidae they are altered by the character of the infraorbital foramina, and in *Dipodomys* especially by the broadness of the prezygomatic features, but the latter are not altogether an accompaniment of bulla development, for this is not pronounced in *Microdipodops*.

As already described, the infraorbital foramen is extremely variable in these major groups of rodents. In murine and cricetine types it is V-shaped and situated dorsally between the zygomatic plate and the rostrum. In sciuromorphs it is minute and circular, near the ventral border of the zygomatic plate, close to the dental arch and usually if not always subtended by a pronounced process for the attachment of the superficial masseter muscle. In hystricomorph types, including the jerboas, it is also minute, but maxillary conditions are complicated by the situation above it of the huge infraorbital vacuity, for the passage of a part of the masseter muscle. In heteromyids and geomyids it is situated far cranialward upon the rostrum. The resulting conditions are more clearly shown by the illustrations than verbal descriptions. Usually in these rodents the infraorbital foramen is bounded completely by the maxilla but occasionally the premaxilla borders it anteriorly.

Of great influence in contributing to the characteristic appearance of the skull of *Dipodomys* is the broad, flaring, superior part of the maxillary zygomatic root. In the less specialized of the heteromyids this is no more pronounced than in the average squirrel, and this is also largely the case in *Microdipodops*, so the character is not necessarily correlated with increased width of skull because of bulla development, but rather does it seem to have been caused partly by orbital requirements, perhaps, and largely because of muscular needs. It should be mentioned that in the wood rat and similar murines and cricetines, the form of the infraorbital foramen and cavernous character of the bone within this renders it unlikely that any distinct maxillary sinus occurs. In the squirrel, pocket gopher, and heteromyids, however, this sinus is large and well formed, being especially extensive in the last group, partly as an accompaniment of the small incisor root. In *Dipodomys* it invades the maxillary root of the zygoma for some distance.

In heteromyids the relation of the zygomatic root of the maxilla to the jugal is not much different from the condition in the wood rat, but in the jerboas, because of the infraorbital vacuity, it is of another pattern and completely surrounds the vacuity. In *Salpingotus* the ventro-caudal termination of the zygomatic root is prolonged caudalward by means of a long process (fig. 24).

Jugals. In the pocket mice examined this bone is of the general type of that of the wood rat, but more slender and occupies the posterior half or three-fifths of the zygomatic arch. In the squirrel it is laminated over the maxillary root and reaches the lachrymal, while in *Dipodomys* and *Microdipodops* it is intermediate between these two conditions, partly overlapping the expanded portion of the maxillary root with the slight expansion more horizontal than vertical. At a point in *Dipodomys* about 4 mm. from the posterior termination there is a sharp spinous process upon the medial aspect, barely indicated in *Microdipodops*. In the jerboas this bone also reaches the lachrymal, is very extensive indeed, and of a right-angular shape, because of the conformation of the infraorbital vacuity. In *Scirtopoda* especially the anterior part is expanded in a posterior direction, which is not pronounced in *Allactaga* nor *Salpingotus*.

Lachrymals. This bone is invariably present, flush with the bony surface in the squirrel but it is more in the nature of a process in the others, and somewhat broader and strap-shaped in the jerboas and *Dipodomys*. The mediorostral border articulates with the maxilla except in *Perognathus* and *Microdipodops*, in which genera it takes the form of a relatively long process, chiefly from the frontal, and is particularly apt to be lost during cleaning of the skull. The squirrel appears to be the only one of these rodents in which the lachrymal canal pierces this bone. In the wood rat the bony canal seems to be absent, the duct passing within the infraorbital foramen and so to the nasal passage through a vacuity in the bone. Neither does a foramen occur in *Scirtopoda* and the route of the duct is not apparent. In *Allactaga* a foramen, evidently for this duct, is found in the frontal nearby, while in the Heteromyidae it pierces the maxilla just below the lachrymal.

Vomer. In rodents with narrow rostrum the internal details of this part of the internal nares are much compressed laterally. The vomer of *Dipodomys* is exceedingly thin but quite extensive vertically. It rests upon the floor of the nasal cavity between the internal vertical plates of the premaxillae and divides above, forming two ascending wings which closely clasp the mesethmoid cartilage. The bone thence (caudalward) arises from the floor of the nasal passage to extend in a slender process which ends at the base of the cribriform plate.

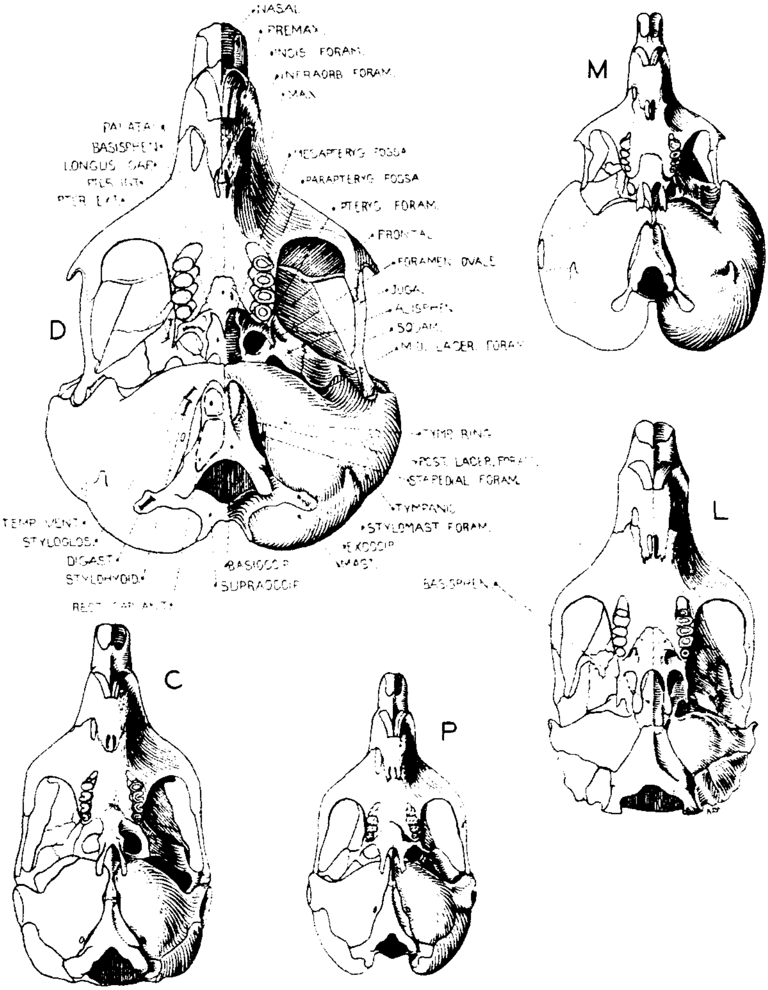
Palatines. The palatal bones of *Dipodomys* unite with one another at an early age, and frequently, at a later stage, with the maxillae and other adjacent bones as well. They form the ventral and lateral borders of the posterior nares and the extreme anterior part of the pterygoid fossae and are considerably different in conformation from conditions in the wood rat. A medial palatal spine, posteriorly directed, may be present or absent but is frequently very long (3 mm.) in *Dipodomys*. The precise position of the posterior nares in relation to the last molar is variable, being situated more rostrally in the wood rat, intermediate in *Dipodomys*, and farther caudalward in the squirrel and jerboas.

Ethmoids. Filling the rostral part of each half of the nasal cavity are the nasoturbinal and maxilloturbinal. The former is attached to the extreme lateral border of the nasal bone and its ventral part is grooved for receiving the border of the maxilloturbinal. The latter is more nearly horizontal, and parallel to the nasoturbinal, than seems to be usual in rodents. It continues rostrally from the maxilla upon the dorsorostral border of the infraorbital foramen, but its anchorage within the nasal passage seems to be more concerned with the premaxilla than the maxilla.

The remainder of the ethmoid complex in *Dipodomys* consists of mesethmoid, cribriform plate, and ethmoturbinals. The sagittally situated mesethmoid bone is rather short, but the mesethmoid cartilage extends therefrom almost to the anterior nares. Caudally the osseous connection is with the cribriform plate, and dorsally with the anterior part of the frontal shield. The cribriform plate, forming the rostral wall of the brain case, is, if anything, a trifle larger than in the wood rat. The perforations for the olfactory filaments are very numerous indeed and the cerebral surface is finely crenulated, to correspond with the turbinal folds, but this is much less marked in other rodents examined.

The fragile convolutions of the ethmoturbinals are bounded caudally by the cribriform plate, medially by the mesethmoid, and laterorostrally by the medial wall of the maxillary sinus, except that the latter does not join the mesethmoid rostrally, leaving a slit which constitutes the olfactory and respiratory parts of the nasal passage. Within this bony capsule the ethmoturbinals are expanded dorsally and more constricted ventrally.

Frontals. A character exhibited strongly by the pocket gopher is the tendency for the early obliteration of the suture between the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LONGUS CAP.
PTER. INT.
PTER. EXT.
MESAPTERYG. FOSSA
PARAPTERYG. FOSSA
PTERYG. FORAM.
FRONTAL
FORAMEN OVALE
JUGAL
ALISPHEN.
SQUAM.
MID. LACER. FORAM.
TYMP. RING
POST. LACER. FORAM.
STAPEDIAL FORAM.
TYMPANIC
STYLOMAST. FORAM.
EXOCCIP.
MAST.
TEMP. VENT.
STYLOGLOS.
DIGAST.
STYLOHYOID.
RECT. CAP. ANT.
BASIOCCIP.
SUPRAOCCIP.
BASISPHEN.
D
M
L
C
P

FIGURE 23. Ventral view (× 1½) of the crania of Heteromyidae: Dipodomys (D), Microdipodops (M), Liomys (L), and of the subgenera Chaetodipus (C) and Perognathus (P) of the genus Perognathus.

frontals. Hence, this character encountered in *Dipodomys*, but less strongly marked, can hardly be correlated with the breadth and flatness of this part of the skull. In the wood rat the frontals are very narrow, considerably broader in the squirrel and the jerboas, and exceedingly broad in *Dipodomys;* but this character is less pronounced in other heteromyids, including *Microdipodops.* Whereas in most rodents the frontals are thin and fragile, in the majority of heteromyids they are thicker, with characteristic cancellous tissue internally and smooth, fine-grained plates upon the ental and ectal surfaces. This is most pronounced in *Dipodomys* and it is apparent that strength is thereby added as the skull becomes broader and flatter. But the reason why *Microdipodops* should lack this character is obscure.

In *Dipodomys* the frontal bones have the general form of a vertical section through a truncated cone. This frontal shield is very broad posteriorly, and regular except at the anterior boundary, which conforms in outline to the irregularities caused by the posterior terminations of the nasals and ascending branches of the premaxillae. An acute, anterior process is also projected upon each side between these ascending branches and the maxillary plates of the zygomatic arches. The angle formed by the frontal shield with its orbital plate is acute and sharp. This plate extends caudalward from the lachrymal, the anterior suture being with this bone and the maxilla, the inferior one with the orbitosphenoid, and the posterior with the squamosal; but the ossification of the inferior portion and of the adjoining orbitosphenoid is incomplete and their precise boundaries in this region are indeterminable.

Sphenoids. The precise limits of the basi-, ali-, orbito-, and presphenoids are not easy of determination in *Dipodomys*, for they appear to be fused completely. The most important details, from the present standpoint, occur in the pterygoid region. In heteromyids and geomyids this is very characteristic, with foramina passing endocranially, one in either pterygoid fossa. The anterior part of the latter is shallower than in the jerboas but deeper than in either the wood rat or squirrel, and in all they extend farther rostrally, as compared with the mesopterygoid fossa, than in the wood rat. In adult skulls of none of these rodents are there separate pterygoid bones but these occur in *Dipodomys* of tender age, and doubtless in comparable skulls of the others as well. It should be mentioned that the choanae or posterior narial opening is more restricted in *Dipodomys* and *Microdipodops* than in the others.

In *Dipodomys* the orbitosphenoid is very fragile and there is usually, if not always, an irregular vacuity between it and the frontal with ossification incomplete. From the endocranial aspect it is seen that the optic foramen is almost as large as the sphenoidal fissure, and that the presence of the pterygoid foramen and foramen ovale are instrumental in forming, endocranially, a strong, bold, alisphenoid bridge between the former and the middle lacerated foramen.

In the jerboas the orbitosphenoid is largely the same save that the vacuity occurs between this bone and the squamosal. In all of them the sphenoidal fissure is combined with the foramen rotundum and is situated between the orbito- and alisphenoid.

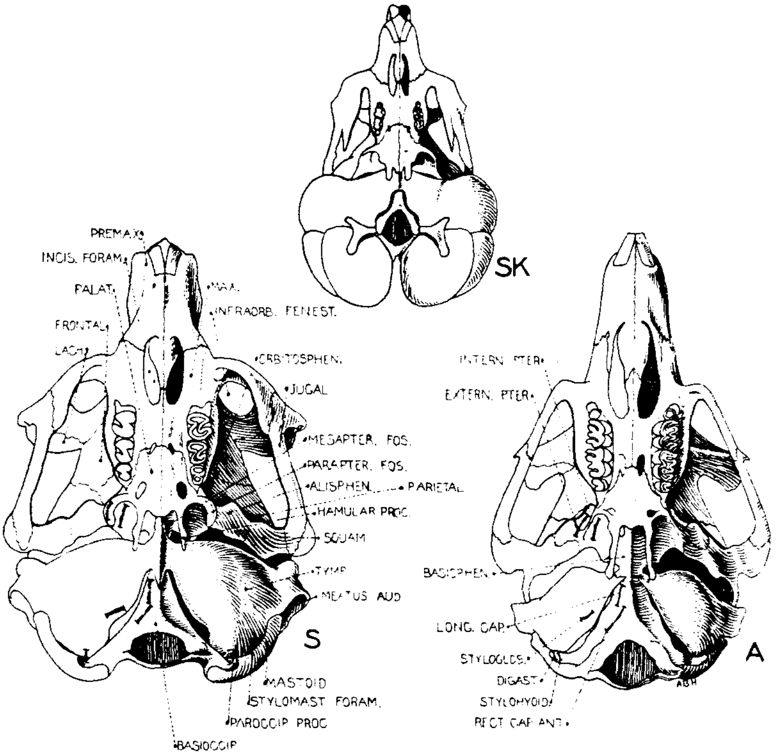
In all of them also the alisphenoid is prominent and constitutes an irregular ridge between the palatal and squamosal. This bone forms most of the parapterygoid plate and is pierced by the foramen ovale.

The basi- and presphenoids are fused, but from an examination of an immature skull of *Dipodomys* it is seen that the former constitutes that part of the bone dorsal to the junction of the horns of the bullae and as far rostral as the level of the pterygoid foramen, while the presphenoid, from the endocranial aspect, ends just short of the cribriform plate. It is not quite so narrow and rod-like as in the jerboas.

Parietals. The tendency in the Heteromyidae is for the parietals to adopt a rectangular shape and without rostro-lateral extension as in so many other rodents (wood rat, squirrel, and even the pocket gopher). Thus the parietal-frontal sutures together form an essentially straight, transverse line, characteristic in less marked form also of the jerboas examined. The outline of the laterocaudal part of the parietal, however, is modified in direct ratio to the development of the bulla, at first slightly so in the pocket mice, and greatly so in the kangaroo rats and mice, becoming almost completely triangular in such forms as *Dipodomys deserti* and *Microdipodops*. In these animals the modification is such that only an exceedingly restricted part of the parietal border is adjoined by the squamosal; and the encroachment of the enormous bullae upon the midline has all but eliminated the parietal-supraoccipital sutures.

In *Allactaga* the parietal-frontal, parietal-parietal, and parietal-interparietal, as well as the frontal-frontal, sutures are of an exaggeratedly dentate type that must be particularly strong, but this character is shared by none of the other species under consideration.

Whereas in heteromyids the reduction of a part of the parietal accompanying bulla development gives the appearance of having been accomplished by a simple horizontal crowding of the bulla, in the jerboas this is not the case. Clearly in these animals the course of development has been in the nature of first the enlargement of the bullae beneath the parietals, as well as the squamosals and occipital, and later the partial atrophy of the portions of these bones so affected.


SK
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PARAPTER. FOS.
ALISPHEN.
PARIETAL
HAMULAR PROC.
SQUAM
TYMP.
MEATUS AUD
S
MASTOID
STYLOMAST FORAM.
PAROCCIP PROC
BASIOCCIP
INTERN. PTER.
EXTERN. PTER.
BASISPHEN.
LONG. CAP.
STYLOGLOS.
DIGAST.
STYLOHYOID.
RECT. CAP. ANT.
ABH
A

FIGURE 24. Ventral view (× 1½) of the crania of Dipodidae: Scirtopoda orientalis (S), Salpingotus kozlovi (SK, redrawn from Vinogradov), and Allactaga (A).

Interparietal. It is difficult to escape the conclusion that in the majority of mammals at least the character of the interparietal is definitely correlated with the development of the temporal fossae. The enlargement of the temporal muscles to the point where there is a sagittal crest, will usually not only be accompanied by reduction of the interparietal, but also by fusion of this element with the parietals and elimination of the sutures involved. Similarly, as a rule, the broadest interparietals are often encountered in mammals having the weakest temporal muscles. *Dipodomys* forms an interesting exception to this, reduction of the interparietal having been clearly brought about by enlargement of the audital bullae.

In the pocket mice the tendency is for this bone to be rectangular, and considerably greater in transverse than in sagittal dimension save in those forms with bullae that are larger than usual (as *Perognathus amplus*). In the majority of kangaroo rats (as *Dipodomys spectabilis*) the interparietal is narrow and shield-shaped, bounded laterally by the ascending branches of the supraoccipital. In *Microdipodops*, with its enormous bullae, this bone is a minute vestige, and in *Dipodomys deserti*, whose bullae approach one another to a greater degree than any other species of the genus, the interparietal seems to have been crowded out completely.

In the jerboas the interparietal is well defined but with nothing distinctive. It may be broadest either cranially or caudally.

Temporals. This part of the skull, consisting of squamous, petromastoid, and tympanic or audital portions, is of the usual complexity.

In the less specialized of the *Heteromyidae* the squamosal is not markedly different from that encountered in such rodents as the wood rat or the squirrel. Seemingly of chief interest is the conformation of the part of the bone about the mastoid process of the squamosal, in so many rodents bordering the prelambdoidal fenestration of this bone. In neither heteromyids, geomyids, nor jerboas is this fenestration present, although in *Allactaga* there does occur a small vacuity between the squamosal and parietal.

In *Liomys* a process of the squamosal which doubtless represents the mastoid process forms the superior lip of the auditory meatus, but in the specimens of *Perognathus* (of both subgenera) examined this process occurs above the meatus and is long and very slender. In *Dipodomys* and *Microdipodops* it is clearly indicated above the zygomatic root of the squamosal, although in these genera it does not extend caudally anywhere near the meatus. It is interesting to note that this process is more conservative than the part of the squamosal which in more generalized rodents usually articulates with the occipital just above the fenestration.

As the bulla develops the more ventral part of the squamosal is also affected, to the extent where the bulla directly adjoins or even overlaps laterocranially the glenoid cavity (*Microdipodops*). In the more specialized sorts the zygomatic process of the squamosal is extremely short—a condition likely correlated with the great width of the posterior part of the skull, as it is a character shared with the jerboas.

It should be noted that in *Dipodomys spectabilis* there is a ridge or shelf visible in ventral aspect between the glenoid cavity and the zygomatic process, marking the attachment of an interesting and peculiar muscle slip, tentatively termed the ventral slip of the temporal. This ridge, and apparently the muscle in well marked form, is absent in *D. deserti* and this difference serves to differentiate adults of the two species at a glance without recourse to other characters.

In the jerboas the termination of the zygomatic process of the squamosal is notably robust but in other respects the ventral aspect of this bone is not noteworthy.

In many mammals the petrosal and mastoid portions of the temporal are so well fused that it is entirely impracticable to define their precise junctions. In those of the character of the wood rat and squirrel, however, as in most other sorts, it is at least easy to differentiate externally between the tympanic bulla and the mastoid, partly by the difference in the texture of the bone and always by the situation of the mastoid between the lateral or ventral part of the lambdoidal crest, the exoccipital, and the tympanic bulla. But it is hardly possible to distinguish between the mastoid and petrous portions of the temporal save in a general sense.

It may be said that both the mastoid and the tympanic bulla in the wood rat and the squirrel are of moderate size, the former with a flat surface and the latter subspherical in form with a fair degree of inflation. In both there is a faint indication of a projecting lip to the auditory meatus.

As the pocket gophers (Geomyidae) are more closely related to heteromyids than are any other living mammals it might be expected that these would show some tympanic resemblance to the conditions in *Dipodomys*. This is not the case and it appears likely that in this detail the two groups have followed opposite courses of development. But if the more generalized members of the Heteromyidae are compared both with *Thomomys* and *Dipodomys* various similarities may be noted. In the former the bulla is flatter than in the average murine rodent and is projected into a flask-like meatus that is relatively of very small diameter. This character of tubular meatus is met in varying degree in the less saltatorial of the heteromyids, apparently occurring in inverse ratio to bulla size. It is present in fair degree in *Liomys irroratus* and to a progressively lesser extent through the hairy and then the silky pocket mice to the kangaroo rats and mice, in which the meatus is of large diameter and is flush with the globular surface of the bulla.

In this connection there should be mentioned the situation as reported in *Liomys canus*, by Allen (1904). It appears that in this species there is a small, semitubular ossicle at the base of the ear cartilage which is hinged to the lip of the meatus by a ligamentous structure. That this is a development for fulfilling some definite audital function is indicated by its presence in other rodents of both Old and New Worlds (*Ctenodactylus*, *Pectinator*, *Chinchilla*). As the cartilage of the auricula is considered to be phylogenetically a continuation of the hyoid complex it would not be particularly surprising to find an occasional auricular center of ossification. I have failed to find it, however, in several species of *Perognathus*.

In *Liomys* the mastoid and body of the audital bulla are both very similar to conditions in *Thomomys* save that the former is relatively a bit larger. The transition from this stage to the extreme situation in *Dipodomys* is then perfectly illustrated by examining successively *Perognathus* (*Chaetodipus*) *baileyi* and *P.* (*P.*) *amplus*. It will be seen that this is accomplished by extreme enlargement of the mastoid accompanied by a much slighter increase in the size of the audital bulla. The course of procedure is for the visible part of the mastoid to increase first dorsally, and then both rostrally and medially. The exoccipital is at first not as much affected as the supraoccipital, which is crowded medialward, and the part of the squamosal above the meatus, which is practically eliminated as the mastoid replaces it. At a later stage (*Perognathus amplus*) the conformation of both ex- and supraoccipital is much affected by crowding of the mastoid, as are both interparietal and parietal. While this change in the mastoid is taking place the visible portion of the audital bulla is altered in but slight degree, relatively speaking. There is some increase of inflation and rostral extension of the anterior cornu to the midline of the posterior interpterygoid fossa. Finally, when a skull of *Microdipodops*, or even *Dipodomys*, is examined, one can not fail to gain the impression that the entire nonsquamous portion of the temporal has developed as a single unit to a single end, and the result is a relatively huge, bulbous structure that has distorted the entire caudal part of the skull.

From an examination of the skulls of *Dipus*, *Scirtopoda* and *Allactaga* at hand it seems clearly indicated that the enlargement of the bulla of the jerboas has followed a somewhat different procedure.

In these the inflation of the audital bulla is more pronounced than in any heteromyid, although in *Allactaga* the cornua are not projected to the midline. In *Allactaga* the mastoid is very small, considerably larger and quite well inflated in *Dipus*, and more pronouncedly so in *Scirtopoda*, the mastoid being about as capacious as the audital bulla. Illustrations of *Salpingotus* seem to indicate that thereafter, in the saltatorial specialization of the acoustic apparatus in jerboas, the audital bulla increases but slightly in size while the mastoid experiences inflation surpassing in degree even the kangaroo mice. The association in this group of the zygomatic root of the squamosal with the audital bulla is much less intimate than in heteromyids.

In *Pedestes* there is still another trend, for its audital bullae are perhaps relatively smaller than in the average rodent, while the mastoids are very large indeed.

By an examination of the external surface of the *Dipodomys* bulla one can distinguish certain landmarks. Thus, in the area between the posterior border of the meatus and the exoccipital process is a faintly defined sulcus, marking the junction of the audital and mastoid portions of the bulla; and along this sulcus is situated the stylomastoid foramen for the passage of the facial nerve. Bordering the foramen is a projection of bone, flush with the surface, that is apparently the homologue of a mastoid process of the usual character. Extending forward from the superior border of the meatus is another irregular line, marking the junction of the mastoid and audital portions. A third sulcus extends from the posterior border of the meatus dorsally and then medially to the vicinity of the posterior part of the interparietal, marking the position of a septum which divides the mastoid into medial and lateral chambers. By close attention it is usually possible to distinguish externally the arc of the tympanic ring, occurring, in ventral aspect, about half way between the pterygoid fossa and the stylomastoid foramen. It usually takes the form of a whitish, curved line and its conformation in *Dipodomys* shows that the tympanic membrane is tilted at an angle about half way between vertical and horizontal. In the squirrel, wood rat and the jerboas it is more nearly vertical. The only other foramen which may be distinguished externally is that for the passage of the stapedial artery, piercing that part of the bulla adjacent to the basioccipital.

By removing the parietals and a part of the frontals from a skull of *Dipodomys* the medial aspect of the auditory complex is exposed. By gross comparison of the details of this animal with those of a wood rat and a cat, it is seen that the tentorium, separating the cerebral from the cerebellar part of the brain-case, is derived in the cat entirely from the parietal. In the wood rat the latter bone plays no part in the formation of the tentorial process but this is derived from both mastoid and petrosal elements, and this is the case in *Dipodomys*. This detail, however, cannot be settled with certainty in the case of the single skull of *Scirtopoda* which was dissected; but it appears likely that both parietal and auditory elements are involved, and that in different mammals the derivation of the tentorium may vary with other details.

The medial aspect of the auditory apparatus is suggestive that this item has both modified and been modified by the conformation of the brain. There is a broad, bony depression between the tentorium and the squamosal to accommodate the occipital pole of the cerebrum, and this is of rather uniform curvature.

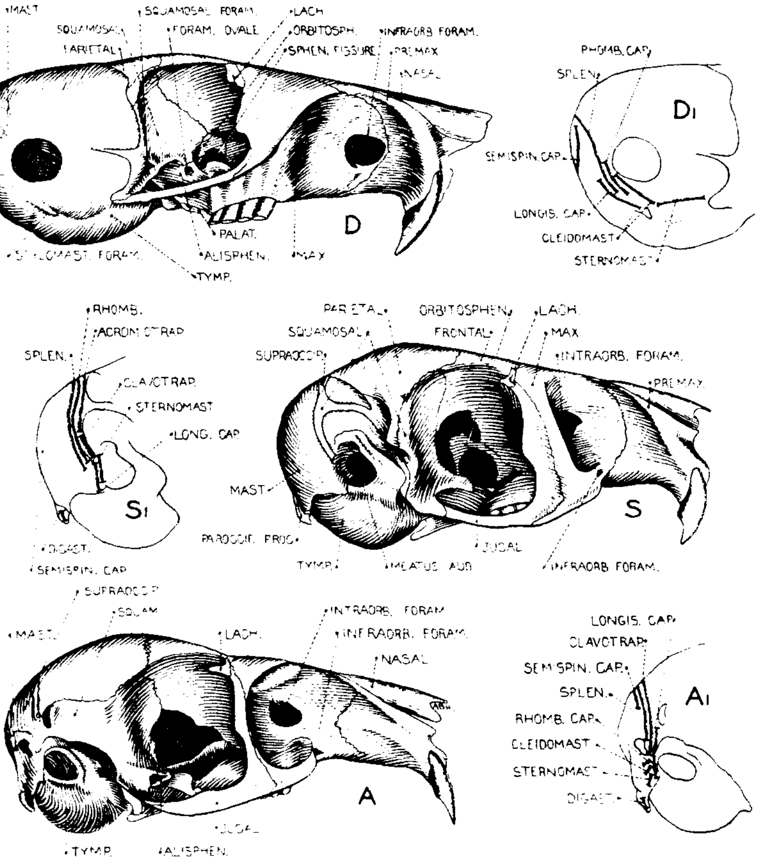
The infratentorial part of the ear bone can be examined only after separation of this portion from the remainder of the skull. When this has been done it is seen that the depression between the tentorium and supraoccipital is broken up into a superior fossa that is shallow and rather broad, for the accommodation of the anseriform and paramedial lobules of the cerebellum, and a smaller inferior fossa, much deeper, into which fits the paraflocculus.

Upon this aspect the tentorium passes ventrally and rostromedially along the tympanic cornu practically to the tip of the latter, which articulates with its fellow of the opposite side. Near its anterior termination the tentorium is thrown into a fold, bridging a large foramen for the passage of the semilunar or Gasserian ganglion of the trigeminus (N. V). Upon the apex of the tympanic cornu is the foramen for the passage of the Eustachean tube.

The ventral part of the supratentorial depression is pierced by a foramen for the emergence of the stapedial artery, which then occupies a sulcus in the bone for some distance rostrally. Investigation discloses the course of this artery through the tympanic structures to be entirely within a bony canal, extending from the neighborhood of the basioccipital as before stated, skirting a part of the rim of the fenestra rotunda at the base of the cochlea, and passing between the crura of the stapes before emerging upon the supratentorial depression.

Rostromedial of the parafloccular fossa there are (in the infratentorial area) two foramina of considerable size, within which one may distinguish branches and cribriform spots. The more dorsal of these foramina accomodates the facial nerve (N. VII), and through them pass the cochlear and vestibular branches of the acoustic nerve (N.

VIII). Effort, expended upon several specimens, both cleaned and in spirits, to follow the nerve passages involved was only partially successful, and it is only after histological study of the skull that the topography of the finer nerve twigs of the inner ear can be determined with exactitude.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SPHEN. FISSURE
PREMAX.
NASAL
STYLOMAST. FORAM.
PALAT.
ALISPHEN.
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SPLEN.
D1
SEMISPIN. CAP.
LONGIS. CAP.
CLEIDOMAST.
STERNOMAST.
RHOMB.
ACROMIOTRAP.
SPLEN.
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STERNOMAST.
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SEMISPIN. CAP.
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ORBITOSPHEN.
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SQUAMOSAL
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SUPRAOCCIP.
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TYMP.
ALISPHEN.
LONGIS. CAP.
CLAVOTRAP.
SEMISPIN. CAP.
SPLEN.
A1
RHOMB. CAP.
CLEIDOMAST.
STERNOMAST.
DIGAST.

FIGURE 25. Lateral view of the skulls of Dipodomys (D), Scirtopoda (S), and Allactaga (A), showing also the areas of attachment of some of the cervical muscles.

As already mentioned the dorsal and caudal parts of the bulla of *Dipodomys* are composed entirely of mastoid elements, which are separated from the tympanic portion of the bulla by a bony septum. The latter passes from the zygomatic root of the squamosal caudally along the superior border of the external auditory lumen and thence ventrally to the stylomastoid foramen. From this point two irregularly semicircular lines diverge to join at the exoccipital process and the small area thus inclosed (fig. 26) marks a portion of the mastoid which has overthrust the tympanic, thus forming a small, superficial sinus. The mastoid is entirely cavernous and is divided into a dorsal and a posterior sinus by a thin, bony septum which extends from the posterior border of the external auditory passage to the region of the interparietal.

The dorsal mastoid sinus is a large chamber whose fragile dorsal boundary is strengthened rostrally and medially by bony buttresses. The medial wall is dihedral, the anterior marking the cerebral and the posterior the cerebellar part of the septum. The sinus communicates with the entotympanic cavity (middle ear) by a ventral fenestration through which may be seen the malleus and incus. On the lateral border of this fenestration is a process, upon the summit of which is a minute foramen, for the passage of the ligament of the tensor tympani muscle. Upon the anterior border of the fenestration the rounded margin gives the appearance of being tubular, but it has no lumen and hence would seem to be rod-like for strength. In the more caudal part of the sinus the most striking feature is a portion of the superior semicircular canal, with its ampulla, and below it, the less well marked rostromedial portion of the lateral canal.

The posterior mastoid sinus is smaller than the dorsal, is rectangular in shape and extends dorsomedially. Its walls lack buttresses. Ventrally it also communicates with the entotympanic cavity by means of a fenestration. The most conspicuous features are the extensive curvatures of the posterior and lateral semicircular canals. From the middle of each of these, as well as from the ventral part of the posterior canal, extend bony struts with rounded margins. The fenestration is bounded by the two most lateral of these three struts, and between them is visible the stapedial canal. Lateral to the fenestration and extending from the vicinity of the lateral semicircular canal is the curvature of the bony canal transmitting the facial nerve, which is, in reality, within the lamination between the posterior mastoid sinus and the tympanic. Just lateral to the base of the canal, from the posterior aspect, is a smaller, accessory fenestration communicating with the fossa incudis of the entotympanic.

The true part that the mastoid element plays in the scheme of the temporal complex has never been determined. Certainly by medical men and in all probability by comparative anatomists as well the mastoid is usually thought to be a part of the skull posterior to the ear, composed internally of cancellous or multicavernous tissue which in man may become distressingly infected. Certainly the story of the mastoid cannot be told on a part of one page, but it may be permissible here to point out that in many mammals having use for an efficient balancing organ in the form of a large tail, such as the wood rat for one, the hard, thin shell of the mastoid is completely filled by the bulbous paraflocculus of the cerebellum. Although there may well be exceptions, on the whole it seems safe to say that only following the relative increase in the size of the mastoid element (*Perognathus*) or decrease in the paraflocculus in such mammals as do not employ a long tail for balancing or some such use (pocket gopher, man) does the mastoid take on a markedly cancellous character, the individual cells of which probably communicate with the middle ear. And in the former eventuality the cancellous character of the mastoid becomes lost following extreme hypertrophy (kangaroo rat), or in some mammals (jerboas) even moderate enlargement, of this element.

An examination of other available Heteromyidae indicates the successive steps probably followed during the development of the *Dipodomys* mastoid. In *Liomys* (as in geomyids) the small mastoid is composed of cancellous bone whose airspaces are all of small size. In *Perognathus baileyi* there is a thick surface area of more fragile, finely cancellous tissue with a deep sinus of considerable size. In *P. amplus*, with its larger mastoid, the cancellous zone is thinner while the deep sinus is larger, although still single. In other species of *Dipodomys* and in *Microdipodops* the individual details vary to some extent but not to an important degree.

In the jerboas it was found that the mastoid of *Allactaga* was, on the whole, very similar in size and general conformation to that of *Liomys*, save that there was quite a large fenestration. This opened within a mastoid that had no sign of cancellous tissue but which was hollow and with an outer wall that was very thin, much as in the wood rat. The mastoid shell thus formed communicated with the endocranium and was completely occupied by a paraflocculus with a constricted base. In *Dipus sowerbyi* the slightly smaller paraflocular fossa was comparable to the extent that it was located within a hollow mastoid, but there was no fenestration communicating with the ectal surface, and only a part of the mastoid was thus occupied. This was the extreme caudomedial part, adjoining the occipital. The more lateral and rostral part of the mastoid, located above and immediately behind the auditory tube, was thin-walled and cavernous, and was separated from the parafloccular diverticulum of the mastoid by a thin septum. This large sinus communicated, in the cleaned skull, with the ectotympanic vestibule, by a circular vacuity situated immediately dorsal to the tympanic membrane. Across this foramen, in the spirit specimen of *Scirtopoda*, there was stretched a delicate, transparent membrane very comparable to a tympanic membrane. It is thus indicated that both *Scirtopoda* and *Dipus*, and probably other specialized genera of jerboas, are equipped with a resonantly cavernous mastoid whose efficiency is augmented by a membrane between it and the ectotympanic vestibule, while in *Dipodomys* the larger mastoid opens into the middle ear.

In *Scirtopoda* the development of the mastoid has been carried still farther than in *Dipus*, chiefly dorsally and to some extent caudally, so that the parafloccular diverticulum is over-ridden (from the lateral side in a caudomedial direction) by the mastoid sinus proper. The latter develops two buttresses, comparable to those in *Dipodomys* but more extensive, which almost separate the mastoid into anterior, middle or dorsal, and posterior sinuses.

In many mammals, as the Carnivora, Insectivora, etc., the tympanic part of the temporal complex is derived from an external ectotympanic annulus, of varying robustness and extent, chiefly for supporting the tympanic membrane, and an entotympanic portion, comprising the remainder of the audital bulla proper. It is the rule for these two elements eventually to fuse, and at times this occurs at a relatively early age; or they may remain separate, as in some insectivores. Occasionally, I believe, the entotympanic portion of the audital bulla remains cartilaginous (as in some insectivores) but it is my understanding that in mammals the ectotympanic ring, supporting the membrane, is always present as an element and is always ossified in the adult. But the ectotympanic does not always occupy its primitive ectal position, as mentioned by van Kampen (1905) and others. In the adult *Dipodomys*, for instance, as in many other rodents, it is found that the annulus has migrated to a position within the entotympanic capsule, or audital bulla as we see it. The original stimulus for this is, of course, unknown, but it may be pointed out that in many rodents the tympanic membrane is disproportionately large as compared to the auditory meatus and that unless the membrane developed entally within an enlarged bulla the tympanic annulus would otherwise make for a disproportionately capacious auditory meatus.

By careful dissection of the bulla of an adult *Dipodomys* it was seen that the tympanic annulus is represented by an extremely tenuous ring of bone bounding the membrane, and that this is so firmly fused to the entotympanic capsule that it is distinguishable only as a slight arcuate ridge. The direction of this, and hence of the inclosed tympanic membrane, is situated with the ventral part of the arc at a point about three-fifths the distance from the stylomastoid foramen to the anterior cornu of the bulla. The direction therefrom is approximately 45 degrees to the horizontal, and it extends to a point near the dorsal border of the lumen of the auditory meatus. The relatively immense size of the membrane will be realized when it is mentioned that in diameter it measures in excess of 6 mm. (in *D. spectabilis*), which is therefore more than three-fifths as large as is this detail in man.

Unfortunately the fetal material for determining the development of the ecto- in relationship to the entotympanic parts of the bulla has been unobtainable. I was able to secure from the National Museum, however, a juvenile but a few days old, and after the cranium of this specimen was clarified it was seen that the ectotympanic ring occupies relatively and approximately the same position as in the adult; that the formation of the posterior portion of the entotympanic has a tendency to precede the extension of the anterior cornu; and that at this age the medial part of the entotympanic is ossified while the lateral part, near the meatus, is as yet without sign of ossification.

Thus in *Dipodomys*, and apparently in other heteromyids and geomyids, the ectotympanic is not ectal in situation, but is covered by entotympanic features, and thus all the superficial details of the audital bulla proper are composed of entotympanic elements.

As previously mentioned the external auditory meatus is somewhat projecting in the less specialized heteromyids but this is not the case in *Dipodomys*, in which the orifice is flush with the surface contour. In all of these the lumen is more or less cylindrical, in *Dipodomys* with a very slight recess dorsally. In the jerboas, however, there is a very definite vestibule or recess, located dorsally and just within the orifice, into which opens the fenestration from the mastoid sinus. As mentioned, in life this is covered by a delicate, transparent membrane. In the kangaroo rat, therefore, the great mastoid sinuses, undoubtedly developed as a resonating apparatus, communicate solely with the middle ear. In the jerboas this apparatus can act as a sounding board by means of a membrane of its own, situated in the outer ear, and independent of the tympanic membrane.

The contour of the audital part of the bulla has already been described. In the animals under consideration the composition of the capsule is of two sorts. In *Thomomys* and the pocket mice (both *Perognathus* and *Chaetodipus*) this is quite thick and finely cancellous, so that the cavernous part of the audital bulla, and hence of the tympanic membrane, is smaller than surface details would indicate. In all the others there is no cancellous tissue, the bony walls being thin, hard and either single, or double (*Liomys*), the inner wall being incomplete and separated from the outer wall by a very shallow space, the two being joined by numerous bony struts. Because of this difference in the thickness of the bulla wall the tympanic cavity of *Liomys* is actually larger than of some pocket mice (*Perognathus*) with larger bullae.

Removal of the inferior part of the audital capsule, and of the tympanic membrane, of these rodents with enlarged tympanic features reveals the details of the middle ear with remarkable clarity. It is seen that in reality the malleus and incus are suspended along the lateral border of the superior mastoid fenestration (between superior mastoid sinus and cavum tympani proper) at only two points. Laterorostrally the point of attachment is the anterior ligament of the malleus, from the anterior process of this bone to a process upon the margin of the fenestration; and mediocaudally by the posterior ligament of the incus, from the short process of the incus into the fossa incudis. This line of suspension of the malleus and incus appears to be correctly placed for giving great delicacy of vibration at the articulation between incus and stapes.

The manubrium of the malleus is long, slightly spatulate, and flattened along the entire border that is attached to the tympanic membrane. It ends laterally in a short processus lateralis. There is a strongly characterized manubrial lamina, absent in many rodents, directed at a right angle to the membrane, and the superior margin is slightly expanded, so that the lamina is slightly bimarginate. At about the middle of the superior border of the lamina is a small process for attachment of the extremely tenuous m. tensor tympani. From the ventral aspect it is seen that a long, sharp, processus anterior is directed craniolaterally from the base of the manubrium, for attachment of the anterior ligament. This process, parallel to the axis of the head of the malleus, is attached, throughout the greater part of its length, to the latter by an extremely delicate lamina. The head of the malleus is rounded and there is no well defined neck or peduncle. The articular surface is broad. In *Microdipodops* the malleus is essentially similar, nor greatly different in the pocket mice, save that the muscular process is at the base of the manubrium. In *Thomomys* the plan is much like *Dipodomys* but with different proportions.

In the jerboas the chief difference in the malleus is in the fact that the part of the head anterior to the manubrium is not rounded, with a distinct, elevated, anterior process. On the contrary it appears as though a long anterior process had supplanted the rounded anterior part of the head of *Dipodomys*. The processus muscularis was at the manubrial base, but no process for the attachment of the opposite end was distinguishable in *Scirtopoda*, although in *Dipus* the process was better defined, with much more capacious foramen for the accommodation of the m. tensor tympani, than in *Dipodomys*. In both of these genera there was an anterior ligament of the malleus and a posterior one of the incus much as in *Dipodomys*, but in addition there was accessory suspension of the ossicular chain by a wisp of tendon extending cranially and another caudally, from the processus lateralis of the manubrium. These were attached to the processes at the ventral terminations of the arc of the mastoid-ectotympanic fenestration. In other words the thin, transparent membrane which occupied this fenestration had a ventral border, between the horns of the surrounding bony arc, that was largely tendinous, and in the middle of this tendon was attached the processus lateralis of the manubrium. The important point in this connection is not that there is accessory suspension of the ossicles in these two jerboas, but rather that in one group of highly specialized saltatorial rodents there has been developed, apparently as an aid to hearing, an accessory membrane of the cavernous part of the mastoid, and that the ossicular chain has taken advantage of this circumstance in a manner that is highly suggestive of the thesis that the accessory membrane aids in transmitting sound through the ossicles to the inner ear.

It seems that the details of the malleus are largely correlated with those of the incus. Both are very different from the condition in the wood rat, for instance, in which the suspension is triangular (three-point) rather than essentially linear (two-point), while the articular process of the incus is more at a right angle with the manubrium, rather than largely parallel, as in *Dipodomys*. This is essentially the same scheme as obtains in the jerboas, and although it may well be that this ossicular arrangement is more efficient in the transmission of certain sounds, than is that of the wood rat, still this can not be considered as a saltatorial modification, for it is encountered in the pocket gopher, among other rodents.

Cockerell, Miller and Printz (1914) have characterized the incus of the kangaroo rat as a very remarkable bone, but I am unable to detect any very significant dissimilarity from this bone in other rodents having the same general type of ossicular suspension. In *Dipodomys* the articular surface, and hence the head, of the incus is of generous size, and the crus breve, attached to the fossa incudis by the ligamentum incudis posterius, is essentially on a straight line with the malleus-incudial axis. The articular process, or crus longum, diverges somewhat from the parallel position with respect to the manubrium of the malleus, but its situation when considered together with the stapes may be judged as parallel, or rather as at a fair right angle with the ossicular axis. There is a disk-like lenticular process attached to the crus longum by an extremely fragile and constricted peduncle, prone to breakage during dissection. This articulates, by a synovial joint, with the equally disk-like capitulum of the stapes.

Broadly speaking this type of incus is encountered in all the mammals mentioned (including squirrel, pocket gopher, and jerboas) except the wood rat, although there is variation in the proportions, especially of the articular process. In the two groups of *Perognathus* (but not *Liomys*) the peduncle of the latter process is excessively flattened, or ribbon-like, and this, in slightly different form, is also a character of the jerboas examined.

The stapes of *Dipodomys* is rather broad and short, of such size that the intercrural fenestra is practically filled by the stapedial canal, and with disk-like capitulum. The processus m. stapedius is very poorly defined, and the basis stapedis bullate to a degree where the form is almost hemispherical. The latter character obtains to various degrees in all heteromyids and geomyids, apparently. In *Microdipodops* the crura of the stapes are quite remarkably elongated —a characteristic which makes the whole bone unusually long with respect to the other ossicles.

The stapedial canal has already been described. In *Dipodomys* this is a completely bony canal for the passage of the stapedial artery, which enters the tympanic part of the bulla near the basioccipital, borders the fenestra rotunda and the base of the cochlea, passes between the crura of the stapes and emerges into the endocranium. It is probable that a stapedial artery exists in every mammal at some stage of its development. This is a development of the second aortic arch and in man persists until the middle of the third month of fetal life. In many other mammals it also disappears at some ontogenic stage, while in still others it persists without calcification, and again, as in *Dipodomys*, passes through a completely bony tunnel, which apparently is integral (fused) with the cochlear and entotympanic elements, as is the case in all the Heteromyidae. A complete stapedial canal is present in the squirrel, and in *Microdipodops* also, although in *Perognathus amplus*, representing the silky pocket mice, there is evidence that in spots the calcification has become arrested before completion. In *Perognathus baileyi*, as representing the hispid pocket mice, the bony canal is capacious and complete practically to the stapes, where it ends abruptly and the unprotected (save by soft tissue) artery thence passes between the stapedial crura. It is then a simple step to the condition as found, for instance, in the wood rat, where the canal has disappeared, leaving merely a well marked sulcus to indicate the path of the large artery, which enters the bulla near the paroccipital process. Still another situation is encountered within this and related groups, however. In *Liomys* the artery has apparently disappeared but the bony "canal," without a lumen, persists as far as the stapes and the position of the original foramen adjoining the basioccipital is indicated by a small fossa. In *Thomomys* this process is carried even farther for the solid "canal" passes uninterruptedly through the stapes.

In the jerboas there is a large stapedial artery and at least a part of the canal appears always to be present. An interesting condition is encountered in this group. The dorsal part of the canal (corresponding to the arterial sulcus in the wood rat) is formed by the petrosal, in these rodents distinct from and unfused with the entotympanic. The more ventral portion, or "roof" of the canal, however, is formed of the entotympanic element and this part may be incomplete, enclosing from a third to practically all of the artery as far as the stapes.

In *Allactaga* the petrosal and entotympanic elements are very sharply differentiated indeed. The view within the entotympanic from the ventral aspect discloses the cochlea, including the margins of both the fenestra and a portion of the sulcus of the stapedial artery, as forming the petrosal element that has intruded within the surrounding entotympanic capsule and remained unfused. This is so to only a slightly lesser degree in *Scirtopoda* and *Dipus*. In *Allactaga* there is a definite tendency for the entotympanic to send a thin lamina of bone to cover the medial part of the stapedial artery and a small

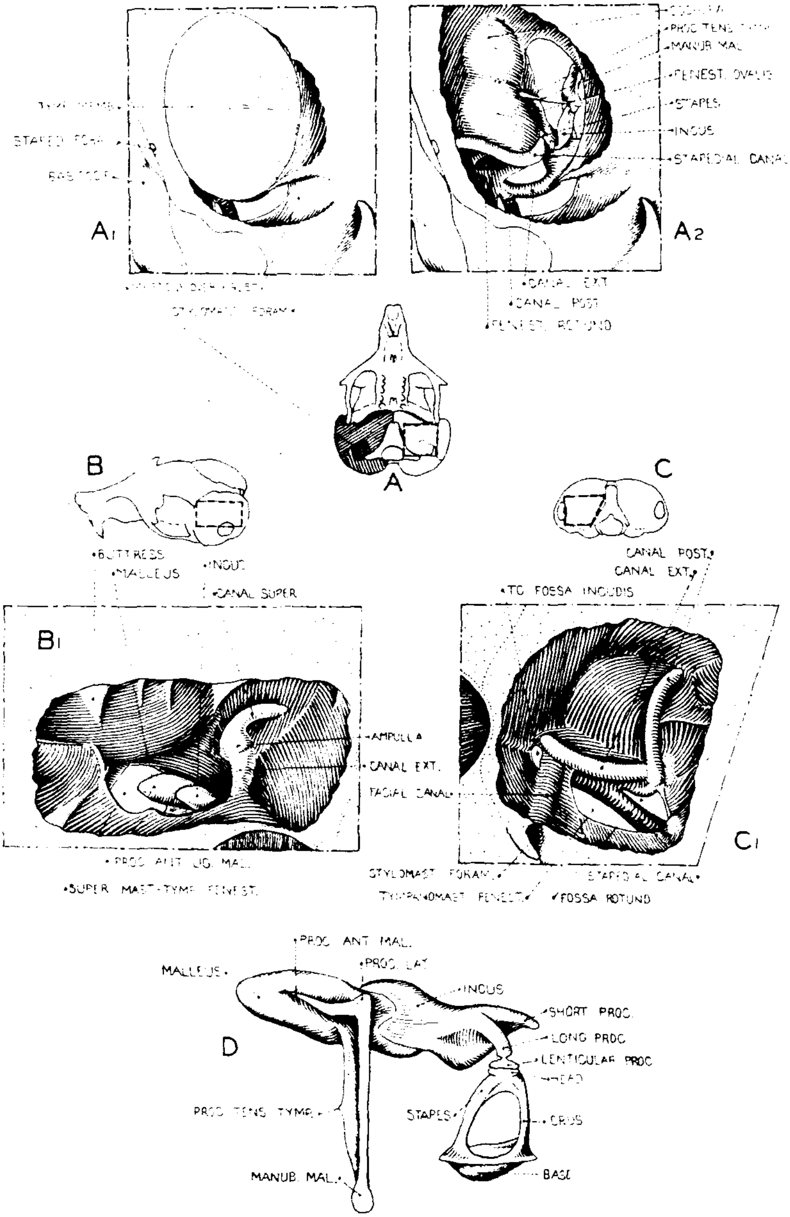
A1
STAPED. FORAM.
COCHLEA
PROC. TENS.
MANUB. MAL.
FENEST. OVALIS
STAPES
INCUS
STAPEDIAL CANAL
A2
CANAL EXT.
CANAL POST.
FENEST. ROTUND.
STYLOMAST. FORAM.
A
B
BUTTRESS
MALLEUS
INCUS
CANAL SUPER.
B1
AMPULLA
CANAL EXT.
FACIAL CANAL
PROC. ANT. UG. MAL.
SUPER. MAST.-TYMP. FENEST.
C
CANAL POST.
CANAL EXT.
TO FOSSA INCUDIS
C1
STYLOMAST. FORAM.
TYMPANOMAST. FENEST.
FOSSA ROTUND.
STAPEDIAL CANAL
PROC. ANT. MAL.
PROC. LAT.
MALLEUS
INCUS
SHORT PROC.
LONG PROC.
LENTICULAR PROC.
HEAD
D
PROC. TENS. TYMP.
STAPES
CRUS
MANUB. MAL.
BASE

FIGURE 26. Details of the middle and inner ear of Dipodomys, showing sketch outline of skulls (A, B, C) indicating areas inclosed in broken line which, in the other items, are shown greatly enlarged and with their bony walls broken away. (A2) is a deeper view of (A1).

portion of the adjacent bone. This is more pronounced in *Scirtopoda*, while in *Dipus* fully half of the ventral part of the cochlea is so covered. The result is that in the jerboas the dorsal wall of the stapedial canal is petrous, and the ventral wall, or that part of it which occurs, is always entotympanic.

In gross features the semicircular canals of *Dipodomys* are hardly worthy of note, save in respect to their quite remarkable freedom from the usual amount of surrounding bony tissue, especially within the posterior mastoid sinus. They are quite similar in conformation to those of other mammals, the superior and posterior canals being in the vertical and the lateral in the horizontal plane, or virtually so. The plane of the superior canal is at an angle to the sagittal plane of about 45 degrees, as is probably invariably the case in the Mammalia.

There are many details of the physiology of hearing about which we know little or nothing. It seems quite likely, however, that in respect to the labyrinth a short cochlea with few coils of the sort found in at least some of the Insectivora represents a considerably more primitive, or less specialized, mammalian condition than does one with more coils. As some of the rodents are equipped with a greater number of coils than representatives of any other mammalian order, so far as known, it appears justifiable to assume that the auditory equipment of such rodents is in *some* respects more efficient than of any other mammals. But whether this takes the form of increased keenness of exactly the human type of hearing cannot be told. In *Dipodomys* there are not an unusual number of cochlear turns but at least it is likely that its great sinuses have introduced into the situation an acoustic element about which we can only speculate.

The cochlea of *Dipodomys*, in ventral aspect, is not merely a slight swelling of the bony surface as is the case in the wood rat, for instance, but is sharply marked off from the surrounding bony features as in the squirrel, and is rather thin-walled. In all the heteromyids and geomyids examined the capsule of the cochlea is apparently fully fused with the bordering entotympanic element. In the squirrel and wood rat (and many other rodents) a sutural line is definable, and in the jerboas this is very distinct, to the extent in *Allactaga* at least where it is probable that the cochlea is attached to the bordering entotympanic only by connective tissue.

At the base of the cochlea caudal to the stapedial canal or artery there is the fenestra rotunda, opening into the scala tympani, and giving access to the scala vestibuli is the fenestra ovalis, into which fits the basis stapedis. In the wood rat, squirrel, and *Dipodomys* there are from two to two and a half spirals of the cochlea. But in other respects the cochlear features of the kangaroo rat are different. In the first place the cochlear capsule in the wood rat and squirrel is short, relatively broad, and subglobular in form. In *Dipodomys* and *Microdipodops*, but not to any marked extent in *Liomys*, *Perognathus* or *Thomomys*, the cochlear capsule is much elongated and has a definite central constriction. It will be found that in all these rodents the part of the scala represented by the lower three-quarters of the first turn is much larger and more capacious than the part more apically situated, and it is this larger, basal portion that is contained, in the kangaroo rats and mice, within the basal expansion of the cochlear capsule, while the remainder is contained within the part above the constriction.

In *Dipodomys* the lamina spiralis ossea was much reduced in breadth as compared with the squirrel and pocket gopher, with the canalis spiralis relatively more capacious. The elongation of the capsule in these two genera was accompanied by spirals that were considerably steeper, and hence much longer in comparison with the width.

In the jerboas there were also slightly more than two spiral turns to the cochlea, which was subglobular and entirely comparable to the condition in the wood rat and squirrel, rather than in the kangaroo rats and mice.

Occipital. The occipital is, of course, composed of supra-, ex-, and basioccipital elements and in *Dipodomys* this complex is hardly less remarkable than that of the temporal.

In the squirrel the occipital is of generous proportions. The mastoid indents the border of the complex between the ex- and supraoccipital elements, and there is a prominent paroccipital process of the exoccipital. In the wood rat the exoccipital is narrower and the supraoccipital sends a long, narrow process ventrally in front of the mastoid, this forming the extreme distal part of the well defined lambdoidal crest. In *Thomomys* this supraoccipital process is represented as much broader, and the crest is correspondingly broad and less sharp. The paroccipital process is also absent as such, but it is represented by a ridge, and the exoccipital covers the entire medial part of the mastoid in a thin lamina. This may very well represent the condition occurring in the ancestral form of the Heteromyidae. In *Liomys* the lambdoidal process of the supraoccipital has become smaller, the exoccipital lamina covering the mastoid has shrunken and retreated medially, and the paraoccipital ridge has assumed the form of a blunt and rather broad process. These steps are carried farther in the hispid pocket mouse (*P. baileyi*) and in addition the increasing size of the mastoid has restricted the parietal to the region entirely rostral to the interparietal. Accompanying this situation the exoccipital has sent a slender process rostrally along the lateral border of the interparietal. In the silky pocket mouse (*P. amplus*) the width of the supra- and exoccipitals is still more reduced, save for the paroccipital process, which extends more lateral than ventral. The anterior process of the supraoccipital, bordering the interparietal, is still present and extremely slender. The culmination of these various steps is encountered in *Dipodomys deserti* and *Microdipodops* (in ratio to bulla development) in which all the occipital elements save the paroccipital process are as much reduced as they could well be, the part of the supraoccipital visible from the dorsal aspect being merely a thin sliver of bone, and the remainder as narrow as the articulation with the atlas will permit.

An analysis of the situation accompanying the reduction of the heteromyid occipital, as followed above, discloses an interesting situation. In the Mammalia the musculature inserting upon the occiput is always confined to this plane. In other words it apparently is incapable of shifting its attachment to the parietals or interparietal, but if there be a need for an altered angle of leverage of the muscles involved this is accomplished by a tilting of the entire occipital plane. But it is entirely feasible for the muscles inserting along the lambdoidal crest to shift from mastoid process of the mastoid to the mastoid process of the squamosal, or from the latter to the supraoccipital border. In *Microdipodops* and *Dipodomys deserti*, however, all of these muscles may be considered as confined to the mastoid alone, for the superior part of the supraoccipital is too insignificant to furnish an appreciable area for attachment.

In the jerboas a somewhat different principle is involved. In *Scirtopoda* and *Dipus* at least, and probably in some others, it is clear that the occipital has not been gradually crowded medially as the bulla enlarged, but rather that the latter has first developed *beneath* the occipital, and that later the overlying occipital lamina has disappeared except along a line corresponding to the position of a lambdoid ridge, which persists as a thin laminar process extending ventrally behind the auditory meatus. This process has disappeared as well in forms with yet larger bullae (*Salpingotus*).

It seems to be beyond question that small saltatorial rodents of the sorts under consideration are in the habit of carrying the body more erect, and hence the head tilted forward at an angle with the body axis, to a considerably greater degree than is the case with the wood rat, for instance. It would therefore be expected, in accordance with the arguments that I have already advanced (1930) in discussing aquatic mammals, that they would have the occipital plane tilted farther to the rear, but comparison with generalized rodents of a variety of sorts shows that this is not the case to any very marked degree. I suspect that this is attributable to a variety of causes, among which is small size, for in the much larger *Pedetes* the backward tilt of the occipital plane is very marked. Then two the smaller heteromyids and jerboas have small, light occipital condyles that are located farther ventral than normal, and it is felt that neither this feature nor that of the unusually rounded conformation of the lambdoidal area can be properly evaluated at this time.

It may be mentioned that *Allactaga* is the only saltatorial rodent examined whose maxillary incisors project to a marked degree. This is popularly presumed to be a development enabling the animal to employ the incisors as a pick for loosening the ground while constucting burrows.

Mandibles.

The mandibles of rodents that are saltatorially modified to the degree that the bullae are of phenomenal size are notable for their weak coronoid processes, following reduction in size of the temporal muscles, and for lateral extension of their angular processes. The latter condition is correlated with the fact that in *Dipodomys* especially, and to a lesser degree in *Microdipodops* and probably *Salpingotus*, the glenoid fossa does not immediately adjoin the zygomatic root of

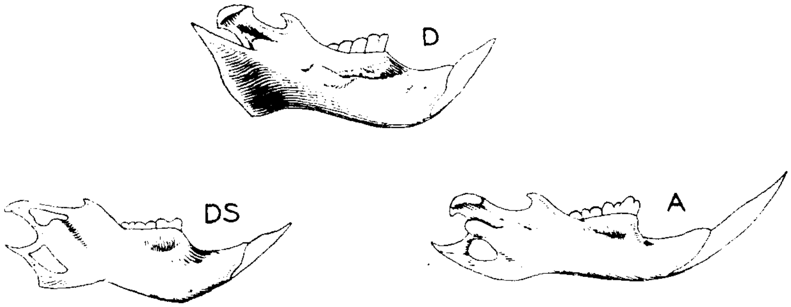

FIGURE 27. Right mandibles (× about 1½) of Dipodomys (D), Dipus sowerbyi (DS), and Allactaga (A).

the squamosal but is somewhat medial to it. Hence, efficiency dictates that in the mandible the insertion of the masseter, upon the angular process, should be somewhat lateral to the condyle. In *Allactaga* the mandible is slender and weak, with low condylar process, the fragility of the posterior part being carried to the point where there is a large fenestration in the bone at the base of the coronoid process. This is also the case in the *Scirtopoda* dissected, while in *Dipus sowerbyi* there is an additional fenestration at the base of the exceedingly weak condylar process (fig. 27). The latter condition is shown to obtain also in *Scirtopoda lichtensteini* at least, according to Vinogradov (1927), but no mandibular fenestrations are shown in the same author's (1922) drawings of *Salpingotus*.

Trunk.

It is extremely likely that if the different segments of the vertebral series could be properly evaluated with each other and with actual body size the results would be very significant in such a study as the present one. The difficulties encountered are very great, however. For example, if the cervical series be compared with trunk length and is found to be relatively shorter in one animal than another, it is impossible to tell whether the result indicates a shorter neck or a longer body, for the reason that there is no standard by which the trunk length can be reduced to an identical value. This difficulty appears so serious that it is considered inadvisable to attach much importance to such comparisons unless they are corroborated by other details, or very seriously to evaluate extensive series of vertebral measurements of all available saltators, on the grounds that they would probably be misleading.

Cervical vertebrae. A mobile head at the end of a neck of considerable length may be employed very effectively as a balance. In aquatic, cursorial, and saltatorial mammals of a high degree of specialization the tendency is for the development of a single, highly efficient equilibrator and the suppression of accessory balancers. Hence, as the tail of *Dipodomys* is so well developed for this purpose one might expect to find the neck somewhat shorter than in the wood rat for instance. That such is not as yet the case to any marked extent may be attributable in part to the fact that the neck in rodents of usual rat-like form is already somewhat shortened. Additional shortening has occurred, however, to a definite degree in the more highly modified *Scirtopoda* (cervical series 8 per cent of the body length; 12 per cent in *Allactaga, Pedetes, Dipodomys;* and 12.5 per cent in *Neotoma*). But perhaps this is not altogether the proper way for comparison, for the atlas and axis have certain rotary functions to perform which leave these two details less affected (at least up to the point where fusion of the vertebrae occurs) by a shortening stimulus than the other five cervical elements. It may be more proper and significant to state that in a *Neotoma* skeleton the five posterior cervicals are almost as long as four (3–4/5) of the anterior thoracics; in *Pedetes* and *Allactaga* the comparable figure is 3–1/3; in *Dipodomys* about 3; and in *Scirtopoda* less than 3.*

In a generalized, quadrupedal rodent there is a very real need for definite mobility of the head and neck while seeking food and during other activities. In a rodent of marked saltatorial adaptation, in which the hind limbs have increased out of proportion to the fore limbs, mobility of the head is in some respects relatively needless and in others disadvantageous. It is of disadvantage to the extent that the effectiveness of the tail as an equilibrator is lessened by a mobile neck: and it is needless because when the animal wishes to indulge in elevation or depression of the head this will be accomplished less by individual movement of this element, as in a generalized form, than by the tendency to accomplish this end by a tilting of the entire body, through the hip joints as a pivot. By this procedure there is little or no necessity for independent movement of the head in the sagittal plane.

Aside from slight difference in the form of the well defined spinous process of the axis, some little variation in the transverse processes of the atlas, and in the degree of fusion of the elements, the cervical series of the genera under consideration do not differ from that of the generalized rodent. In both the latter and the saltatorial forms the last five cervicals lack spinous processes save in *Pedetes*, in which these are slightly developed. The spine of the axis is well developed in all, but least so in *Allactaga*. All have ventral lamellar processes of the sixth cervical, chiefly concerned with the longus colli muscle; but in *Scirtopoda* these processes are weak and ill defined. Lateral prominences consist of a dorsal series, or parapophyses, and a ventral series of longer "transverse" processes, or diapophyses.

Lyon (1901) found that "in *Dipus* atlas and axis, free dorsally and laterally only, below they are fused into one piece." In all of the specimens examined by me, however, the atlas has been free of the axis. In two specimens of *Scirtopoda* at hand the five central cervicals are completely fused, with the axial spine developed so as to surmount the entire complex, but Flower (1874) and Lull (1904) have stated that the last six are fused. It was found that in *Dipodomys spectabilis* the second and third, or second to fourth, were fused, but in some individuals of other species (as *D. merriami*) the fifth was also involved. It thus seems clear that the degree of fusion of the cervicals within this genus is individually variable though with specific trends one way or the other. That some fusion should occur in *Dipodomys* and that this character should be more marked in *Scirtopoda* is entirely expectable, this corresponding with the degree of saltatorial adaptation in other respects. But it is not according to expectation to find that *Allactaga*, which appears to be much more adept at saltation than *Dipodomys*, has all of this series free, as has *Pedetes*.

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* This detail could be determined only approximately for these two genera because of the fused cervicals.

Thoracic vertebrae. The fact that in *Neotoma* and many, if not most, other rodents of a generalized nature there are thirteen thoracic vertebrae, while in all the saltatorial rodents (as well as in *Lepus*) examined there were 12, probably indicates that in leaping rodents the tendency is for a shortening of the thorax, but not of the individual vertebrae involved. The proportion of thoracic to body length accordingly varies in the smaller saltatorial sorts from 30 (*Allactaga*) and 32 (*Dipodomys* and *Scirtopoda*), to more than 35 (*Pedetes*, as in *Neotoma*) per cent of the body length. Actually, however, if one take only the first twelve thoracics in the wood rat it is found that in all these smaller rodents this series averages very much the same, and certainly the differences are too slight to be of much significance.

In *Neotoma* the first thoracic is devoid of a spine, while that of the second is long. Succeeding ones are progressively slightly smaller, and all slope rearward as far as the tenth, which is vertical. In *Dipodomys* the second is not the longest of the series and all are exceedingly short as far as the ninth thoracic. The latter is somewhat longer, and the remaining thoracics are abruptly much longer. In this animal the eleventh is the vertical vertebra. In the jerboas this plan is different to the extent that the posterior thoracics are not abruptly longer. In fact all the thoracics are very low and poorly defined, the anterior being practically absent and only those in the extreme posterior part of the series have really appreciable definition. In these it is the twelfth spine that is vertical. In *Pedetes*, however, the vertical spine is the eleventh, as in *Dipodomys;* but in this animal the spines are all well defined and more uniform than in any of the small saltators examined.

It is thus indicated that in these saltatorial rodents there is a definite tendency for the vertical or anticlinal spine to shift caudally, in ratio to the degree of saltation, from the tenth thoracic in the generalized rodent (*Neotoma*, *Sciurus*, etc.) to the eleventh in sorts better modified for leaping (*Dipodomys*), and finally to the twelfth (jerboas), indicating a corresponding shift in the center of motion of the vertebral column and back musculature.

In the rodents under consideration the lateral muscular processes of the thoracic vertebrae, comprising met-, an-, and diapophyses, are essentially as in the wood rat, the conformation differing only in minor detail. Whereas in *Neotoma*, however, the differentiation of the transverse processes of the anterior thoracics into separate met- and anapophyses may be said to occur upon the eleventh vertebra, this takes place upon the ninth or tenth in the saltatorial forms. Hence this feature does not vary according to the position of the anticlinal vertebra.

Lumbar vertebrae. Complementary to the fact that in all rodents herein considered there are 19 thoracicolumbar vertebrae, there are 7 lumbars in all the leaping sorts and 6 in the wood rat. The fact that in the jerboas and *Pedetes* the proportion of the lumbar series to body length is about 40 per cent and about 34 in *Dipodomys*, while in *Neotoma* this figure is 33 (37 per cent allowing for 7 instead of 6 vertebrae), seems to indicate that no safe correlation with saltatorial habits can be accepted for this item, save, of course, that with any diminution in the number of thoracic vertebrae there will be a compensational lengthening of the distance between thorax and pelvis.

From *Neotoma*, and to a considerable extent from *Sciurus*, the lumbar region of *Dipodomys* and *Pedetes* differs in having the spines higher (especially posteriorly) and narrower in a sagittal direction, the metapophyses higher anteriorly and lower posteriorly, anapophyses relatively better defined (but disappearing from last two or three), and the posterior diapophyses considerably better developed. On the whole these same trends are encountered in the jerboas in all details mentioned save in the anapophyses, which are smaller.

Sacral vertebrae. There are usually 4 or 5 sacral vertebrae in in *Dipodomys*, 3 in *Pedetes*, *Sciurus* and *Thomomys*, and 4 in *Neotoma* and the jerboas. The sacral length accordingly varies from 13 per cent of the body length in *Pedetes* to 25 per cent in *Dipodomys*, so it is unlikely that the number of vertebrae fused into the sacral complex has any direct relationship to saltatorial habits. Width of sacrum appears to be relatively greater in the jerboas than in the wood rat, and greater still in *Dipodomys*. It is believed that this feature, indicating breadth of a part of the back musculature, is probably a saltatorial modification in the kangaroo rat's special equipment for saltation, while approximately the same end may be accomplished in a different manner in the case of the jerboas.

The above reasoning probably applies, in different degree, to the case of the sacral spines. In *Neotoma* the sacral spines are essentially the same as the lumbar ones. In *Pedetes* the three spines of this series are firmly fused together. In *Dipodomys* the first 2, and in the jerboas the first 3, sacrals are devoid of spines. It is believed that the partial absence of sacral spines is merely the reflection of the presence of a spinalis element to the back musculature, stretching between the last lumbar and the last sacrals or first caudal, and while this may be partially attributable to saltatorial habits in these particular animals, the presence of a spinalis in this region can hardly be included in a list of saltatorial modifications.

Caudal vertebrae. The caudal length is amenable to the stimuli derived from various modes of locomotion to a greater extent than any other series of the vertebral column. Strictly cursorial mammals, or those which are even partially saltatorial, often have a tail of insufficient massiveness to constitute an effective equilibrator or counterpoise, this function being assumed by the head and neck; or the tail may be virtually absent (*Lepus*). In strictly saltatorial sorts, however, the tail must be of sufficient length and mass to function as an efficient and almost instantaneous counterpoise, while the head and neck will virtually cease to function in any such capacity. Theoretically it is possible that a saltatory mammal with no tail to speak of and a long, sinuous neck might leap about with the agility of a jerboa, but no mammal has assumed this form, unless one except the giraffe, which is too cumbersome to be saltatorial.

In *Neotoma* there are about 28 to 30 caudal vertebrae, about 30 or 31 in *Dipodomys*, 26 or 27 in *Pedetes*, and 26 to 30 in the jerboas. It is thus seen that there has been no definite increase in *number* of vertebrae, and comparison shows that increase in tail length has been secured by elongation of the individual vertebrae. Thus in a typical skeleton of *Neotoma* the tail is 145 per cent of the body length, about 160 per cent in the much heavier *Pedetes*, 234 in *Dipodomys spectabilis*, 241 in *Allactaga*, and 272 per cent in *Scirtopoda*. This is entirely according to expectation, at least in respect to the lighter rodents, whose long leaps have some features in common with the flight of an arrow.

Many of the chevron bones had been lost during cleaning and no study was made of this feature. It was found, however, that they do not cease between the ninth and tenth caudals, as said by Parsons (1898) to be the case in *Dipus*, but extend quite to the end of the tail in the jerboas as well as *Dipodomys*. They become better defined in the saltators, through *Dipodomys* and then the jerboas, as representing the lighter forms. There is, however, no tendency in these mammals for the chevrons to become long and to simulate the form of a spinous process. But in the heavier *Pedetes*, as in the kangaroo, this tendency is very pronounced and in the former the chevrons near the tail base are as long and as well defined as the spinous processes of the anterior thoracics.

As saltatorial modifications increase there is needed an augmented ability to move the tail more strongly in a variety of directions, as a counterbalance, and the stronger muscular equipment needed in this connection is reflected in these saltatorial rodents by longer processes upon the anterior part of the caudal series. This is especially marked in *Pedetes*, possibly in compensation for its relatively shorter tail.

Sternum. The manubrium is individually somewhat variable in form but does not depart in essential details from that characteristic of most rodents. The number of sternebers is also variable individually. Since reporting on the wood rat (1926) I have examined skeletons of this genus having both 3 and 5 of these elements, and there is probably equal variation in the saltators. Such differences usually mean merely that there have been fusion of two centers, in the case of a reduced number, or separation of an additional center, where augmented. The xiphoid bone was rather short in all instances except the wood rat.

Costae. As already mentioned, there were 13 pairs of ribs in the wood rat and 12 in the saltators. In *Pedetes* the first 11 ribs had both capitular and tubercular attachment to the vertebrae, the first 10 in *Neotoma* and *Sciurus*, 9 in *Dipodomys* and 8 in the jerboas. Reduction in the number of ribs with tubercular attachment is supposed to allow an increase in the mobility of the chest wall, but it is difficult to understand how this feature would be of benefit to saltation.

Extremities.

Clavicle. This bone was present in all individuals but it had no unusual characters.

Scapula. There is but slight relative difference in scapular length among those species considered, but there is considerable variation in other details of this bone. Whereas *Neotoma* has the supraspinous fossa about equal in width to the infraspinous, in the saltators the former is much the narrower, especially in *Dipodomys*, *Pedetes*, and *Scirtopoda*, a character which accompanies reduced width of the supraspinatus muscle. The latter fact seems not particularly significant, for the function of this muscle is intimately connected with that of the infraspinatus. The narrower scapula of these saltators is accompanied by a falciform glenovertebral angle, and this in turn is accompanied by a pronounced broadening of the teres major attachment over its lateral surface. And by the falciform projection of the bone the serratus magnus is given better leverage for depression of the posterior part of the scapula, and hence anterior elevation of the arm, than would otherwise be the case. Increase in the effectiveness of the teres major might very easily be translated into a better ability in digging than the size of the anterior limb might indicate.

The acromion is narrower in the saltators and the coracoid relatively shorter, especially in *Dipodomys*, than is the case in either *Neotoma* or *Sciurus*. The latter feature apparently reflects a corresponding weakness in the development of the short head of the biceps, but as the latter was present in the kangaroo rat and absent in the jerboas it is surprising that the coracoid was weakest in the former.

In most mammals the anterior limb is shorter than the posterior, and plays a lesser rôle in locomotion. Even in the strictly cursorial sorts the hind limbs usually furnish considerably more than 50 per cent of the propulsive force, while the anterior members are more exclusively concerned with support. It must also be borne in mind that even those rodents which we are accustomed to consider as fairly generalized in major features are to a slight extent saltatorial; this has had an undetermined effect upon limb length.

When the arm length, considered in this instance as comprising the combined measurements of humerus and radius, are compared with body length it is found that any reduction in relative arm length that may have occurred cannot be appreciated in this manner, for the percentages are 46 for *Dipodomys*, 45 for *Neotoma*, 42 for *Scirtopoda* 41 for *Allactaga*, and 40 for *Pedetes*. What has actually occurred is probably some slight shortening of the arm in the saltators, which gives the appearance of being enhanced both by a shortening of the body and by a lengthening of the hind limb. Certainly, however, the arm has not lost as much as the leg has gained in functional length.

Humerus. In comparison with the radius the brachial segment is of the same or greater length in *Neotoma* and *Pedetes*, and definitely shorter (44 to 46 per cent) in the other saltators. As this detail varies largely according to group in other mammals there is no reason for assuming it to have adaptational significance in the present case.

In *Dipodomys* the humerus shows practically no bicipital groove. In comparison with *Neotoma* the deltoid insertion is more spinous, with less of the form of a crest or ridge, reflecting the more circumscribed origin of the deltoid muscle in *Dipodomys* (and the jerboas). The anterior part of the greater tuberosity is very poorly defined, for some obscure reason, but the lateral part is much as in the wood rat. The lesser tuberosity is slightly higher. The lateral epicondylar ridge is relatively broader and the trochlea is located a bit more medial in respect to the humeral axis. There is an entepicondylar foramen in *Dipodomys*, as in the wood rat, and in the former only among the animals dissected is there a well formed sesamoid bone within the original tendon of the supinator and located directly lateral to the radial head.

The jerboas agree in having a bicipital groove, with the anterior part of the greater tuberosity well defined but lower than in the wood rat, while the entepicondylar ridge is not quite so broad but extends farther proximally. There is no entepicondylar foramen. In a specimen of *Pedetes* the latter is present, although it did not occur in an adult which Parsons (1898) dissected. In this animal the insertion of the deltoid was indicated not by a sharp process, but by a broad, blunt rugosity.

Radius and *ulna.* Length of forearm has already been given. Otherwise there are few details of any significance. In *Neotoma, Dipodomys* and the jerboas these bones are all very similar, and all show the same tendency to develop a thin lamella of bone upon the radial border of the ulna to supplant in large degree the interosseous membrane. This is not the case in *Pedetes*, however. In *Dipodomys* the "falcate process" of the ulna is knob-like with a slightly constricted neck, so that at a casual glance it is easily mistaken for a carpal bone. Adjoining it, but upon the volar aspect, is a knob-like process of the radius. Neither character is present in the jerboas.

Manus. The manus of *Dipodomys* is entirely comparable to that of *Neotoma*, differing from the latter only in details that are relatively minor. It consists of a broad scapholunar, ulnare (triquetrum), a large pisiform extending well to the volar surface of the lateral border of the hand; a large, long, and strong medial sesamoid which completely bridges the long flexor tendons of the hand and which articulates with the pisiform; a centrale, first (multangulum majus), second (multangulum minus), and third (capitatum) carpale, and an unciform (hamatum). In *Allactaga* the second carpale was more medial than strictly proximal to the second metacarpal. In this animal there was a small sesamoid on the dorsum of the manus, located between the radius and the medial part of the scapholunar. It was not possible to determine this point in the more completely cleaned skeleton of *Scirtopoda*. The latter, however, exhibited the character of the second carpale described for *Allactaga*, but the scapholunar was narrower (transversely), more spherical, and less strap-shaped, while the ulnare was correspondingly broader.

No study of the disarticulated bones of the manus was undertaken, and the only features apparent in the articulated skeletons as worthy of note were the facts that in *Allactaga* the claws were shorter and the digits correspondingly longer, especially the second phalanges, than in *Scirtopoda*. In addition there was, in *Dipodomys* and the jerboas, a definite process, proximally directed, upon the dorsum of the pollex at the base of the nail. In *Dipodomys* this has the appearance of being a sesamoid bone between the first and second phalanx, while in *Scirtopoda* it is better developed and seems to be completely fused with the terminal phalnax. Neither manus was articulated in the skeleton of *Pedetes*.

Pelvic girdle. The components of the innominate bone are thoroughly fused in all the specimens examined, so that the precise limits of the individual bones cannot be defined. The attachment of the innominate to the vertebral column is at the anterior part of the sacrum, and the angle of the pelvic axis with that of the column is about the same in all animals examined.

The total length of the innominate is not very significant in the present study, for length of ilium especially is dependent upon many diverse factors. The innominate is about 48 per cent of the body length in *Dipodomys*, 47 in *Scirtopoda*, 42 in *Pedetes*, 39 in *Allactaga*, and 35 in *Neotoma*, and it is not possible to make much use of such an array of figures, especially when it is considered that body length is also reflected thereby. A much more significant comparison is that of the postacetabular to the preacetabular measurement of the pelvis. The former, expressed in percentage of the latter, is

66 per cent in *Neotoma*, 62 in *Pedetes*, 71 in *Dipodomys*, 81 in *Allactaga*, and 94 in *Scirtopoda*. These figures may, of course, indicate that there has been experienced by the saltatorial forms either a shortening of the ilium, a lengthening of the ischium, or both. Comparison of specimens seems to indicate that there has been a definite alteration in both elements, but that that of the ilium is the more pronounced, and that the degree of change is in direct ratio with saltatorial specialization.

But this feature may be further analyzed. Is it the part of the ilium from acetabulum to sacral attachment, concerned chiefly with the mechanics of pelvic and hind limb support, or the portion from sacral articulation to the iliac spine, more concerned with muscular and ligamentous considerations, that is chiefly concerned? If the distance from the center of the acetabulum to the center of pelvo-sacral articulation be designated as the postsacral portion of the ilium, and the distance from the latter point to the tip of the iliac spine as the presacral, then the percentage of the postsacral to presacral measurement is 70 per cent for *Neotoma*, 96 for *Dipodomys*, 110 for *Allactaga*, 112 for *Pedetes*, and 117 for *Scirtopoda*. These figures are very definitely suggestive of the thesis that as saltatorial specialization of rodents increases there is alteration in the (chiefly) mechanical requirements, that will shorten the distance between the hip joint and the sacral articulation, and that this is correlated with (or apparently followed by, in this group) a further but less pronounced, shortening of the anterior ilium.

I am entirely unable to convince myself that a long ilium, projecting far forward of the hip joint, can have been developed, or retained, because of active muscular advantage alone, for there are no really important muscles attached to the anterior ilium that would not seem to function approximately as well if this bone were shorter. But the *passive* muscular, and ligamentous, advantage should be relatively great. Surely muscles and ligaments of fixation would operate to better advantage from a relatively long presacral extension of the ilium in furnishing leverage to counteract any strain put upon the pelvis by postsacral stress. This is an interesting possibility to consider but it can by no means be accepted until more is known of the contrary conditions encountered in some of the primates, as shortly to be discussed by W. L. Straus, Jr.

Other major details of the ilium comprise the vertical dimension, or width, and the degree of lateral divergence of the spines. In *Dipodomys* the distance between the ilial spines is about the same as in *Allactaga*, but they flare more in the latter for the reason that the sacral width is less. These details are largely dependent upon the conformation of the lateral part of the longissimus dorsi, attached to the medial ilium. In *Scirtopoda* this detail was relatively poorly developed and, as might be expected, the interspinal measurement is less. Vertical measurement of the ilium is apparently correlated with requirements chiefly of the glutei minimus and quartus (scansorius) and of the iliacus, although the other muscles attached to the ilium undoubtedly have some effect in this connection. Attention should be called to the fact that in *Scirtopoda* the gemellus superior reaches practically to the spine of the ilium.

The relative size of the superior and inferior gluteal fossae is a puzzling detail. In *Neotoma* and *Dipodomys* the superior is very noticeably the larger, while in the jerboas and *Pedetes* this is so of the inferior fossa. I am unable to correlate this with muscular attachments for the reason that the gluteus minimus may occupy one fossa in one genus and the other in another. The muscles which have their chief attachment upon the spine of the ilium are the glutei superficialis and medius, and to a lesser degree the iliacus, but the differences existing seem not to be correlated with variations in the shape of this bony detail.

The rectus femoris process of the ilium was slightly smaller in *Dipodomys* than *Neotoma*, still more reduced in *Scirtopoda*, and absent as a process in *Allactaga* and *Pedetes;* but corresponding differences in the muscle were not detected.

There are various factors that must be considered in discussing the posterior pelvis. For instance it is almost impossible to place a proper valuation on symphyseal details. We know nothing about the requirement of strength in this structure, or the amount of spreading that takes place in the different sorts of mammals during parturition. Needless to say the latter factor is *the* most important consideration in the shaping of the pelvis, for the variant developing a pelvic architecture which does not allow the passage of the young is summarily eliminated, and no contrary stimulus, no matter how vigorous, can have any effect on the species.

In this connection the details of the symphysis pubis may or may not be of importance, according to genus. Presumably, as in some other rodents (guinea pig, pocket gopher), there is a softening of the connection and a lengthening of the connecting fibers to various degrees, according to group requirements, when the young are born. At any rate the symphysis is about as long in *Dipodomys* (3.7 mm.)

as in the wood rat, longer in *Allactaga* (6.4 mm.), and unusually extensive in *Scirtopoda* (11.4 mm.). In the latter genus the pelvic arch is pronouncedly and acutely V-shaped, but U-shaped in *Dipodomys*, with *Allactaga* occupying an intermediate position.

The relatively longer postacetabular measurement of the innominate in the saltators allows for a somewhat more effective leverage for the muscles attached posteriorly, with function of retracting the thigh and flexing the shank, only the former of which actions is primarily helpful during saltation. The mechanical and numerous muscular stimuli that combine to cause the variation in the minor details of the ischium and pubis are so inextricable that it seems unwise to attempt their interpretation. It may be pointed out, however, that while in *Neotoma* the superior tuberosity of the ischium is directly vertical to the inferior tuberosity, as is the case in *Scirtopoda*, in *Allactaga* the inferior tuberosity is anteriorly, and in *Dipodomys* posteriorly, situated in relation to the superior tuberosity. All four saltators, including *Pedetes*, are characterized by the lateral projection of a spinous process upon the superior tuberosity of the ischium, which, however, in *Dipodomys* assumes more the form of a crest. This is most likely attributable to the increased efficiency, and stronger, more tendinous origins, of these components of the hamstring muscles thereto attached, and which, in connection with fixing action by the extensors of the shank, are of major use in the retraction of the thigh. In *Pedetes* the ischial tuberosity is projected dorsally so as almost to be falciform. Both *Dipodomys* and *Pedetes*, but not *Neotoma* nor the jerboas, have a sharp ischial spine, marking the anterior limit of the obturator notch, upon the superior border of the ischium. This is undoubtedly correlated with the fact that in the two genera the obturator internus is double, and the notch is a necessity for the proper guidance or confinement of the tendon passing from the anterior division of this muscle.

Before considering the individual bones of the posterior extremity it will be desirable to make some comparison of the limb as a whole. In the case of mammals with non-elongated carpal and tarsal elements the usual intermembral index 1/* is significant, and for mammals of the general organization of the wood rat this would doubtless prove to be entirely satisfactory. As a matter of interest it may be given for the others. The intermembral index is accordingly 74 in *Neotoma*,

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* 1/ $\frac{\text{humerus} + \text{radius} \times 100}{\text{femur} + \text{tibia}}$ = intermembral index.

96 in *Lepus*, 46 in *Dipodomys*, 48 in a large kangaroo (*Macropus*) of undetermined species, 39 in *Pedetes*, 38 in *Scirtopoda*, and 36 in *Allactaga*. But whereas in these genera the humerus plus radius does equal functional length of arm (if one except the short distance from the radius to the palm), the femur plus tibia is not the functional length of leg, for these mammals are largely digitigrade, at least in locomotion. Theoretically the leg length should be the distance from the head of the femur to the knee, thence to ankle and finally from the ankle to point of the anterior foot from whence is applied the final leaping impulse. This is some point between the anterior end of the metatarsus and the tip of the toes, impracticable of precise determination. So for saltatorial mammals the intermembral index should be modified by adding to the sum of femur and tibia the distance from the center of the ankle joint to a point representing the average anterior termination of the three or four longest metatarsals. The index is then 60 for *Neotoma*, 35 for *Dipodomys*, 30 for *Pedetes*, and 25 for both *Allactaga* and *Scirtopoda*. This probably represents with a satisfactory degree of accuracy the respective saltatorial attainments of the genera in question.

Femur. Comparison indicates that the percentage of the femur to body length is about 31 in *Neotoma*, 42 in *Pedetes*, 43 in *Dipodomys*, 47 in *Allactaga*, and 49 in *Scirtopoda*. But it must be remembered that this also reflects relative difference in body length. A more satisfactory treatment would seem to be to express the proportion of the femur in percentage of the sum of femur, crus and pes reduced to 100. The value of the femur is then 35 in *Neotoma*, 28 in *Dipodomys*, and 24 in *Allactaga* and *Scirtopoda*, indicating relative reduction in the length of this segment accompanying elongation of the limb.

Compared with *Neotoma* the femur of *Dipodomys* has the greater trochanter higher with respect to the head, and the third trochanter as well is situated more proximally. This indicates an increase in the leverage of the gluteus medius complex and probably a tendency toward the speeding up of the action of the gluteus superficialis and adductor brevis, inserting upon the third trochanter. Similarly compared the femur of the jerboas has a relatively larger neck situated more nearly at a right angle to the shaft, which latter is more slender. The greater trochanter is also higher, and this is especially marked in *Pedetes*. In the saltators there seems to be a tendency for the trochanteric or obturator fossa to extend toward the lesser trochanter, culminating in *Pedetes* in which the fossa includes the base of the latter trochanter, thus theoretically increasing the strength of movement of the femur to the rear during leaping. The absence of a third trochanter in *Pedetes* and the jerboas is a group character and has nothing to do with saltation.

There were many minor differences in the attachments of the thigh musculature, but these are difficult to evaluate precisely and it is felt that none of them were of outstanding, or at least clearly apparent, significance in regard to saltation. Some consideration, however, should be given to probable trends.

As is well known a high degree of cursorial or saltatorial specialization is conducive to the confinement in a single plane of the movements of the joints of the limbs employed in propulsion, and the reduction of possible movement in other directions. This means that, theoretically, all muscles that can aid in simple flexion and extension of the limb segments will be developed, while those which normally accomplish abductive, adductive and rotational action of the member will tend toward reduction unless they prove to be of importance for fixation during action of the flexor and extensor groups of muscles. In addition it would be expected that there would be a simplification of the musculature and a trend toward the consolidation or partial fusion of the components of the several groups—a tendency toward the fusion of the elements of the quadriceps femoris, or of the adductor mass, for instance.

It may be noted that the psoas minor was weak in all specimens dissected, while the psoas major was well developed, but to a lesser extent in *Scirtopoda*. The vastus medialis was especially extensive in *Dipodomys* and smaller in the jerboas. The tendency shown for the fusion of some of the shorter hip muscles is often encountered in rodents, but it should be noted that the obturator internus was double in both *Dipodomys* and *Pedetes*, although weak in the jerboas. It appears that there is a tendency in the saltators, most pronounced in *Allactaga*, for partial fusion of the adductor mass, and in the animal mentioned one head of the semimembranosus is also involved. In the jerboas the adductor magnus is relatively more robust at origin and appears to be better situated for extending the thigh, while the reverse is the case with the gracilis, which originates farther forward and seems less of an extensor and more of an adductor of the knee than in *Dipodomys*. The biceps mass is relatively more extensive in the jerboas, but it is difficult to see any functional advantage to be derived from the fact that in this group the biceps has split up into several divisions.

Crus. In the three genera dissected there were five sesamoid bones of the knee in addition to the patella. As usual there was a minute capsular or semilunar sesamoid upon either side, a lateral sesamoid concerned with the gastrocnemius lateralis, a medial one connected with the medial head of this muscle and with the plantaris, and in addition a popliteal sesamoid upon the medial side of the joint, within the tendon of origin of this muscle. There is apparently an increase in the size of the gastrocnemial sesamoids in direct ratio to saltatorial modifications, and they are especially large in *Pedetes*. In an articulated skeleton of the latter all five bones are in place. The gastrocnemial sesamoids are relatively very large indeed, with broad articular surfaces directed toward the articular surface of the condyles, and the lateral one has separate, well marked processes for the attachment of the gastrocnemius and of the plantaris. The popliteal sesamoid also has a broad articular surface.

In all the animals under consideration the crus or shank comprises a single bone, the reduced fibula being firmly fused, at least distally, with the large tibia. The comparative lengths of femur and tibia are especially prone to variation according to specialized habits and this feature is particularly significant in relation to saltatorial ability.

Even the most primitive of mammals are often definitely specialized in some direction and it is somewhat difficult to give instances of what is meant by a generalized type. But it seems probable that in a hypothetical, composite picture of generalized mammals the femur would be a trifle longer than the tibia. That this is actually the case with the wood rat is convenient for purposes of comparison, but after all *Neotoma* is both saltatorial and arborial to some degree.

If tibia be compared to body length the resulting percentage will be found to be about 30 in *Neotoma*, 52 in *Pedetes*, 57 in *Dipodomys*, 62 in *Scirtopoda*, and 64 in *Allactaga*, indicating an actual, progressive elongation of the crus as leaping ability increases. Expressed in percentage of total limb length (the sum of femur, tibia and pes) it will be found that this value for *Neotoma* is about 33, 35 for *Dipodomys*, again 33 for *Allactaga*, and 30 for *Scirtopoda*. So the relative alteration of this segment is slight and of doubtful import.

The free fibula, or rather the part of it above its fusion with the tibia, is 61 per cent of the tibial length in *Neotoma*, 49 in *Pedetes*, 44 in *Scirtopoda*, and 39 per cent in *Dipodomys* and *Allactaga*. So the tibia-fibular juncture is, in the main, relatively higher in the more saltatorial rodents. In the smaller saltators it could not be demonstrated satisfactorily that the lateral malleolus is composed of the distal fibula (although undoubtedly it is), but in *Pedetes* this was clearly apparent (as in *Neotoma*) and the element was here attached to the tibia only by connective tissue for a distance of two or more centimeters.

As compared with *Neotoma* the crus is relatively more slender in all of the saltators considered (except *Pedetes* ?). *Dipodomys* has the anterior tibial fossa deeper and the posterior shallower, but the proximal part of the fibula and its head are broader, and this anatomical character is largely the case in the jerboas as well, but the posterior fossa is even weaker. As compared with *Dipodomys* the tibia of the jerboas is absolutely more robust, but not relative to length.

In regard to the muscles and mechanics of the thigh the only thing apparent in the present connection is that there is a gradual recession of the hamstring attachments toward the knee accompanying saltation. In other words, in *Neotoma* the semitendinosus or biceps, or both, extend along the shank to a point about 70 per cent the distance from the knee to the heel, while in the saltators this is reduced to less than 50 per cent; indeed but little more than 40 in the case of *Scirtopoda*. It is also apparent that the muscles of the crus proper have shorter muscle bellies and longer tendons in the saltators. This fact is difficult of precise presentation by percentage, but by defining the spot where the muscular part of the gastrocnemii may be said to cease, calling the part of the crus above this spot the muscular part of the shank, and the distance therefrom to the heel as the tendinous part, it will be found in *Dipodomys* that the former comprises 56 per cent of the length of the triceps surae. In *Allactaga* the muscular part is 53, and in *Scirtopoda* 50 per cent of the whole.

Aside from the differences in the muscles of the shank imposed by the digital variation, it is worthy of note that in the jerboas the peroneus longus was very small, doubtless as an accompaniment of the reduction in possible pronation of the joint following saltatorial habits. As suggested by osteological details the tibialis anticus was very robust in *Dipodomys* and *Scirtopoda*, and even larger in *Allactaga*, while the flexor tibialis and tibialis posticus were very weak indeed.

Pes. No rodent is completely digitigrade. All rest with the feet flat upon the ground but as saltatorial ability increases and the foot becomes more springy, an animal will undoubtedly be more prone to stretch upward upon its toes in order to reach above its head to smell or touch some object, than will one not so specialized. And the longer foot will give to it the advantage of being able to stretch farther than the unspecialized sort. But it may be stated as a generality that during speedy locomotion all rodents fitted for running, as well as leaping, are digitigrade in respect to the pes. That is, when running swiftly the final inpulse for projecting the body forward is applied not by the heel, but by the toes not only in the jerboas but in the kangaroo rat, the wood rat, and even in man. Whether the digits themselves are chiefly concerned, however, or merely the distal ends of the metetarsals, is uncertain. For convenience, however, the latter is assumed to be the case, and the effective length of the foot in these animals is therefore presumed to be the distance from the center of the ankle joint to the end of the second longest metatarsal.

In *Neotoma* the pes is but 29 per cent of the body length, while in *Pedetes* this rises to 55, in *Dipodomys* to 58, and in *Allactaga* and *Scirtopoda* to 82 and 95 per cent respectively. So it is seen that when so compared the pes is more than three times longer in this three-toed jerboa than in the wood rat. When the pes is compared with the femur and the tibia the relative positions of the animals are the same. When considered in percentage of total leg length (sum of femur, tibia and pes) it is found that this value is about 32 for *Neotoma*, 37 for *Dipodomys*, 42 for *Allactaga*, and 46 for *Scirtopoda*, showing the greatest relative as well as actual elongation of any of the limb segments accompanying saltatorial specialization.

The chief elements of the tarsus of *Dipodomys* consist of calcaneum, astragalus, navicular, first, second and third tarsales, cuboid, and several sesamoid bones. In comparison with *Neotoma* the tarsus of this genus differs in the following repects:—In the former animal the part of the calcaneum which appears from the lateral aspect to be articulated with the astragalus is somewhat raised. In *Dipodomys* it is sharper and higher, and this may be termed the lateral articular process of the calcaneum. In dorsal view the broad, shelf-like, lateral process of the calcaneum of the wood rat, beneath which pass the peroneal tendons, is much reduced in the kangaroo rat, accompanying presumed reduction in pronation, and it has shifted to the anterior end of the bone. The astragalus has become reduced in length so that anteriorly it does not terminate even with the end of the calcaneum but considerably to the rear, and there is a slight reduction in the size of tarsale 2. The medial tarsal sesamoid, upon which the tibialis posterior inserts, is considerably smaller. In at least some individuals of both kangaroo rat and wood rat there is a minute accessory sesamoid immediately forward of the other, which appears not to be associated with any muscle but rather with the ligamentous structure of the ankle. In *Neotoma* the posterior process of metatarsal 5 is long and falciform, completely covering the peroneus longus canal, but in *Dipodomys* it is much shorter and does not cover the canal. The reduction of this process has probably accompanied a diminution in the importance of the peroneus brevis which inserts thereon, as well as an increase in the tendinous character of the abductor ossis metatarsi quinti. The first metatarsus of *Dipodomys spectabilis* is much reduced but still is considerably longer than any bone in the second or third row of the tarsus.

Ventrally upon the pes of *Dipodomys* the navicular projects in a sharp ridge between tarsale 1 and the calcaneum, which prominence is entirely lacking in *Neotoma*. The sustentaculum tali is much narrower in *Dipodomys*, while the ventral surface of the cuboid is broader. Resting upon (plantarward of) the entoventral process of metatarsus 5 is a small sesamoid from which arise those slips of the flexor digitorum brevis which pass to digits 4 and 5. I have discovered a small sesamoid to occur also in the same position in at least some specimens of *Neotoma*, and the definition of this doubtless increases with age.

In most features, except for a longer cuboid, the tarsus of *Pedetes* resembles that of *Dipodomys*.

In the skeletons of both jerboas before me there is a sesamoid bone, measuring 6.5 mm. in length in *Allactaga*, and 3.5 mm. in *Scirtopoda*, still attached to the heel, its original position having been in the distal part of the calcaneal tendon. In the spirit specimens dissected this part of the tendon had not yet become calcified, but this evidently takes place completely as age advances, and has followed toughening of the tendon. In the jerboas the greater calcaneal process or heel is actually longer (5.7 mm. in *Scirtopoda*, 5.3 in *Allactaga*, and 4.6 in *Dipodomys*), giving greater leverage to the tendon. The lateral articular process of the calcaneum is higher and sharper in both jerboas than in the kangaroo rat, and the trochlear part of the astragalus is more nearly cylindrical (a greater part of a circle). From the dorsal aspect the rest of the tarsus is essentially as in *Dipodomys*, save that in the jerboas the first tarsale is actually longer, with a sharp terminal process plantarward in *Scirtopoda*, with which there articulates the medial tarsometatarsal sesamoid. No muscles are attached to this but against its flattened lateral surface there pass the tendons of the flexores digitorum breves. In *Allactaga*, however, the sesamoid similarly situated, but larger, was located within the original fasciculi of the short flexors to digits 1, 2, and 3.

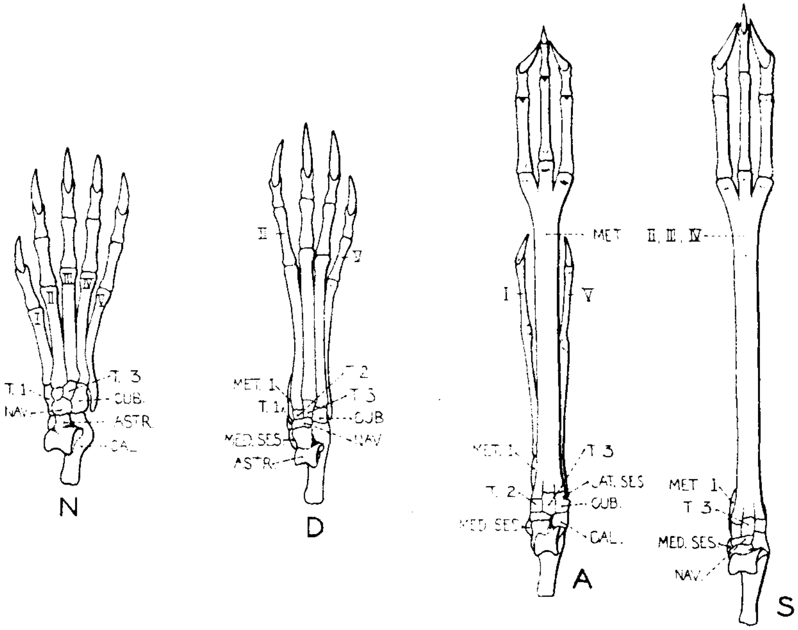
Upon the lateral side of the foot the abductor ossis metatarsi quinti was reduced in *Dipodomys* and still more so in the jerboas, especially in *Scirtopoda*, in which it was merely a tendinous band. In *Dipodomys* and *Allactaga* it passed from the calcaneum to metatarsus 5 as usual (probably with fibers to the cuboid as well in the latter genus). In *Scirtopoda*, without a fifth toe, it passed to the deeper of the lateral tarsal sesamoids, which appears to be the vestige of a metatarsus 5, and is so termed. In *Dipodomys* the 2 contrahentes appeared to originate from the plantar aspect of the navicular, but there were none in the jerboas.

In *Allactaga* there was a large tarsometatarsal sesamoid at the base of the fifth digit. This was within the original fasciculi of the short flexors to digits 4 and 5, but the true origin of these slips seems to be more precisely from the large plantar process of the cuboid. In *Scirtopoda* this was not the case, and the short flexors of all three digits seemed to take origin directly from this sesamoid, which was located superficially between the cuboid and the vestigeal fifth metatarsal. In *Dipodomys* the short flexors appeared to originate as follows: that to digit 2 from the navicular; that to digit 3 from the cuboid; and those to digits 4 and 5 from the lateral sesamoid.

It should be mentioned that as saltation advances and the chief action of the ankle joint is confined more strictly to a single plane, there is a progressive narrowing of the entire tarsus; and that the length of tarsus remains absolutely about the same, except for a slight lengthening of the heel.

The metatarsus of *Dipodomys*, as compared with that of *Neotoma*, is somewhat simpler, the bones are less irregular at base but more crowded together, and in cross section the metatarsal complex is more arched. Otherwise (except for the absence of the fifth) the metatarsi of the two are very similar, not only in detail but in proportions. In *Dipodomys* the percentage of the third when compared to the entire foot length is but two or three points greater than for *Neotoma*. So it is seen that the greater length of foot in *Dipodomys* is gained by elongation both of the metatarsus and digits in practically equal degree. This is particularly well illustrated when the length of metatarsus 3 is compared with the effective leg length. In *Neotoma* this percentage is (in two specimens) between 10 and 11, in *Dipodomys* 18, in *Allactaga* about 28, and in *Scirtopoda* 30, showing that the elongation of this detail progresses at a more rapid rate, as specialization advances, than any other part of the leg.

The step toward saltatorial specialization illustrated by the jerboa metatarsus as compared with that of the kangaroo rat is a great one. In the jerboas and most of their close allies there is a central "canon bone," formed by the fusion of the second, third and fourth metatarsals. In addition to this there may also be a shorter fifth toe (*Scarturus*), or both a fifth and a first (*Allactaga, Euchoreutes*). I do not regard the phylogeny of the foot in these several types of saltators as having been at all uniform, but on the contrary, as involving several processes.



III
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T. 3
LAT. SES.
CUB.
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A
II, III, IV
MET. 1
T. 3
MED. SES.
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FIGURE 28. Dorsal view (× 1) of the bony details of the pes of Neotoma (N), Dipodomys (D), Allactaga (A), and Scirtopoda (S).

The foot of *Scarturus* I regard as being most comparable to that of *Dipodomys*. As in the latter genus it appears likely that the hallux dropped out at an early stage of the more extreme part of its pedal evolution, and that the fifth digit kept relative pace with the others to about the stage now occupied by *Dipodomys*. In the case of the five-toed jerboas the hallux, for some particular reason, was not lost as in *Dipodomys*, but developed at about the same rate as the fifth digit, probably to form a fairly symmetrical pes with five distinct metatarsals. It seems likely that from this stage (comparable with that of the dipodomyid pes) the saltatorial evolution of the three middle digits of the ancient jerboa stock developed at an augmented pace, rapidly becoming elongated and cemented into a single bone, while the first and fifth toes (where these are now present) ceased further to develop and diminished in importance. Unless these are actually put to some use, such as for scratching (in the case of the hallux) or as an aid in acting as a "snowshoe" in soft sand, these accessory digits are undoubtedly in course of diasppearance.

From the above it should not be postulated that *Scirtopoda* and *Dipus* necessarily passed through the same stage as now occupied by *Scarturus* or *Allactaga*. That it may have done so is readily granted, but to me it appears more likely that at the stage comparable to existing *Dipodomys* it had lost both first and fifth digits, as *Salpingotus* has done in advance of the fusion of the three metatarsals.

In *Allactaga* the first phalanges of both first and fifth digits are of about the same length, while the metatarsus of the fifth is somewhat longer than that of the first; but because the first tarsale is so long the claw of the hallux is actually in advance of that of the fifth. In this genus the fifth metatarsal is only 37 per cent of the third, while in *Dipodomys* the former is but a couple of millimeters the shorter.

In the two jerboas under consideration the three main metatarsals are completely fused, save as they tend to split along their original lines of juncture following extreme dessication of the skeleton. The resulting tarsal shaft is even more slender than the distal tibia. Near the base of the toes the bone broadens greatly, and separates into tripartite divisions, each bearing a capitulum for the articulation of a digit. The separation of the three divisions extends a short distance proximal to the end of the bone, undoubtedly for the purpose of affording a surface for the attachment of ligamentous vestiges of the interossei muscles. This point should be noted. The degree of suppression of these ligaments, and of the fusion of the bony surfaces concerned, is a criterion of the specialization for saltation in that it reflects the amount of independent digital movement that is retained.

The metatarsophalangeal sesamoids are slightly smaller in *Dipodomys* than in *Neotoma*, but are much enlarged in the jerboas. It appears certain that the size of these is not determined so much by their acting as anchors for those of the short foot muscles as are attached to them, as it is by their function of pulleys, over which glide the long tendons to the digits.

Except in the matter of length the digits of *Dipodomys* are not materially different from those of *Neotoma*, save that digit five is relatively shorter in the former. The case of the hallux in the kangaroo rat is of especial interest, however. In certain groups within this genus a rudimentary external hallux is invariably present; and species so characterized were formerly designated generically as *Perodipus*. Other groups invariably lack the hallux, and in still others this feature is present in some individuals and lacking in others; or it may be present upon one foot and absent from the other. As has been pointed out by Grinnell (1922) the reduction of the hallux is least advanced in kangaroo rats of the *ordii* and *microps* groups. "In the *agilis* group, two of the constituent races show occasional individuals in which the claws or phalanges are absent on one or both of the first hind toes. In the *heermanni* group, one race shows occasional lapses, and in two races (*californicus* and *eximius*) the claw and the phalanx which bears it is absent normally, though in one race it is present rarely on one or both hind feet. In the *merriami* and *deserti* groups the process of loss is complete to the extent that only a rudiment of the first metatarsal is present." The latter is not the condition in *D. spectabilis*.

In *Dipodomys*, as in the jerboas, the conformation of the phalanges of digits 2 and 4 is such that the tips, and hence the nails, are slightly inclined toward digit 3, as indicated not only by the position of the digits but by the facts that their nails are a bit more worn upon the side adjoining digit 3. This clearly suggests that there is a slight tendency in the kangaroo rat, more accentuated in the jerboas, for the animal to group the three central digits as closely as possible.

In the two sorts of jerboas under consideration the bones, and even the nails, of digits 2 and 4 are more robust than of 3. The first phalanx of the third is also slightly longer than of the other two digits, and this is apparently accentuated by the greater slenderness of this digit and by the fact that its metatarsophalangeal joint is situated farther forward. The terminal phalanx of the third digit is also extended so that its axis is at a right angle with the axis of the second phalanx. This condition has apparently been brought about by hypertension of the tendon of the extensor digitorum longus to this digit, and relaxation of the tone of this branch of the flexor tibialis. This tendency, but in much reduced form, is exhibited by digits 2 and 4 also, especially in *Allactaga*. The result is that the unusually globular, terminal pads of the digits are applied to the ground in a triangular pattern, and the nails are well elevated.

As it may be considered that all the short muscles of the jerboa foot have either disappeared or become tendinous there is possible no independent movement of the toes. In other words such movement of the toes in relation to the metatarsus as may take place is involuntary, as far as the foot is concerned, and is governed involuntarily by extension and flexion of the entire pes through the musculature of the crus. It is unlikely that this would result in any definite abduction of the toes. On the contrary the tendency appears to be for adduction of the digits and a grouping of the digital pads into a single complex. Concomitant to this is a clear trend toward the twisting of the second and fourth digits so that their nails incline toward that of the third, precisely as in *Dipodomys* save to a more pronounced extent. Undoubtedly this feature is correlated with a tendency toward grouping of the three digits, or in other words a tendency for the three to function largely after the fashion of a single, more robust digit.

Conclusions.

On the whole, the characters which fit a rodent for a saltatorial existence are seldom of a sort which can be placed in a table, as present or absent. Mostly they comprise slight differences in proportion, increase in the length of the pes, reduction in size of the lateral digits, and so forth. But rarely can there be listed such a definite item as fusion of the metatarsals. It seems clear that specialization for a saltatorial existence involves alteration of the physiological mechanism of nerve impulse and muscle reaction to fully as definite a degree as it does change in gross bodily form, and the former cannot be evaluated.

Speaking in general terms the impression is given by a study of the anatomy of the rodents dissected in the present instance that saltatorial specialization affects both the heteromyid and dipodid stocks in the following particulars:

There is some degree of broadening of the face, and a progressive weakening of the masticatory musculature, but the first of these details at least is only secondarily attributable to saltation, because of bullae enlargement.

There is experienced an increase in size of the auditory apparatus, with enlargement of the mastoid chiefly, and of the tympanic part of the ear to a lesser extent. If such enlargement be but moderate the external ear may also be very large. If increase in size of the mastoid features be of phenomenal degree, then the external ear will be either small or of medium size.

There is an increase in the shortness of the neck, already short in rodents.

There appears to be a shortening of the trunk with increased saltation, although this is difficult of proper determination.

There is an apparent increase in the strength, and in the complexity, of the long back musculature, associated with its use in connection with the hind limbs in leaping.

There is a decrease in the robustness of the arm, possibly with a slight relative shortening of this member, and the latter character is accentuated in appearance by the increase in length of the hind limb.

There is experienced a progressive increase in the postacetabular length of the innominate as compared with the preacetabular measurement, as well as a relative shortening of the presacral part of the ilium.

There is a progressive elongation of the hind limb, especially distally. Thus the relative elongation of the pes is greatest, of the shank less, and of the thigh least of these three segments: or, expressed in another way, if the sum of the thigh, shank and foot length be taken as 100, then it will be found that with saltatory specialization there will be progressive shortening of the thigh, and elongation of the foot, while the length of the shank in relation to total limb length experiences a very slight reduction.

There is, probably, a slight reduction in the importance of the fibula as compared with more generalized rodents, but this is not pronounced.

There is a phenomenal increase in the length of the pes, involving both metatarsus and digits. The breadth of the pes decreases and there is compaction of the bony elements, this, in the most specialized sorts, resulting in fusion of the central metatarsi. There is usually a tendency toward reduction and eventual elimination of the side digits (with accompanying change of tarsal details) but the final disappearance of these may be long delayed. The muscles of the foot proper will become tendinous.

The perissodactyl or odd-toed type of pes is eventually attained, save that in the most highly specialized sorts the third digit is more slender than the second and fourth. The two latter digits have their nails inclined toward the third, so that there is a tendency for the grouping of the digits into a functional unit. The result of these hind limb specializations is the assumption of a bipedal, saltatorial, digitigrade gait. There is no discernible indication that these rodents will eventually become monodactyl, or that when resting they will ever cease to be plantigrade.

In anatomical details the most specialized of the jerboas are apparently far in advance of the kangaroo rats in saltational features. Whether the former also surpass greatly in saltatorial performance remains to be determined.

It is interesting to note that the most generalized of the Heteromyidae are not discernibly fitted for leaping; hence, that within this family saltatorial specialization varies from very poor to very good. In the Dipodidae, on the other hand, although one member (*Sicista*) of a nearly related family is generalized, saltatorial specialization, on the whole, varies from good to astonishingly good. In other words dipodids are considerably better fitted for leaping than are heteromyids. The facts are suggestive of the thesis that, providing other factors were equal, North America has not maintained an arid, "desert" type of plains country where this now occurs for as long, or at least as uninterruptedly, as has the Old World. Or, stated differently, the Dipodidae have evidently been exposed to saltatorial influences for a longer period of time than the Heteromyidae.

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THE ECLIPSING BINARIES RW ARAE AND SW OPHIUCHI.

By Martha B. Shapley.

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THE ECLIPSING BINARIES RW ARAE AND SW OPHIUCHI.

BY MARTHA B. SHAPLEY.

Read by Title November 9, 1932.

A large number of observations of eclipsing variable stars have been made from plates of the Harvard photographic collection by Mrs. Esther Robinson, working under a grant to Dr. Harlow Shapley from the Permanent Science Fund administered by the American Academy of Arts and Sciences. Mrs. Robinson has had much experience in the estimation of the brightness of variable stars on photographic plates, and the light curves that have been derived from her measures are of considerable accuracy, apparently as satisfactory as can be obtained from in-focus photographic images.

The light curves derived from the observations were placed at my disposal for the calculation of the orbital elements and for general discussion of the various characteristics of the binary systems. The present communication gives the results for RW Arae and SW Ophiuchi, two systems which have in common the character of constant maximum light, with a barely measurable indication of a secondary eclipse. The light curves for these binaries show no trace of ellipticity or reflection effects.

RW Arae. The variability of RW Arae was discovered by Miss Cannon in 1910 and an approximate period determined. The new photographic observations, derived from 866 plates of the Harvard collection, permit the revision of the formula for primary minimum as follows:

$$\text{minimum} = \text{J. D. } 2419755.6911 + 4^{d}.3675094E \pm 0.0000025E \text{ (p. e.)}$$

In the course of the discussion of the normal points and supernormal points I have adjusted the initial epoch which was used in the least squares solution for the light elements so as to have it coincide with the mid-point of primary minimum. Table I contains information concerning the comparison stars for both variables, and Table II gives data of a general nature for the variables themselves. The 866 observations for RW Arae have been grouped into the normal points in Table III; successive columns give the serial number of the normal with the number of observations, the mean phase computed with the elements given above, the mean photographic magnitude, and the residuals from the theoretical light curves which I have derived from the computed orbital elements listed in Table IV. The first seventeen normal points from Table III refer to the primary minimum, and from fifty-two to sixty-three the normals fall in the region of secondary minimum. Since the exposure times average approximately one hour, the phases in the third column are geocentric.

The mean photographic light curve given by Table III has been satisfactorily represented. The star proves to be a binary system in which the bright component has about one-third the diameter of the faint star but is more than fifty times as bright per unit surface area. This conclusion is based on the hypothesis of complete darkening toward the limb according to the usual cosine law. An alternative solution, assuming uniform distribution of light over the disk of the star, gives a binary system in which one component is only a sixth the diameter of the other but has five-sixths the total light of the system. Thus the conception of uniform brightness is less satisfactory than the hypothesis of complete darkening.

The two computed curves for RW Arae are shown in Figure 1 with the observations of primary minimum. In Figure 2 (a) all the normal points of Table III are given, together with the computed curve. The maximum is essentially flat, from which we conclude that the stars are spheroidal. There is in the observations a slight suggestion of a secondary minimum midway between the successive primaries, in agreement with the computed value of $0^m.01$. The maximum light observations preceding the secondary minimum show as a whole a brighter average magnitude than those between the secondary and the following primary. This phenomenon perhaps indicates that the preceding side of the bright component is brighter than its following side.

The principal characteristics of the RW Arae system are shown in diagram in Figure 2 (b): the components are well separated; the small bright star has about one and one-half times the diameter of the sun; the larger, fainter component has about five times the sun's diameter; and the inclination is relatively low for a total eclipse.

The orbital data for RW Arae, on the two assumptions of uniform distribution in brightness and total darkening at the limb, appear in Table IV. The semiduration of totality for "darkened" stars is but a third the duration of the alternative assumption and results in a rounded minimum and a slightly better fit to the observations. In

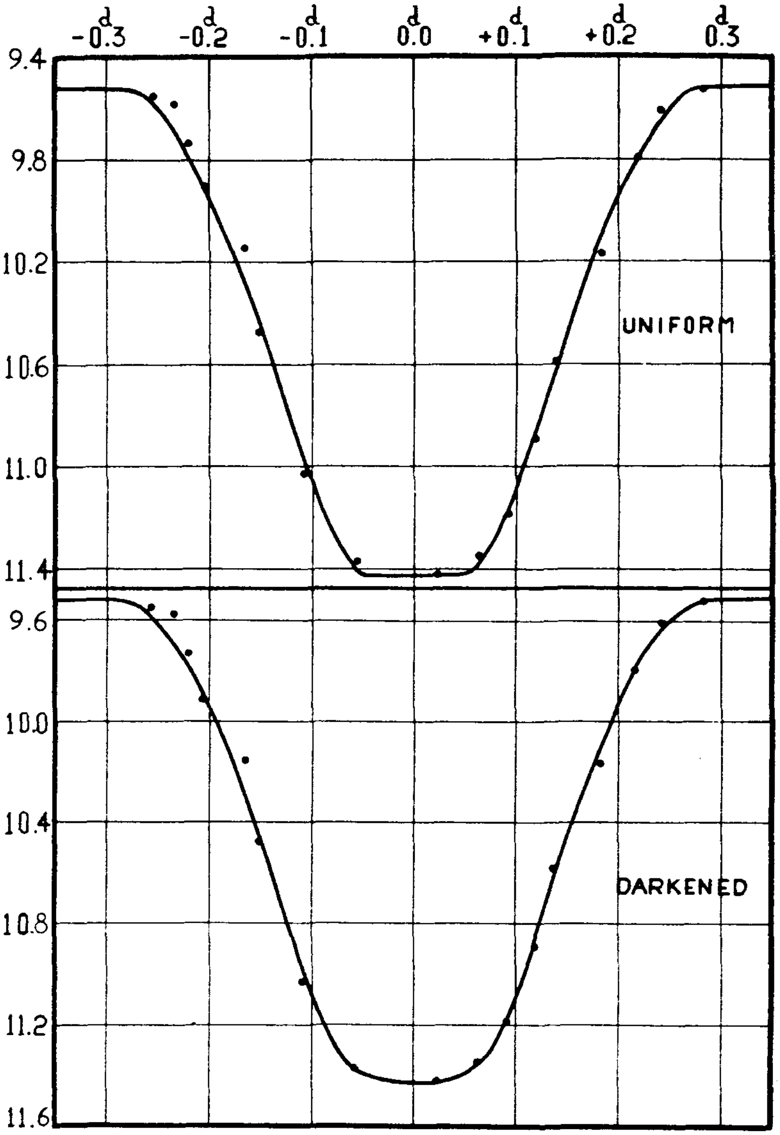

Figure 1. RW Arae: Computed curves and observations during primary minimum. Ordinates are stellar magnitudes.

the less probable "uniform" solution the bright star is exceptionally small compared with the fainter one, but its surface brightness is nearly one hundred and fifty times as great per unit area, indicating a high degree of density, but perhaps not abnormally so for a star of spectral class A0. With this exception the two solutions are similar. It should be noted that the absolute magnitude, M_2, of the component for which we have the spectral class, is within the range of values commonly found for spectrum A0; it should also be noted that the faint absolute magnitude and the low density of the large component emphasize the anomalous position of many of the dark components of eclipsing binaries in the spectrum-luminosity (Russell) diagram. Both the absolute magnitudes and the mean densities are accurately determined as to order of magnitude, notwithstanding uncertainty concerning other characteristics of the binary. But we should expect a star of the third magnitude (absolute) to have a mean density of at least one hundred times the value here computed. The spectral class of the large star unfortunately remains unknown; probably it is similar to that of the sun.

The symbols in Table IV are those conventionally used in the calculation of eclipsing star orbits, with subscript 1 referring to the larger, darker component.

SW Ophiuchi. The eclipsing variable SW Ophiuchi was also found at the Harvard Observatory. When announced in 1908 by Miss Leavitt an approximate period was given. The light elements, based on the new observations derived from 631 photographs, are as follows, after adjusting the initial epoch to the eclipse:

$$\text{minimum} = \text{J. D. } 2420740.4691 + 2^{d}.4459310E \pm 0.0000016E \text{ (p. e.)}.$$

Information concerning the comparison stars and the variable itself has been given in earlier Tables. The sixty-seven normal points into which the observations have been assembled appear in Table V which follows the form of Table III described above. Normals one to twenty refer to primary minimum; forty-one to fifty to the region of the secondary minimum. Since the computed curves based on the uniform and darkened hypotheses are almost identical, the residuals, O—C, in Table V are given only for the uniform solution. The phases are heliocentric. Figure 3 shows the computed curves and observations during primary minimum; Figure 4 (a) gives the whole computed curve with the observed data, and (b) represents a diagram of the system. The maximum light observations indicate the absence

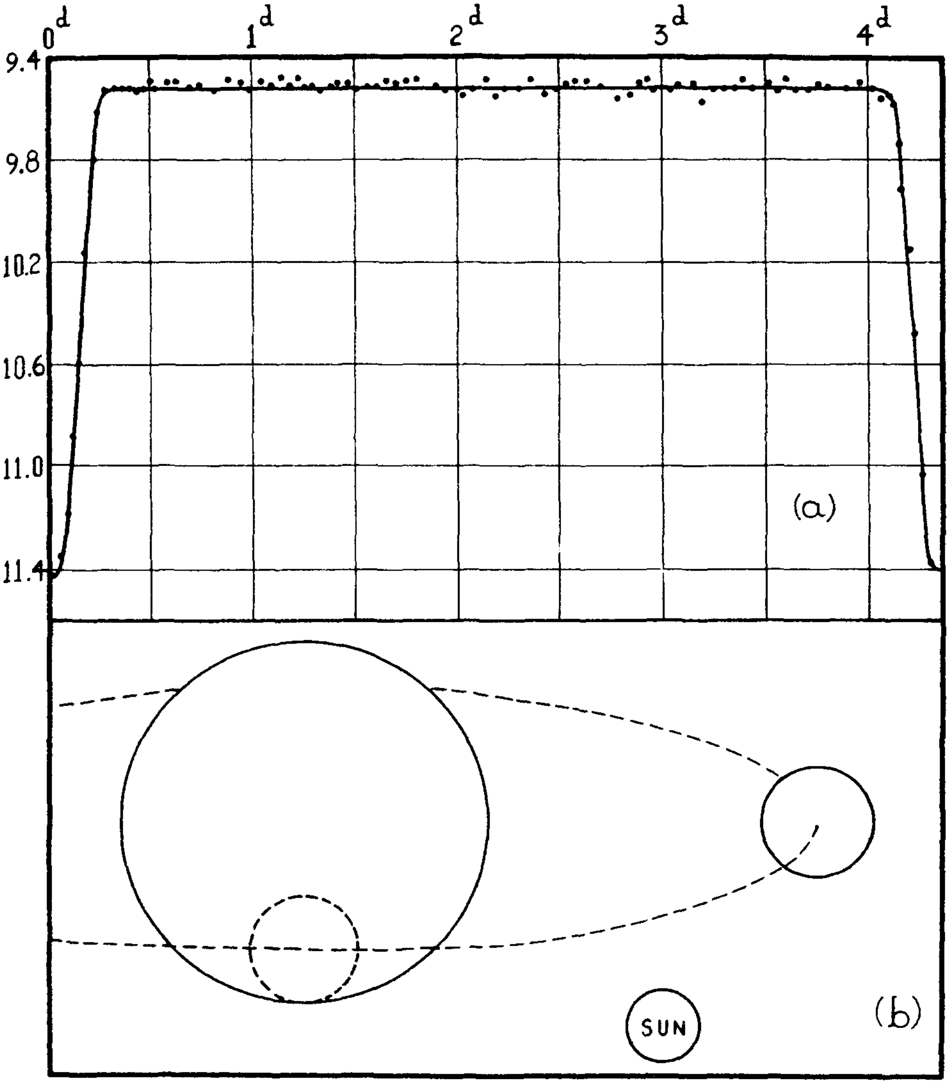

FIGURE 2. RW Arae: (a) Normal points from Table III, and computed curve. (b) Diagram of the system (darkened). Ordinates are stellar magnitudes.

of appreciable ellipticity and show faintly the secondary eclipse which is computed to be three hundredths of a magnitude.

The system of SW Ophiuchi consists of two stars the fainter of which has nearly twice the diameter of its bright companion, and both are considerably larger than the sun. It is a remarkably normal eclipsing binary, in size, orbit, density, magnitude, surface brightness, and parallax. The orbital data are given in Table VI in the usual form. Perhaps the most striking point in Table VI is the unusual

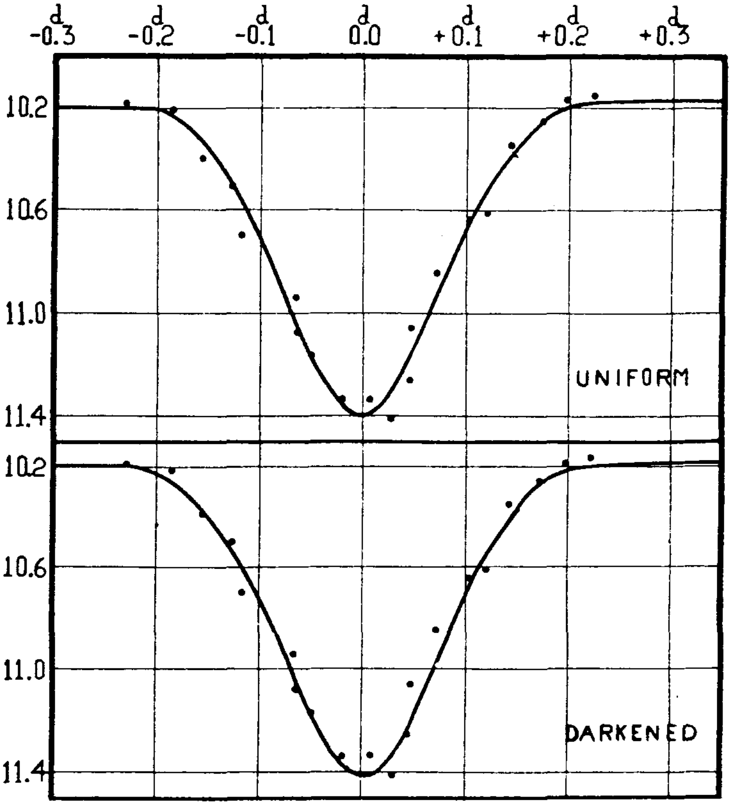

FIGURE 3. SW Ophiuchi: Computed curves and observations during primary minimum. Ordinates are stellar magnitudes.

similarity of the uniform and darkened columns. The observations are equally well represented by the two hypotheses.

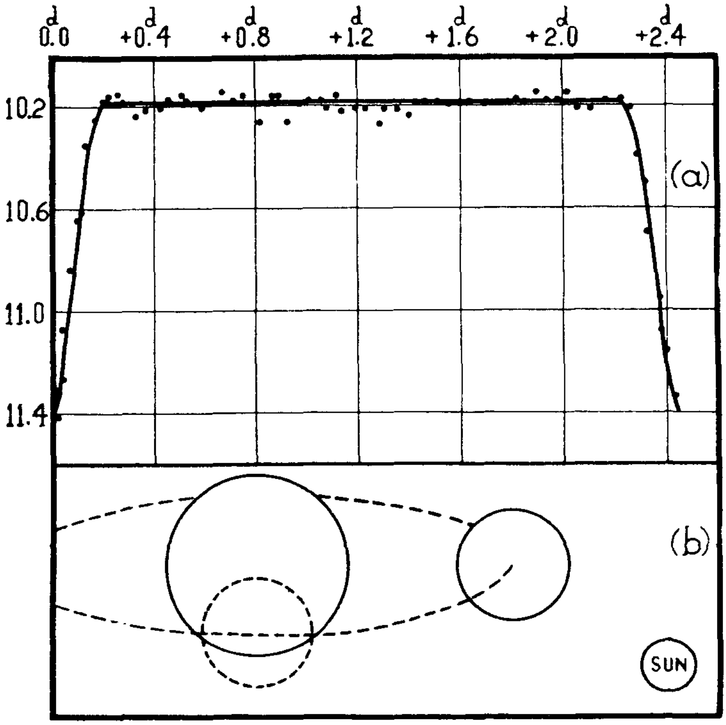

FIGURE 4. SW Ophiuchi: (a) Normal points from Table V, and computed curve. (b) Diagram of the system. Ordinates are stellar magnitudes.

Summary. The eclipsing binary RW Arae has been observed on 866 photographs from the Harvard collection and the material made the basis for the calculation of orbital elements and other data. The elements and the light curve show that there is a total eclipse involving two stars, well separated, differing considerably in size and surface brightness. The stars are spheroidal, and there is a slight secondary minimum. The assumption of a stellar disk darkened toward the edge agrees with the observations better than the alternative assumption of uniform brightness.

The low density combined with faint absolute magnitude of the large dark companion presents a puzzling phenomenon that is not observed among isolated stars and is in conflict with the usual views on the spectra and luminosities of stars.

Six hundred and thirty-one observations of the eclipsing variable SW Ophiuchi yield orbital data showing a system of fairly close stars partially eclipsing each other in a period of about two and a half days; the smaller and brighter component is three-fourths eclipsed at primary minimum. The uniform and darkened solutions are nearly identical and give results that are entirely normal in a binary of spectral class A0.

October 1, 1932.

TABLE I.

THE COMPARISON STARS.

Variable	Comparison Star	H.D.	R.A. 1900	Dec. 1900	Pg. Mag.	Spectrum
RW Arae	a	157559	17h 18m.8	−57° 1′	8.06	F0
	b	159236	17 28.4	−57 3	8.62	A0
	c	159020	17 27.2	−57 4	9.68	A0
	d		17 28.2	−56 52	10.65	F8
	e		17 25.9	−56 55	11.38	F5
	f		17 26.2	−56 57	11.60	too faint
SW Ophiuchi	a	147080	16 14.6	− 6 43	9.97	F8
	b	146623	16 12.1	− 6 26	10.51	F0
	c	146816	16 13.1	− 6 45	10.91	G5
	d		16 13.2	− 6 50	11.55	K

TABLE II.

GENERAL DATA ON THE TWO SYSTEMS.

	RW Arae	SW Ophiuchi
R.A. 1900	17h 26m.3	16h 11m.1
Dec. 1900	−57° 5′	−6° 44′
λ	302°	334°
β	−14°	+28°
Pg. mag. max.	9.52	10.19
Spectrum	A0	A0
Primary eclipse	total	partial
Shape of stars	spheroidal	spheroidal
No. of observations	866	631

TABLE III.

NORMALS OF RW ARAE WITH RESIDUALS FROM COMPUTED CURVES.

Number	Number of Observations	Mean Phase	Mean Magnitude	O—C	
				U	D
1	5	$-0^{d}.257$	9.55	$0^{m}.00$	$-0^{m}.03$
2	6	.234	9.58	− .10	− .10
3	4	.220	9.73	− .04	− .03
4	5	.205	9.91	+ .01	+ .01
5	5	.165	10.15	− .13	− .13
6	7	.151	10.48	+ .02	+ .02
7	5	.109	11.03	+ .05	+ .05
8	1	−0.057	11.37	− .04	− .01
9	2	+0.023	11.43	.00	.00
10	2	.063	11.35	− .03	− .01
11	4	.093	11.19	+ .03	+ .03
12	5	.119	10.89	+ .04	+ .01
13	5	.137	10.59	− .02	− .01
14	6	.181	10.17	+ .05	+ .05
15	7	.218	9.80	.00	.00
16	8	.241	9.61	− .01	− .03
17	6	.281	9.53	+ .01	.00
18	11	.321	9.52	.00	
19	11	.364	9.52	.00	
20	10	.396	9.52	.00	
21	10	.426	9.53	+ .01	
22	10	.464	9.52	.00	
23	11	.495	9.49	− .03	
24	10	.525	9.53	+ .01	
25	11	.579	9.50	− .02	
26	11	.630	9.50	− .02	
27	10	.690	9.52	.00	
28	10	.738	9.51	− .01	
29	11	.811	9.53	+ .01	
30	12	.880	9.49	− .03	
31	12	.945	9.50	− .02	
32	10	0.983	9.53	+ .01	
33	12	1.036	9.50	− .02	
34	10	.091	9.51	− .01	
35	10	.142	9.49	− .03	
36	10	.178	9.51	− .01	
37	11	.215	9.49	− .03	
38	11	.250	9.52	.00	
39	13	+1.291	9.51	−0.01	

TABLE III—*cont.*

Number	Number of Observations	Mean Phase	Mean Magnitude	O—C	
				U	D
40	11	$+1^d.334$	9.53	$+0^m.01$	
41	10	.378	9.51	− .01	
42	9	.419	9.50	− .02	
43	10	.470	9.50	− .02	
44	11	.512	9.53	+ .01	
45	11	.559	9.52	.00	
46	11	.607	9.52	.00	
47	10	.665	9.50	− .02	
48	10	.706	9.51	− .01	
49	10	.746	9.50	− .02	
50	10	.812	9.49	− .03	
51	10	.888	9.52	.00	
52	10	.945	9.53	+ .01	+0.01
53	12	1.988	9.53	+ .01	+ .01
54	10	2.032	9.55	+ .03	+ .02
55	10	.080	9.53	+ .01	.00
56	11	.136	9.49	− .04	− .04
57	10	.193	9.56	+ .03	+ .02
58	10	.234	9.53	.00	− .01
59	10	.303	9.53	+ .01	.00
60	11	.365	9.49	− .03	− .04
61	11	.433	9.55	+ .03	+ .03
62	10	.486	9.53	+ .01	+ .01
63	10	.534	9.51	− .01	−0.01
64	10	.583	9.50	− .02	
65	10	.641	9.50	− .02	
66	11	.708	9.52	.00	
67	11	.792	9.56	+ .04	
68	11	.835	9.55	+ .03	
69	11	.886	9.50	− .02	
70	10	.928	9.49	− .03	
71	10	2.965	9.53	+ .01	
72	10	3.040	9.53	+ .01	
73	10	.084	9.51	− .01	
74	10	.154	9.51	− .01	
75	10	.193	9.58	+ .06	
76	10	.246	9.53	+ .01	
77	12	.305	9.53	+ .01	
78	11	.347	9.53	+ .01	
79	10	+3.387	9.49	−0.03	

TABLE III—*cont.*

Number	Number of Observations	Mean Phase	Mean Magnitude	O—C U	O—C D
80	15	+3.445	9.53	$+0^m.01$	
81	10	.525	9.51	− .01	
82	11	.562	9.53	+ .01	
83	10	.605	9.49	− .03	
84	11	.653	9.53	+ .01	
85	10	.713	9.53	+ .01	
86	10	.761	9.51	− .01	
87	11	.807	9.52	.00	
88	12	.898	9.52	.00	
89	11	3.959	9.50	− .02	
90	10	4.024	9.53	+ .01	
91	10	+4.073	9.56	+0.04	

TABLE IV.

ORBITAL ELEMENTS OF RW ARAE.

	Uniform	Darkened
Range, primary..........	$1^m.91$	$1^m.91$
Range, secondary........	$0^m.006$	$0^m.015$
t'....................	$0^d.273$	$0^d.289$
t''...................	$0^d.051$	$0^d.017$
M_1...................	+3.37	+2.61
M_2...................	+1.67	+0.91
m_1...................	11.43	11.43
m_2...................	9.73	9.73
L_1...................	0.172	0.172
L_2...................	0.828	0.828
J_2/J_1...............	149.	53.4
i.......................	70° 30′	75° 40′
r_1...................	0.418	0.358
r_2...................	0.075	0.107
$\bar{A}$...............	14.1	14.1
α_0..............	1.00	1.00
ρ_1................	0.003	0.005
ρ_2................	1.17	0.404
π...................	$0''.0024$	$0''.0017$

TABLE V.

NORMALS OF SW OPHIUCHI WITH RESIDUALS FROM COMPUTED UNIFORM CURVE.

Number	Number of Observations	Mean Phase	Mean Magnitude	O—C
1	10	$-0^{d}.228$	10.18	$-0^{m}.01$
2	10	.183	10.21	− .03
3	6	.153	10.39	+ .04
4	5	.127	10.50	.00
5	6	.113	10.69	+ .09
6	5	.067	10.95	− .02
7	6	.062	11.08	+ .02
8	5	.049	11.16	.00
9	6	−0.018	11.34	− .03
10	5	+0.009	11.34	− .06
11	7	.031	11.41	+ .11
12	5	.047	11.26	+ .09
13	5	.049	11.06	− .10
14	6	.072	10.84	− .11
15	5	.103	10.64	− .03
16	6	.120	10.61	+ .06
17	6	.142	10.35	− .06
18	7	.174	10.25	− .01
19	10	.198	10.18	− .02
20	11	.224	10.16	− .03
21	10	.261	10.15	− .04
22	10	.284	10.18	− .03
23	11	.333	10.24	+ .05
24	10	.374	10.22	+ .03
25	12	.415	10.21	+ .02
26	11	.459	10.18	− .01
27	10	.507	10.16	− .03
28	11	.534	10.18	− .01
29	10	.586	10.21	+ .02
30	10	.637	10.19	.00
31	10	.674	10.15	− .04
32	11	.719	10.18	− .01
33	10	.749	10.16	− .03
34	10	.790	10.19	.00
35	12	.824	10.26	+ .07
36	13	.869	10.16	− .03
37	10	.893	10.16	− .03
38	12	+0.933	10.26	+0.07

TABLE V—*cont.*

Number	Number of Observations	Mean Phase	Mean Magnitude	O—C
39	10	$+0^d.977$	10.19	$0^m.00$
40	9	1.012	10.18	− .01
41	10	.057	10.18	− .01
42	11	.081	10.21	+ .01
43	10	.108	10.16	− .04
44	10	.137	10.22	+ .01
45	10	.188	10.21	− .01
46	10	.228	10.21	− .01
47	11	.268	10.27	+ .05
48	11	.307	10.21	.00
49	10	.365	10.21	+ .01
50	10	.399	10.24	+ .05
51	10	.433	10.18	− .01
52	12	.462	10.18	− .01
53	10	.506	10.18	− .01
54	12	.565	10.19	.00
55	12	.643	10.18	− .01
56	11	.697	10.19	.00
57	12	.744	10.19	.00
58	10	.793	10.19	.00
59	11	.821	10.18	− .01
60	11	.860	10.19	.00
61	11	.901	10.15	− .04
62	10	.936	10.18	− .01
63	10	1.982	10.18	− .01
64	10	2.023	10.15	− .04
65	10	.061	10.21	+ .02
66	11	.117	10.21	+ .02
67	11	+2.169	10.18	−0.01

TABLE VI.

ORBITAL ELEMENTS OF SW OPHIUCHI.

	Uniform	Darkened
Range, primary	$1^m.22$	$1^m.22$
Range secondary	$0^m.03$	$0^m.03$
t'	$0^d.199$	$0^d.242$
M_1	+2.76	+2.59
M_2	+0.42	+0.25
m_1	12.65	12.65
m_2	10.31	10.31
L_1	0.101	0.101
L_2	0.899	0.899
J_2/J_1	25.5	24.7
i	76° 18′	74° 50′
r_1	0.334	0.354
r_2	0.197	0.212
$\bar{A}$	9.6	9.6
α_0	0.75	0.75
ρ_1	0.016	0.013
ρ_2	0.22	0.17
π	0″.001	0″.001

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THE PURIFICATION AND PHYSICAL PROPERTIES OF ORGANIC COMPOUNDS I. THE INTERPRETATION OF TIME-TEMPERATURE CURVES IN FREEZING POINT DETERMINATIONS AND AS A CRITERION OF PURITY.

BY EVALD L. SKAU

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THE PURIFICATION AND PHYSICAL PROPERTIES OF ORGANIC COMPOUNDS I. THE INTERPRETATION OF TIME-TEMPERATURE CURVES IN FREEZING POINT DETERMINATIONS AND AS A CRITERION OF PURITY.[1]

By Evald L. Skau.[2]

The need for a sound and practical criterion of purity for organic *liquid* compounds has long been very evident. Such a criterion is of fundamental importance in any investigation which requires the use of highly pure materials: for example, in the exact determination of physical constants. The method here described and which we shall classify with what we shall call "*non-comparative criteria* of purity" is applicable to all organic compounds which can be made to crystallize and whose melting points lie between liquid air temperature and about 250° C.

The methods now most generally in use for the estimation of the purity of *liquids* are unsatisfactory and indefinite. A comparison of some physical constant or constants, such as density, boiling point, viscosity, refractive index, or freezing point, with the accepted values in the literature presupposes that the latter values were accurately determined on pure materials, a supposition which is seldom warranted but often made. Another test of purity applied to liquids is the constancy of the values for these physical constants during a fractional distillation. The frequent occurrence of azeotropic mixtures,[3] however, renders this method very untrustworthy. Probably the most satisfactory test now in use is the constancy of the temperature during freezing but this as ordinarily carried out can not be considered a good criterion of purity since the size of the sample used, the relative temperature of the surroundings, and other factors have not been properly standardized.[4] Furthermore, great experimental difficulties are usually encountered in applying this method when the liquid in question is hygroscopic or absorbs carbon dioxide from the atmosphere.

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[1] Contribution from the Research Laboratory of Physical Chemistry, Massachusetts Institute of Technology, No. 252.

[2] National Research Fellow (1926–28).

This paper was written during the tenure of a Guggenheim Fellowship at the Universities of Munich and Frankfort a. Main and the Bureau International des Étalons Physico-Chimiques, Brussels.

[3] See the long series of papers by Lecat, Ann. Soc. Scient. Bruxelles (1926–1930), and also his book, "L'Azeotropisme," Brussels (1918).

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[4] The length of the flat, $(t_2 - t_1)$, on a cooling curve, that is, the length of time taken for the sample to drop through the smallest temperature range detectable with the thermometer used, $(\theta_2 - \theta_1)^\circ$, is a function of many variables. This follows from a consideration of Newton's Law of Cooling assuming ideal radiation and true equilibrium throughout.

Newton's Law states that

$$\frac{dH}{dt} = aK\,(\theta - \theta_S) \qquad (1)$$

where H is the heat content of the sample, a is the area of its radiating surface, K is a constant depending on the particular set-up, and θ and θ_S are the temperatures of the sample and of the surroundings respectively. But

$$dH = mC d\theta \qquad (2)$$

where m is the weight of the sample and C is its specific heat over the small temperature range in question (involving the heat of fusion per gram of the substance). Therefore

$$\frac{d\theta}{dt} = \frac{aK}{mC}\,(\theta - \theta_S). \qquad (3)$$

As a first approximation we can consider a, K, m, and C as constants for a given set-up over the small temperature range θ_2 to θ_1. Under these conditions equation (3) can be integrated and rearranged to give

$$t_2 - t_1 = \frac{mC\,(\theta_2 - \theta_1)}{aK\,(\theta - \theta_S)} \qquad (4)$$

That is, the length of the flat varies proportionally as m, C, and $(\theta_2 - \theta_1)$, and inversely as a, K and $(\theta - \theta_S)$.

The following facts also should be noted: a is a function not only of m but also of the shape and dimensions of the sample tube; and the amount of impurity causing the drop in the temperature of the sample from θ_2 to θ_1 is also a function of the heat of fusion of the sample (from the van't Hoff Equation,

$$\frac{d\ln x}{d\theta} = \frac{\Delta H}{R\theta^2},$$

where x is the mole fraction of the form crystallizing out, ΔH is its heat of fusion, and $R = 1.99$ cals.).

From these considerations it is evident that *the length of the flat can be varied almost at will* by choosing the proper conditions.

In the case of *solids*, on the other hand, considerable progress has been made toward estimating purity but, unfortunately, it is not always given due consideration. The comparison of the freezing point, alone or along with other physical properties, with the values recorded in the literature is many times considered sufficient. The failure to change the freezing point by successive recrystallization of the compound from a variety of solvents is probably the most common and satisfactory general criterion used. It must be borne in mind, however, that its sensitivity depends on the efficiency of the recrystallization, and especially, on the accuracy of the freezing point determination, and the completeness of the removal of the solvent.

In the case of solids as in the case of liquids, a "long flat" on the cooling curve is no proof of high purity unless it is referred to the time required for *complete* freezing under standardized conditions.[5]

Non-Comparative Criteria of Purity. The various criteria of purity can be divided into two classes: *comparative criteria* and *non-comparative criteria.* By a *comparative criterion* we mean one which depends on the comparison of one or more properties of the sample (a) with those obtained by other authors or (b) with those obtained for other samples which have been gotten by some fractionating process. By a *non-comparative criterion of purity*, on the other hand, we mean one by which the estimation of purity involves the measurements made on the original sample only; in this case the only comparison is with a standard or an ideal which is supposedly the same for all pure substances.

All the methods mentioned above are of the comparative type, with the exception of the length of the flat on a cooling curve, this being a rather inefficient form of a non-comparative method. Let us now consider these non-comparative methods since the method here described falls in that class.

Perhaps the most positive single evidence of purity can be obtained from the shape (1) of the volume-temperature curve at constant pressure, (2) of the volume-pressure curve at constant temperature, or (3) of the heat content-temperature curve, when they are determined by a static or integral method in the region where solid changes to liquid. In fact these curves may be used to estimate the amount of impurities present.[6] Serious experimental difficulties are involved in getting the data for the isotherm or the isobar, however, so that the specific heat method, (3), is probably the best of the three. Unfortunately this method is likewise unsuitable for a general laboratory method for estimating purity, due to the excessive time required to get the necessary data.

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[5] See Hüttner and Tammann, Z. anorg. Chem., *43*, 218 (1905); White, Amer. Jour. Sci., [4], *28*, 453 (1909), Z. anorg. Chem., *69*, 305 (1911); and Andrews, Kohman and Johnston, J. Phys. Chem., *29*, 914 (1925).

Johnston and coworkers,[7] however, studied the cooling curves obtained in a special apparatus in the case of solid compounds which had been proved to be sufficiently pure for their specific heat determinations, and found that the flat (constancy of temperature within 0.10°) lasted until the sample was three-quarters solidified.

In the basis of this work and that of White,[8] we have constructed a time-temperature-curve apparatus which is applicable to solids and liquids whose freezing points lie above liquid air temperature and we have made a study of the effects of small amounts of impurity not only on cooling curves but also on heating curves. A comparison of the curves so obtained brings out the fact that the nature of the flat obtained is only one of the characteristics which can be used to detect the impurity, and that it is, in fact, probably neither as reliable nor as sensitive as the nature of the end of the completed cooling curve and especially the nature of the approach to the flat in the heating curve when the conditions are properly standardized. We are also able to point out many advantages of the use of heating curves over the use of cooling curves as a criterion.

The distinct advantage of this time-temperature-curve method for the detection of impurities is that it is a non-comparative method so that, within certain limits to be mentioned, the results obtained for a single sample are conclusive in themselves even though the history of the sample be unknown.

The apparatus here described can be used over a wide range of temperatures and the sample may be handled entirely in vacuo, so that absorption of moisture, carbon dioxide or oxygen from the atmosphere may be eliminated. The apparatus may also be used for the determination of freezing points and transition points at atmospheric pressure or in vacuo.

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[6] Tammann, "Aggregatzustände," Verlag von Leopold Voss, Leipzig (1922) pages 83-90; Block, Z. phys. Chem., *78*, 385 (1912); Bogojawlenski, Schr. d. Dorpeten Naturf. Ges., *13*, 1 (1904); Andrews, Lynn and Johnston, Jour. Amer. Chem. Soc., *48*, 1274 (1926).

[7] Andrews, Kohman and Johnston, *loc. cit.*; Andrews, Lynn and Johnston, *loc. cit.*

[8] White, Z. anorg. Chem., *69*, 305 (1911); J. Phys. Chem., *24*, 393 (1920).

APPARATUS.

The apparatus (see Figure 1) consists essentially of a cooling curve apparatus similar to that previously described,[9] set in an air cryostat which is similar in principle to that used by Keyes and his co-workers.[10]

The sample, 0.5 to 0.7 grams, is hermetically sealed in a specially constructed tube, A, (about 3 cm. long) made of very light 5.5 to 6.5 mm. glass tubing (about 0.07 mm. wall thickness),[11] in order that its heat capacity may be negligibly small with respect to that of the sample, and so constructed that the temperature of the sample may be read by means of a single-junction thermocouple, one element of which, G, fits snugly into a thin-walled capillary tube, N, containing some fluid (ether, CCl_4 or vaseline, as the case may be), and extending down through the center of the sample to within about 4 mm. of the bottom. The sample tube is fastened by means of a wire (not shown in the diagram) to two little knobs protruding from the sides of the thin-walled glass tube K through which the thermoelement enters. This tube extends up through the loosely fitting fibroid stopper L, which is connected in turn with the long fibroid handle M, by means of which the whole may be removed from the apparatus. A wad of cotton, P, is pressed into the mouth of the flask to keep out moisture.

The sample tube is suspended in the shield B, which is a hollow copper cylinder closed at the bottom and wound with a 30-ohm Nichrome resistance H. The latter is insulated from the copper by a thin layer of mica and connected to a variable source of current. The thermoelement, F, runs down through the 1.5 mm. hole, drilled longitudinally, and the junction is silver-soldered into a transverse hole opposite the sample as shown in the diagram.

The shield is set on a mat of shredded asbestos in C, an unsilvered double-walled vessel (length, 45 cm.; inside diameter, 4.5 cm.), so connected through tube E with a mercury diffusion pump and a gas reservoir that the pressure between its walls may be varied at will. D is a silvered Dewar tube with a transparent longitudinal slit so set that the sample is visible through a corresponding slit (6 by 25 mm.) through the shield, a suitable light being placed behind the apparatus. C is held in place partly by the cork O at the bottom and partly by a

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[9] Skau and Saxton, Jour. Amer. Chem. Soc., *50*, 2693 (1928).

[10] Taylor and Smith, Jour. Amer. Chem. Soc., *44*, 2450 (1922); Keyes, Taylor and Smith, Jour. Math. Phys. Mass. Inst. Tech., *1*, 211 (1922). See also Cardoso, Jour. chim. phys., *15*, 317, (1915).

[11] "Wafer weight" or "Friable" Tubing, No. 50105, obtainable from Kimble Glass Co., Vineland, New Jersey, U. S. A.

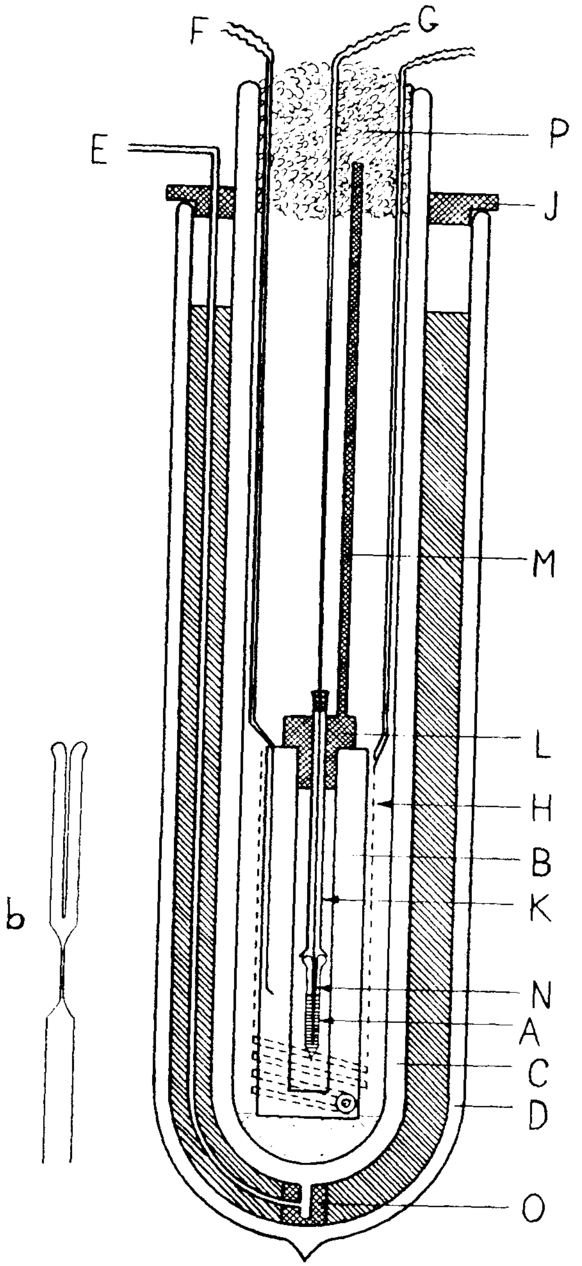
F
G
E
P
J
M
L
H
B
K
N
A
C
D
O
b

FIGURE 1. Time-Temperature Curve Apparatus.

ring of transite board, J, stuck fast to the glass with Khotinski cement and fitting loosely in the mouth of D.

The temperature of the shield and of the melt were read by means of the thermoelements F and G respectively (cold junctions at 0°) the measurements being made to the nearest microvolt, about 0.03°, with a Leeds and Northrup Type K potentiometer. Both thermocouples were constructed of No. 36 copper and No. 30 constantan, B & S gauge. Thermocouple G was calibrated at the freezing point of water, the freezing point of mercury (– 38.87°), the sublimation point of carbon dioxide (– 78.51°), and at the boiling point of liquid oxygen (– 183.00°).[12] The last two points were established by measuring the vapor pressure of the carefully purified substances in a constant temperature cryostat in which the thermoelement was immersed.[13] In each case the purity of the sample was tested by evaporating to one-half volume and then redetermining the vapor pressure. The calibration at the mercury point was carried out in the same apparatus and under the same conditions as the chlorobenzene samples.

PROCEDURE.

Filling the Sample Tube. The sample tube is thoroughly cleaned with boiling concentrated nitric acid, well rinsed, and dried by heating in vacuo, before the constriction is made as shown in Figure 1, b. The procedure for filling depends somewhat on the purpose of the experiment. If it is desired to test a certain substance without further purification and it has already been exposed to the atmosphere, or if, as in the case of the samples used for this investigation, exposure to the atmosphere for a short time has no appreciable effect, the substance may simply be poured into the tube. Otherwise the sample is introduced into an all-glass apparatus with or without some purifying or drying agent, and, after evacuation, condensed into the sample tube which is meanwhile dipped in liquid air. In any case, in sealing off the sample tube all precautions should be taken to avoid overheating the sample even in the vapor state; it is thus advisable to cool the sample in liquid air while gently heating the constriction before sealing, the upper part of the tube being meanwhile stoppered or connected with a vacuum as the case may be.

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[12] "International Critical Tables," New York, Volume I, page 53.

[13] The details of this method have been fully described by Keyes, Townshend and Young, J. Math. Phys. Mass. Inst. Tech., *1*, 302, (1922); see also Loomis and Walters, Jour. Amer. Chem. Soc. *48*, 3101 (1926), and Henning, Ann. Physik, *43*, 282 (1914).

Adjustment of the Apparatus. To obtain a heating or a cooling curve for a substance whose freezing point is below room temperature, flask D is filled with liquid air, and dry CO_2-free air is admitted through E. The temperature of the shield falls rapidly until at the proper time the space between the walls of the vessel C is evacuated.[14] By adjustment and occasional readjustment of the voltage applied across the terminals of the heater, H, the temperature of the shield is now caused to rise or fall at the desired constant rate, which is maintained throughout the curve (from 0.3 to 0.5° per minute for cooling curves and from 0.25 to 0.35° for heating curves). Readings of the melt temperature are recorded for every minute and, during the more important parts of the curve, for every half-minute; readings of the shield temperature are made for every minute at first but later only occasional readings are necessary.

Cooling Curves. The shield temperature should be started from two to three degrees above the temperature at which the crystallization is to be started, so that the rate of fall of the shield and the melt is then roughly the same. Crystallization is then initiated by twirling the sample tube in the apparatus for a few seconds. This can be done by rapidly rolling the upper end of the fibroid handle, M, between the thumb and the index finger after first lifting it about one centimeter. If there is a sudden rise in temperature, this twirling is repeated occasionally during the next minute to ensure dissemination of the crystals which have formed throughout the melt. If there is no break, however, the sample is allowed to cool through another 0.5° before repeating the procedure. After crystallization has thus been started, readings are continued, *without stirring the sample*, until the temperature of the melt has again come to within two or three degrees of the shield temperature, which, in the meanwhile, has been kept falling at the initial rate. Due to the absence of stirring after the first minute, it is desirable to supercool the sample to a temperature from three to five degrees below the freezing point before starting the crystallization as will be explained later.

The above method of starting the crystallization sometimes fails. A more generally applicable method is to remove the sample tube from the apparatus, by means of the fibroid handle, M, dip the very tip of the tube in liquid air for a fraction of a second, and reinsert it in the apparatus, the whole process being completed in two or three seconds.[15] The sample tube is then twirled during the first minute to mix the crystals with the supercooled liquid as above. This was the method used for obtaining the curves reported here.

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[14] This evacuation of flask C is not absolutely necessary. If it were not evacuated, the radiation from the shield would be more rapid; therefore, more heat would have to be supplied by the heater to establish the desired rate of fall or rise, and a corresponding amount of liquid air would be wasted.

Heating Curves. The shield is first cooled to 3° or 4° below the freezing point, as determined by a rough preliminary cooling curve, and kept constant at this temperature. The sample is then completely frozen and inserted in the apparatus at a considerably lower temperature. When the sample has come to within 0.1°–0.2° of the temperature of the shield the latter is caused to rise at the desired rate, which is maintained throughout the curve.[16]

MATERIALS.

The meta-dinitrobenzene was kindly supplied us by Andrews, Lynn and Johnston, and was of the same purity as the sample which they used in their determination of the heat of fusion by a method which involved the construction of the heat-content temperature curve for the substance.[17]

The chlorobenzene was a pure sample "for thermocouple calibration" supplied through the courtesy of Professor Jean Timmermans of the University of Brussels.

The meta-dichlorobenzene used was the middle fraction of a vacuum distillation of the product purchased from the Eastman Kodak Co. It still contained a small amount of impurity, as shown by a heating curve, its freezing point being − 25°. The impurity was not great enough to affect the results of these experiments appreciably as the compound was used only to prepare a 0.005 molal solution of dichlorobenzene in chlorobenzene.

Thus, assuming that this sample contained as much as 1% of impurity, which is hardly probable, the maximum error caused in the final figure for the composition would be only 0.005 molal percent. If the impurity consisted mainly of the ortho and para isomers, as is quite likely, the error involved would be even smaller since the impurity would then have the same molecular weight and would lower the freezing point of the chlorobenzene to the same extent as the meta-dichlorobenzene.

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[15] It is of course true that this method involves the transfer of some heat to or from the sample and that usually a larger amount of the sample has to crystallize to raise the temperature to the freezing point, thus causing the sample to appear less pure if the length of the flat is taken as the only criterion of purity. A pure compound, however, should obviously give the same freezing point by this method as by the other.

[16] A little time may usually be saved by dropping the shield temperature to just below that of the melt when the two are only about a degree apart, and then starting the curve from this point.

[17] Andrews, Lynn and Johnston, *loc. cit.*

EXPERIMENTAL RESULTS.

Cooling and heating curves were run on approximately 0.5 gram samples of (I) pure chlorobenzene, (II) chlorobenzene containing 0.50 mole percent of meta-dichlorobenzene, and (III) a partially purified sample of chlorobenzene which happened to be available. Typical results given in Tables 1 and 2 are represented graphically in Figures 2 and 3 respectively.

In the tables, the columns labelled I, II and III give the temperatures of the corresponding samples at the times shown in the first column. The curves have been shifted along the time axis to make them more readily comparable; i. e., in Table 1, so that they all coincide at the time of initial crystallization when the temperature of the sample rises to the freezing point, and in Table 2, so that the shield temperatures of curves II and III lag about two minutes behind those for curve I. The figures in parenthesis are the temperatures of the shield. The temperatures are all given in microvolts as read, since the corresponding unit in degrees is an inconvenient figure, 0.0293° for readings between − 1600 and − 1700 microvolts. A calibration correction of − 1 microvolt must be added to the figures given so that the corresponding temperatures may be read directly from the standard conversion tables.[18]

In the graphical representations the shield temperatures for Curves I alone have been drawn (the heavy straight lines) since for practical purposes the slopes for the other two curves corresponding may be considered the same. In Figure 2 therefore they would all practically coincide and in Figure 3 the shield lines for samples II and III would both be represented by a line roughly parallel to that for sample I shown but with each point shifted about two minutes to the right.

The cooling curves obtained for chlorobenzene (sample I) showed that it was slightly purer than the meta-dinitrobenzene sample so that we have used the former as our standard for a pure substance.

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[18] "International Critical Tables," New York, Vol. I, page 58.

TABLE 1.

Cooling Curves.

Time (min.)	I microvolts	II microvolts	III microvolts
10	1859	1853	1865
10.5	1647	1658	1675
11	1647	1655	1675
11.5	(1960)	(1977)	(1973)
12	1647	1655	1675
12.5	(1977)	(1988)	(1987)
13	1647	1655	1675
13.5	(1993)	(1999)	(2000)
14	1647	1655	1675
14.5	(2006)	(2013)	(2013)
15	1647	1655	1678
15.5	(2020)	(2027)	(2024)
16	1647	1656	1680
16.5	(2034)	(2042)	(2036)
17	1647	1659	——
17.5	(2049)	1660	1685
18	1647	1661	(2055)
18.5	(2062)	(2070)	1690
19	1647	1665	(2070)
19.5	(2075)	(2082)	1699
20	1648	1675	(2083)
20.5	(2092)	(2094)	1710
21	1648	1690	1717
21.5	(2104)	1699	1727
22	1651	1710	1735
22.5	1655	1722	1746
23	1660	1734	1756
23.5	(2136)	1752	(2123)
24	1690	1769	1784
24.5	1751	1786	1802
25	1797	(2152)	1828
25.5	(2165)	1835	1853
26	1887	1878	1875
26.5	(2178)	1897	(2165)
27	1970	1930	1922
27.5	(2195)	(2184)	(2180)
28	——	1975	1965
28.5	(2207)	(2197)	(2194)
29	——	2015	1997
29.5	——	(2208)	(2205)

TABLE 2.

Heating Curves.

Time (min.)	I microvolts	II microvolts	III microvolts
4.5	(1785)	——	——
5	1785	——	——
5.5	(1777)	——	——
6	1780	——	——
6.5	(1768)	(1781)	——
7	1775	1778	——
7.5	(1779)	(1775)	(1778)
8	1770	1776	1777
8.5	(1749)	(1764)	(1771)
9	1762	1770	1773
9.5	(1740)	(1758)	(1763)
10	1753	1765	1768
10.5	(1732)	(1751)	(1752)
11	1745	1757	1764
11.5	(1725)	(1743)	(1745)
12	1735	1753	1760
12.5	——	(1732)	(1736)
13	1727	1747	1755
13.5	(1708)	(1723)	(1728)
14	1718	1745	1751
14.5	——	(1716)	(1719)
15	1710	1739	1747
15.5	(1690)	(1709)	(1712)
16	1702	1734	1743
16.5	——	(1700)	(1705)
17	1692	1730	1739
17.5	(1672)	(1692)	(1696)
18	1683	1726	1735
18.5	——	(1683)	(1687)
19	1675	1721	1731
19.5	(1653)	(1675)	——
20	1671	1717	1727
20.5	——	——	(1671)
21	1666	1712	1725
21.5	(1635)	(1657)	(1661)
22	1662	1707	1721
22.5	(1628)	(1650)	——
23	1660	1704	1717
23.5	(1621)	(1641)	(1645)

TABLE 2.—*cont.*

Time (min.)	I microvolts	II microvolts	III microvolts
24	1658	1698	1714
24.5	(1612)	(1633)	(1636)
25	1657	1694	1710
25.5	(1603)	——	(1628)
26	1655	1690	1708
26.5	——	(1618)	——
27	1654	1686	1706
27.5	(1583)	(1607)	(1613)
28	1654	1684	1705
28.5	——	(1600)	(1605)
29	1653	1680	1701
29.5	(1570)	(1592)	(1598)
30	1651	1678	1700
30.5	(1563)	(1584)	——
31	1651	1676	1698
31.5	(1552)	(1577)	(1580)
32	1650	1674	1696
32.5	——	(1569)	(1570)
33	1650	1671	1695
33.5	(1534)	(1562)	(1561)
34	1650	1670	1693
34.5	——	(1554)	(1553)
35	1649	1669	1691
35.5	(1518)	(1546)	(1545)
36	1649	1667	1690
36.5	——	——	(1536)
37	1648	1665	1688
37.5	(1501)	(1530)	(1527)
38	1648	1664	1687
38.5	——	(1522)	(1518)
39	1648	1662	1685
39.5	(1483)	(1514)	(1512)
40	1648	1661	1684
40.5	(1475)	(1506)	(1505)
41	1647	1661	1682
41.5	(1468)	(1498)	(1499)
42	1647	1660	1681
42.5	(1460)	(1490)	(1492)
43	1646	1658	1681
43.5	(1450)	(1481)	(1484)

TABLE 2.—*cont.*

Time (min.)	I microvolts	II microvolts	III microvolts
44	1646	1657	1679
44.5	(1437)	(1471)	(1475)
45	1645	1656	1678
45.5	——	(1461)	(1468)
46	1645	1655	1677
46.5	(1416)	(1451)	(1461)
47	1645	1655	1676
47.5	1645	(1442)	1676
48	1643	1655	1676
48.5	1641	1654	1675
49	(1395)	1652	1673
49.5	1639	1650	1670
50	1635	1648	1667
50.5	1632	1646	1665
51	1630	1643	1660
51.5	(1377)	(1404)	(1420)
52	1618	1632	1650
52.5	(1365)	(1397)	(1410)
53	1597	1604	1633
53.5	1582	(1390)	(1401)
54	1557	1524	1596
54.5	(1353)	(1383)	(1391)
55	1510	1471	1558
55.5	(1346)	(1376)	(1382)
56	1465	——	1517
56.5	(1339)	——	(1373)
57	1432	——	——
58	1404	——	——
58.5	(1320)		

COMPARISON OF THE TIME-TEMPERATURE CURVES.

A comparison of the results obtained for samples I, II and III (see Figures 2 and 3) shows that *the effect of impurity on the shape of the cooling and heating curves is much more pronounced when the compound is very nearly pure.* Thus, the effect of the first 0.5 molal percent added to the pure compound was almost as great as over three times as much impurity.

Cooling Curves. Curve I, Figure 2, shows the type of cooling curve obtained with the purest sample of chlorobenzene. From minute

10.5 to minute 21, the fall in temperature is only one micro-volt. The temperature falls 35 microvolts during the next three minutes and then drops abruptly and practically linearly toward the shield temperature. The maximum rate of fall is reached at about the 24th minute.[19]

Although the freezing point of sample II is only about 8 microvolts, 0.23° C., lower than that of the pure standard the characteristics of the cooling curve (Curve II, Figure 2) have been very decidedly altered. (1) The "flat", that is, the constancy of the temperature to within one microvolt, lasts only about one-half as long, from minute 10.5 to minute 16. (2) The subsequent falling off of the temperature is much more gradual so that the maximum rate of drop is not reached until about 9 minutes later. (3) The drop to the temperature of the shield is less steep, and therefore (4) this part of the curve has a fairly definite point of inflection not perceptible in Curve I, and (5) Curve II crosses Curve I.

Although Sample III contains enough impurity to lower the freezing point of the pure standard about three and one-half times as much, 29 microvolts (0.85° C), as in the case of Sample II, the characteristics of the cooling curve (Curve III, Figure 2) are not very different. The "flat" still lasts 3.5 minutes, from minute 10.5 to minute 14, the rounding off of the curve from this point is only a little more decided, and the drop to the temperature of the shield is only slightly slower so that in this region it runs almost parallel to Curve II though they finally do cross. Here also the maximum rate of fall of the temperature is at about the 24th minute.

Necessity for Sufficient Supercooling. It must be kept in mind that during the cooling curve the sample is never in true equilibrium except immediately after the crystallization causes the rise in temperature of the supercooled liquid to the freezing point. At this point only is the sample at the same temperature throughout. In the absence of stirring,[20] the part of the sample next to the walls of the tube

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[19] Curve I (Figure 2) and Curve I (Figure 3) are quite satisfactory approaches to the ideal cooling and heating curves for a pure substance. For a thorough discussion of the reasons for the deviations from these ideal curves in practice, the reader is referred to papers by White [Amer. Jour. Sci., [4], *28*, 453 (1909); Z. anorg. Chem., *69*, 305 (1911); and J. Phys. Chem., *24*, 393 (1920)]. See also Andrews, Kohman and Johnston, *loc. cit.*, Steiner and Johnston, J. Phys. Chem., *32*, 914 (1928), and Tammann, "Aggregatzustände," Leipzig (1922).

[20] White, J. Phys. Chem., *24*, 393 (1920), pointed out that the necessity for stirring can be escaped, and to advantage, by using a sample tube of small diameter.

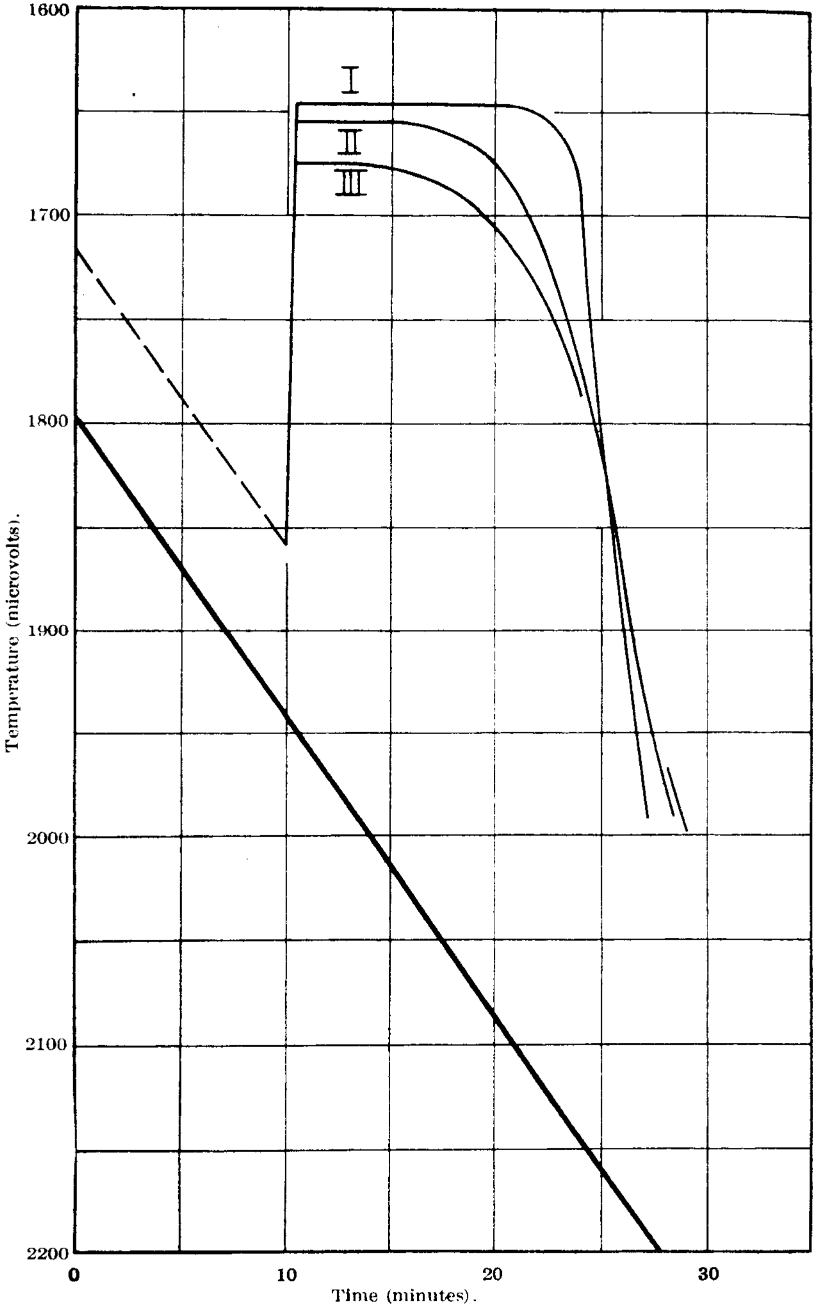

FIGURE 2. Cooling Curves.

freezes further and thus forms a solid shell around the sample, a shell which then cools below the freezing point and therefore conducts the heat away from the layer immediately inside so that that likewise freezes completely, giving up its heat of fusion, and becomes a part of the outer conducting shell. The temperature at the center of the sample remains constant until the shell has grown in that far.

In some cases the crystals formed may segregate at the top or at the bottom of the sample, depending on the relative densities of the liquid and solid phases, and then grow downwards or upwards as the case may be. The resulting cooling curves are of course abnormal and erratic. To avoid such a condition it seems better to use a rather large degree of supercooling so that enough crystals are formed at once to form a network throughout the liquid, thus preventing segregation at the top or the bottom.[21]

Heating Curves. The type of heating curve obtained with the purest sample of chlorobenzene is shown by Curve I (Figure 3). After the first few minutes the temperature of the melt rises parallel to that of the shield (represented by the heavy straight line) and at about minute 19 there is a fairly sharp rounding off after which the temperature of the sample rises more and more slowly until, between minute 35 and minute 47.5, the rise is only 4 microvolts. There is then a decided rise toward the temperature of the shield.

It should be noted particularly that *the first signs of melting appear approximately at the time when the shield reaches the temperature of the "flat."* This is exactly the behavior which would be expected for a pure compound considering the conditions under which the heating curve was obtained.[22] In comparing this curve with those for the samples

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[21] When the supercooled liquid is very viscous, as is usually the case for substances freezing at low temperatures, the crystals cannot be efficiently mixed with the liquid as a whole and one has to depend on the growth or propogation of the first crystals formed. The rate of this propogation increases with the degree of supercooling until a maximum rate is reached considerably below the freezing point. It should also be noted that, especially with low rates of crystallization, for an impure substance there may be a partial segregation of impurities in the portions remaining liquid longest. See Tammann, "Kristallisieren und Schmelzen," Leipzig (1903) pages 131–137.

[22] During such a curve, there is a temperature gradient from the outside of the sample to the thermocouple in the center, and the outside reaches the melting point first, shortly after this temperature has been reached by the surroundings. The outer layer of solid then liquefies and gradually grows toward the center where the temperature of the sample is being measured. See White, *loc. cit.*, and Tammann, *loc. cit.*

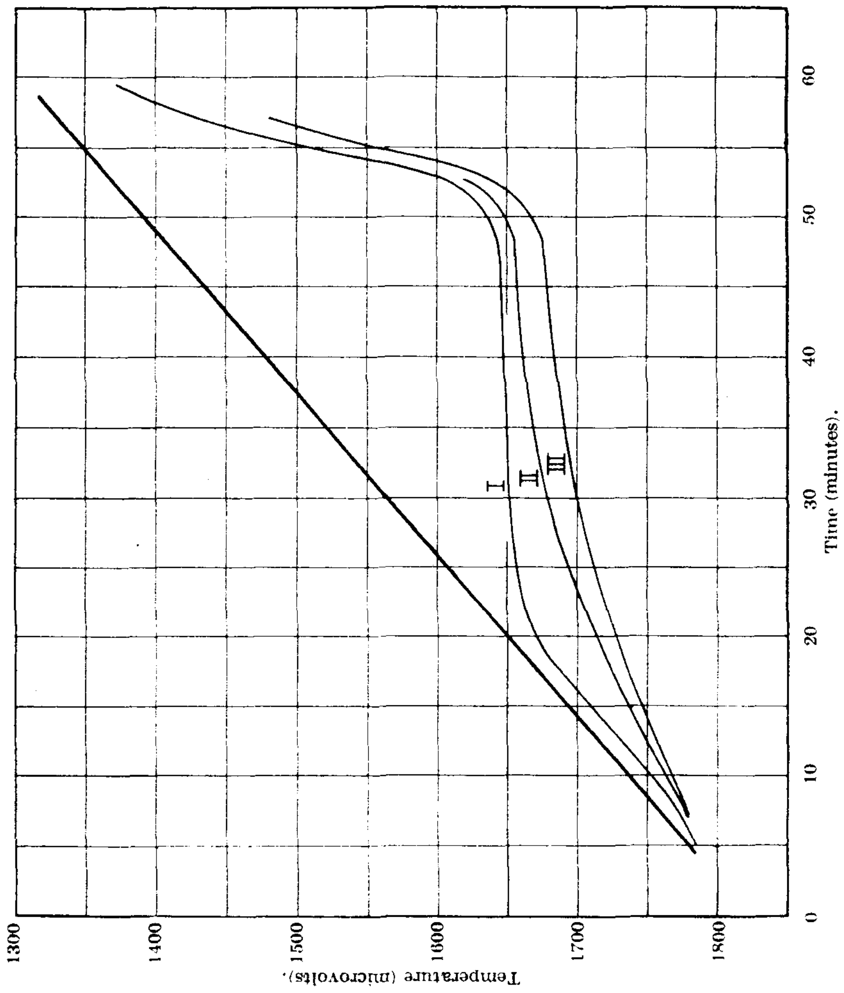

FIGURE 3. Heating Curves.

containing impurity[23] (Curves II and III, Figure 3), three important differences are obvious. (A) The temperature of the impure samples does not tend to follow that of the shield exactly but continues to digress from it from the very beginning and increasingly so,[24] thus causing the approach to the flat to be much more gradual. (B) The constancy within four microvolts just before the final break lasts in Curve II for only 5.5 minutes, from minute 43 to minute 48.5, (as compared with 12.5 for Curve I), and in Curve III for 4.5 minutes, from minute 44 to minute 48.5. (C) The break at the end of the slow rise seems in each case to come within 2 microvolts of the freezing point as determined by the corresponding cooling curves. This will be discussed further below. In the case of effect (A) and (B) it should be noticed that, as in the case of the corresponding cooling curves, there is almost as great a difference between Curves I and II as between I and III although the percentage of impurity in III is roughly three times as great as in II.

SPECIFICATIONS FOR COOLING AND HEATING CURVES FOR A PURE COMPOUND.

On the basis of the above experiments the following are the important characteristics of cooling and heating curves, obtainable under the same conditions in this or a corresponding apparatus, which are suggested for use in applying this method as a criterion of purity using the pure chlorobenzene as a standard.

Cooling Curves. (1) The "flat," that is, that portion of the curve where the temperature drops through only one microvolt, should last through three-quarters of the time between the initial crystallization and the time when the melt temperature has reached its maximum speed downwards. (2) Within the three minutes following, the temperature should drop at least 1° (about 35 microvolts). For substances having a very low heat of fusion per gram, the temperature difference between the shield and the sample as measured at this point would be much smaller and therefore the drop during the next three minutes would be correspondingly smaller. This condition would, however, be indicated by a much shorter duration of the whole curve. (3) The ensuing drop to the temperature of the shield should be very rapid and almost linear when plotted on a scale of 2.5 minutes and 25 microvolts to a centimeter.

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[23] In making this comparison it should be kept in mind that all the points for the shield and the melt temperatures have been shifted to the right about two minutes in the case of Curves II and III, in both the tabulation (Table 2) and the graphical representation (Figure 3).

[24] This is exactly what would be expected since the specific heat of the impure sample increases rapidly with the temperature in the region just below the melting point (see White, *loc. cit.*). On the basis of Newton's Law of Cooling and the conditions under which these curves were obtained, it can be shown theoretically that the difference between the shield temperature, θ_S, and the melt temperature, θ_m, should be proportional to the specific heat of the sample, so that the value for $(\theta_S - \theta_m)$ reached would remain practically constant up to the melting point for a pure compound, whereas the presence of impurities would cause a gradual increase in $(\theta_S - \theta_m)$ in this range as here observed.

The presence of impurities is indicated by a shorter flat which is followed by a much more gradual drop to the maximum rate of fall. This maximum rate is less rapid than in the case of a pure compound and this part of the curve shows a more or less distinct point of inflection.[25]

Heating Curves. (1) The temperature of the sample should soon assume a practically constant rate of rise equal to that of the shield and in the case of an *extremely* pure sample should not deviate therefrom before the temperature of the *shield* has reached the melting point. (2) The "flat," with a constant temperature within five microvolts, should last through about one-half of the period during which actual melting is taking place. If the substance has a very low heat of fusion per gram this "flat" may be much shorter; thus, this criterion alone is not as conclusive as the approach to the "flat."

If the sample is not pure, the temperature of the sample does not rise parallel to that of the shield. Its rate of change grows increasingly less than that of the shield and thus the curve is much more rounded in its approach to the "flat," which is shorter than for the pure compound.

THE ADVANTAGES OF HEATING CURVES OVER COOLING CURVES AS A CRITERION OF PURITY.

The above results seem to indicate that heating curves are perhaps to be preferred to cooling curves in testing the purity of a compound.

In the first place, they are more easily obtained since the procedure is more automatic, there being no necessity for the rather troublesome supercooling, inoculating and stirring.

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[25] The "flat" in this case is of course lower than for the pure compound but this cannot be observed from the one curve alone. The same is true for the heating curves.

Secondly, they have a wider range of applicability, for in the case of some compounds it is impossible to get normal cooling curves on account of their failure to crystallize at the proper time or on account of their slow rate of crystallization.[26] On the other hand, it is always possible to get a heating curve of a compound if it can be made to crystallize at all.

Thirdly, heating curves are probably more sensitive to very small amounts of impurities, for in the range just below the freezing point, where the impurities have the greatest effect on the specific heat, deviations from the normal curve for a pure compound can be much more readily and definitely detected because (1) the difference between the shield and the melt temperatures is only a degree or two instead of in the neighborhood of twenty degrees as in the case of the cooling curves and (2) the curve for the shield supplies the comparison line for the pure compound whereas in the corresponding range for the cooling curve, the sharp drop at the end, the slope of the reference line representing the pure compound can not be very definitely described.

Fourthly, in the case of compounds which transform as soon as they liquefy and where, therefore, the use of the cooling curve method is out of question, the heating curve method is, in some cases at least, still quite practicable; for example, in the case of transanisaldoxime.[27] This is undoubtedly due to the fact that during a heating curve the transformation takes place only in the melted portion which grows inwards from the outside of the sample tube and that the temperature gradient in the solid core remaining for the duration of the melting is therefore not greatly affected.

LIMITATIONS OF THE HEATING CURVE AND COOLING CURVE METHODS.

If the heating curve obtained for a sample by this method gives indications of impurities, the presence of impurities is quite definitely proved. It has been our experience, however, that the same statement can not be made unconditionally for cooling curves, as these are much more likely to show abnormalities due, for example, to the effectiveness of the inoculation and to the ease with which the supercooled liquid crystallizes, as will be discussed below.

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[26] This happens much more often for liquids than for solids. It was shown by Tammann and Hesse, Z. anorg. allg. Chem., *156*, 245 (1926), that the higher the melting point, the greater is the temperature interval in the supercooled liquid state within which the working of the viscous mass by blowing, drawing, etc. is possible; that is, within which it may be expected to attain equilibrium between the liquid and the solid phases with the most efficient stirring.

[27] Skau and Saxton, J. Phys. Chem., January, 1933.

As has been pointed out before, however,[28] if a sample gives a cooling curve or a heating curve which corresponds with that for a pure compound, the proof that the sample is pure is still not absolutely established. It can merely be stated that the composition of the solid phase and of the liquid phase remain identical throughout the freezing or melting, and therefore the possibility of a eutectic mixture and of certain supposedly rare mixed-crystal types are not excluded. Ordinarily, however, enough is known about the sample from other data to reduce the probability of its being a mixture of such exact composition as, for example, a eutectic mixture.

DETERMINATION OF FREEZING POINTS OR TRANSITION POINTS.

The freezing point of a sample can theoretically be found directly from the cooling curves or heating curves which have been run to test its purity. Further, it can usually be read much more easily from the cooling curve than from the heating curve due to the fact that in the former case the temperature remains constant at the freezing point for a considerable length of time, at the beginning of which the solid and liquid phases are presumably in true equilibrium. In the case of heating curves, on the other hand, the temperature is not perfectly constant, nor is the sample as a whole at equilibrium at the time the freezing point is reached. In fact, in the latter case the exact point taken as the freezing point has to be more or less empirically chosen; on the basis of this and of considerable unpublished work the temperature of the sample two minutes before the beginning of the change in direction of the curve at the end of the "flat" can be taken as the freezing point.

In general it has been our experience that if crystallization takes place rapidly[29] the values obtained by the two methods agree within two or three microvolts, the value for the heating curve almost always being the higher. Thus, in the case of Samples I, II, and III, the values are −1645, −1655, and −1676 microvolts, respectively, by the heating curves, and −1647, −1656, and −1675 by the cooling curves. It is not conceivable that the freezing point obtained from the cooling curve can be too high since the surroundings are always at a lower temperature than this and the sample approaches the freezing point from a lower temperature. On the other hand, the value obtained from the heating curve seems never to be too low.[30] [31] Thus, in this case, for example, we can say the freezing point of pure chlorobenzene lies between −1645 and −1647 microvolts, −45.20° and −45.26°. This agrees well with the value assigned to it by Timmermans, −45.2°.[32]

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[28] The reader is referred to a paper by E. W. Washburn [Jour. Ind. Eng. Chem., 22, 985 (1930)] in which various criteria of purity are discussed.

[29] In such cases the process of purification, during a fractional distillation, for example, can be followed by freezing point determinations, of the type described below, until successive fractions show only slight differences, and then the shape of the completed curves can be determined for the final fractions.

The agreement between heating and cooling curves is not always as good as in the above case, however, and it is usually the cooling curve value which is at fault. The value from heating curves is usually reproducible to within two microvolts at least and, on the other hand, no general statement can be made in regard to the reproducibility obtainable from cooling curves except that it is dependent on a fairly rapid rate of crystallization under the conditions of these experiments.[33] When the substance crystallizes much too slowly under the given conditions, as often happens, the apparent freezing point may vary from curve to curve and the shape of the curves may indicate a high percentage of impurity even though the substance is pure.

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[30] This was pointed out, for example, by White, *loc. cit.*; see also Tammann, "Aggregatzustände."

[31] The fact that the cooling curve for sample III gave a value which was higher than that for the heating curve can not be considered as an exception to this statement since the difference was only one microvolt, and therefore within the accuracy of our readings.

[32] Timmermans, van der Horst and Onnes, Archives Neerlandaises des Sciences Exactes et Naturelles, III A, *6*, 180 (1922). See also Communications from the Physical Laboratory of the University of Leiden, Supplement No. 51b, page 35 (1924). The more recent value of Timmermans, −45.35° [Timmermans, Communications from the Physical Laboratory of the University of Leiden, Supplement No. 84 page 3 (1928)] has been retracted by him in favor of his previous value. (Private Communication.)

[33] It should be remembered that this crystallization takes place without vigorous stirring and that therefore if the supercooled liquid is viscous it is more difficult to attain the true equilibrium temperature between the solid and liquid phases. Even in the case of chlorobenzene, Sample I above, where the crystallization takes place rapidly, one of the cooling curves gave a value of −1649 instead of −1647 microvolts. The shape of the curve, however, is not perceptibly different so that its use as a criterion of purity is not affected.

In determining the freezing point by cooling curves, therefore, a number of rapid curves should be run on the sample, setting the shield temperature at intervals of 50 or 100 microvolts below the freezing point for successive curves, inserting the melted sample in the apparatus, and starting the crystallization when the temperature of the melt has come to within 50 or 100 microvolts of that of the shield, which should be kept roughly constant. Thus, various degrees of supercooling can be tried and the highest value obtained for the freezing point will obviously be the nearest to the true value. Four such determinations can usually be carried out in a half hour, since the curves do not have to be completed; the sample being remelted outside the apparatus and a new curve started as soon as the temperature of the beginning of the "flat" has been measured. The freezing points obtained by this method would of course not be expected to be accurate if much impurity were present.[34]

If the original cooling and heating curves give freezing points which agree within one or two microvolts, the true freezing point can be taken as the mean of these values. If, as often happens, the difference is greater, the true freezing point must lie between the heating curve value and the highest value obtained from a series of rapid cooling curves; and if the difference is very decided or if the cooling curve values do not agree among themselves, the true freezing point probably lies nearer the heating curve value.

The same type of curves can be run to find the transition temperature of one solid form into another, but in this case, as has been previously been pointed out for the case of carbon tetrachloride,[35] the cooling curve method is much more likely to give low results than in the case of a solid-liquid equilibrium.

DISCUSSION.

As was pointed out by Washburn,[36] we can never be absolutely certain that a given compound is pure; we can merely reduce the probability of its being impure to a minimum. Let us consider from this point of view the advantage of the time-temperature-curve method for detecting impurity.

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[34] Nernst, Z. physik. Chem., *15*, 681 (1894).

[35] Skau and Meier, Jour. Amer. Chem. Soc., *51*, 3517, (1929).

[36] Washburn, *ibid.*

If a liquid has been purified by fractional distillation, and finally a sample has been obtained which boils at a constant temperature, and which, on further fractionation, gives fractions all of which have this same boiling point, we can conclude that the sample is either a pure compound, an azeotropic mixture, or an ideal solution of two or more substances having identical boiling points; and that thus the probability that it is impure is equal to the probability of it being a constant-boiling mixture. This probability can not be reduced by showing that other physical constants also are identical for the various fractions, or, in other words, by applying any number of additional comparative criteria, except insofar as these constants may be more sensitive to impurity than the boiling point; another method of fractionation may now be resorted to, e. g., fractional crystallization without a solvent. If the fractions thus obtained are identical so far as the physical constants are concerned we can state that the sample is either a pure compound or, that which is highly improbable, that it is a constant-boiling mixture which freezes completely at a constant temperature.[37]

The same degree of certainty in regard to purity may be obtained much more simply by the use of a non-comparative method; for example, by showing that the original sample which boils at a constant temperature gives time-temperature curves corresponding to those for a pure compound. In other words, the occurrence of this type of time-temperature curve indicates at once that the sample freezes at a constant temperature and thus obviates the necessity of carrying out the fractional crystallization which had to be resorted to in the other case in order to reach approximately the same degree of certainty.

Advantages of the Apparatus. The following are the outstanding advantages of the apparatus here described for use as a criterion of purity: (1) It is applicable to both liquids and solids.[38] (2) The sample necessary is very small, 0.5 to 0.7 grams. (3) The sample is always visible. (4) The samples can be handled entirely in vacuo in permanent sample tubes and contamination is thus impossible. This is of great importance not only in the case of compounds which are hygroscopic (like ethyl alcohol) or which absorb carbon dioxide from the atmosphere (like certain amines) or which oxidize in air (like polyphenols), but also in the case of any liquid freezing below 0° C., for at low temperatures the condensation of moisture and of carbon dioxide as well as the solution of air in the sample are very grave sources of contamination. (5) The complete time-temperature curve can be measured and thus the shape of the curve rather than just the length of the "flat" is obtainable. (6) Any number of determinations can be made on the same sample. (7) The curves obtained should be readily reproducible by other investigators. Since the temperature gradient and rate of temperature fall can be designated and since stirring is unnecessary, the conditions can be quite exactly defined. (8) The apparatus has been "calibrated" for use as a non-comparative criterion of purity using as a standard a sample which was proved by its temperature-heat-content curve to be sufficiently pure. (9) The apparatus can be used for the determination of freezing points and transition points with an accuracy of a few hundredths of a degree.

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[37] Since the composition of the solid phase must be the same as that of the liquid out of which it has crystallized.

[38] It has also been used for determining the freezing point and the purity of samples of substances which are gases at room temperature, which were supplied to us through the kindness of Dr. H. S. Davis. See Davis and Schuler, Jour. Amer. Chem. Soc., *52*, 721 (1930).

ACKNOWLEDGEMENT.

This opportunity is taken to thank Professor Frederick G. Keyes for the kind interest he has taken in this investigation.

SUMMARY.

An apparatus for obtaining time-temperature curves has been described in which the sample may be handled in vacuo if desired. The interpretation of the curves obtained with this apparatus (1) as a criterion of purity and (2) for accurate freezing point determinations have been discussed on the basis of experimental curves for a standard sample of high purity as well as for samples containing known amounts of impurity, and the advantages of heating curves over cooling curves have been pointed out. Criteria of purity have been classified as comparative and non-comparative criteria and the relative merits of the two classes have been discussed.

TRINITY COLLEGE,
HARTFORD, CONN.

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RECORDS OF MEETINGS, 1931–32.

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RECORDS OF MEETINGS.

One thousand one hundred and ninety-first meeting.

OCTOBER 14, 1931.—STATED MEETING.

The Academy met at its House at 8.25 P. M.

The PRESIDENT in the Chair.

There were present eighty-six Fellows and Associates and nine guests.

The records of the meeting of May 13 were read and approved.

It was announced that Mr. Robert P. Bigelow had been appointed Corresponding Secretary *pro tem.* in the absence on leave of Mr. Tenney L. Davis.

The Corresponding Secretary reported the receipt of letters accepting Fellowship from George W. Anderson, Edward C. Armstrong, Stephen V. Benét, C. Harold Berry, John D. Black, George Blumer, Roland W. Boyden, H. Addington Bruce, Richard C. Cabot, William B. Castle, Karl T. Compton, Melvin T. Copeland, Samuel H. Cross, William L. Crum, Eugene F. DuBois, Sidney B. Fay, Fred T. Field, James Ford, Robert Frost, William K. Gregory, George R. Harrison, Raymond D. Havens, Robert S. Hillyer, George L. Hosmer, Manley O. Hudson, Nathan Isaacs, Henry James, Truman L. Kelley, R. Bruce Lindsay, George F. McEwen, Francis P. Magoun, Jr., Paul Manship, Daniel G. Mason, Frank J. Mather, Wesley C. Mitchell, William DeW. Mitchell, Edmund M. Morgan, Kenneth B. Murdock, William A. Noyes, Jr., Otto Oldenberg, Herbert Parker, Thomas N. Perkins, Thomas R. Powell, Robert K. Root, George Rutledge, Henry K. Sherrill, Jeremiah Smith, Jr., Pitirim A. Sorokin, Oliver M. W. Sprague, Donald S. Tucker, Charles A. Weatherby, and George W. Wickersham; and of letters accepting Foreign Honorary Membership from Robert S. Conway, Ludwig Diels, Herbert A. L. Fisher, John Galsworthy, August Krogh, Sylvain Lévi, Antoine Meillet, Edgar A. Peers, Charles S. Terry, George M. Trevelyan, and Karl W. Wagner.

He announced that the Council had made the following grants from the Permanent Science Fund:

1. To Dr. Leonard S. Hsu, of the Princeton-Yenching Foundation, one thousand dollars, for researches on the population problems in China;

2. To Professor J. LeRoy Conel, of Boston University, two hundred dollars, for aid in financing a fishing expediton to collect naturally deposited eggs of *Myxine glutinosa* for embryological study;

3. To Professor Edgar Altenburg, of Rice Institute, Houston, Texas, five hundred dollars, to aid his researches on the effect of ultra-violet rays upon the germ-plasm of *Drosophila melanogaster*.

The Corresponding Secretary reported that the Academy had received from His Majesty the King of Siam a gift of the latest edition of the translation from the Pali into Siamese of the Phra Tripitaka, sacred Buddhist canon and commentaries thereon, and from His Excellency Chao Phya Abbai Raja a set of three volumes of the translation into Siamese of the Paramatthamanjusa Visuddhi Maggatika, commentaries on the Visuddhi-Magga, and that the President had addressed a letter to His Excellency the Siamese Minister to the United States asking that the thanks of the Academy be transmitted to the distinguished donors of these valuable gifts.

It was announced that the President had appointed delegates to represent the Academy as follows:

To the dedication of the Lakeside Hospital, Western Reserve University, at Cleveland, June 16, 1931, Dr. Thorald H. Sollmann;

To the inauguration of William Pearson Tolley as President of Allegheny College on October 9, 1931, Mr. George Wharton Pepper, who was obliged to decline;

To the centennial celebration of Denison University, on October 16–18, 1931, President Ernest H. Wilkins of Oberlin;

To the inauguration of Frank Porter Graham as President of the University of North Carolina on November 11, 1931, Professor William McDougall.

The Corresponding Secretary called attention to a printed announcement of the Ninth International Congress of Pure and Applied Chemistry to be convened at Madrid, April 3–10, 1932.

The President announced the death of seven Fellows:—Francis

Greenleaf Allinson (Class III, Section 2), Solon Irving Bailey (Class I, Section 1), Daniel Chester French (Class III, Section 4), Edward Skinner King (Class I, Section 1), Clifford Herschel Moore (Class III, Section 2), George Foot Moore (Class III, Section 2), George Fillmore Swain (Class I, Section 4); and of two Foreign Honorary Members:— Louis Dollo (Class II, Section 3) and Richard Wettstein (Class II, Section 2).

The Fellows elected in May were then presented to the Academy.

The Rumford Medals were presented to President Karl Taylor Compton of the Massachusetts Institute of Technology for researches in thermionics and spectroscopy, after an introduction by Mr. Arthur E. Kennelly, Chairman of the Rumford Committee.

The following communication was presented:

Mr. Karl T. Compton: "Thermionics as a Scientific Tool."

Five papers were read by title:

"Bermuda during the Ice Age," by Robert W. Sayles;

"On the Apsidal Motion of a Faint Eclipsing Binary," by Harlow Shapley;

"The Indigenous Linear-leaved North American Pondweeds of Potamogéton § Axillares," by M. L. Fernald;

"Physical Properties of Single Crystal Magnesium," by P. W. Bridgman;

"Volume-Temperature-Pressure Relations for Several Non-Volatile Liquids," by P. W. Bridgman.

The meeting was dissolved at 10.10 P. M.

One thousand one hundred and ninety-second meeting.

NOVEMBER 11, 1931.—STATED MEETING.

The Academy met at its House at 8.35 P. M.

The President in the Chair.

There were present thirty-two Fellows and Associates and one guest.

The records of the meeting of October 14 were read and approved.

The Corresponding Secretary reported the receipt of a letter from Mr. Paul Elmer More, accepting Fellowship, and of letters declining Fellowship from Messrs. Zechariah Chafee, Jr., and Calvin Coolidge.

He also made the following announcements from the Council:

That the vacancies in the Standing Committees had been filled as follows, until the next election:

Library Committee, William Y. Elliott, of Class III
House Committee, Davis R. Dewey, of Class III
Committee on Meetings, Charles J. Bullock, of Class III

That the Academy had been represented at the funerals of Dr. Samuel W. Stratton and Mr. Charles F. D. Belden, and that a letter of condolence had been sent to the family of Mr. Roland W. Boyden;

That Dr. Henry B. Bigelow had been appointed a delegate to represent the Academy at the Fifth Pacific Science Congress, to be held at Victoria and Vancouver probably in 1933;

That the Academy had received an invitation to be represented at the Seventh American Scientific Congress, to be held in Mexico City, February 5 to 19, 1932.

The Corresponding Secretary read a letter from H. A. Barton, Director, announcing the organization of the American Institute of Physics, through the coöperation of the American Physical Society, the Optical Society of America, the Acoustical Society of America, and the Society of Rheology.

The President announced the death of two Fellows:—Roland William Boyden (Class III, Section 1), and Samuel Wesley Stratton (Class I, Section 4); and of an Associate, Charles Francis Dorr Belden.

The President reported that he had received from the Secretary of the Commonwealth a certified copy, dated May 1, 1931, of the Act of the General Court, Chapter 255 of the Acts of 1931, approved April 29, 1931, amending the Charter of the Academy so that the number of Fellows who may be inhabitants of Massachusetts is increased from four hundred to six hundred.

On behalf of the Council the Corresponding Secretary presented two proposed amendments to the Statutes as follows:

1. Amendment to Chapter II, Article 1, as amended April 8, 1931:
    After "Class IV, Section 2," strike out "History and Archæology" and insert "History, Archæology, and Anthropology."
2. Amendment to Chapter II, Article 5:
    In line 2 after "dollars" insert "unless previously as an Associate he has paid an Admission Fee of like amount," so that the first paragraph of the article shall read:

Article 5. Every Fellow resident within fifty miles of Boston hereafter elected shall pay an Admission Fee of Ten Dollars, unless previously as an Associate he has paid an Admission Fee of like amount.

Mr. Edwin B. Wilson raised the point that these amendments were simply corrections of engrossing and therefore did not need to be referred to a committee to be reported at a subsequent meeting of the Academy. The point of order was sustained and it was

Voted, That the amendments be adopted.

The following communications were presented:

Dr. J. F. van Bemmelen, of Groningen: "Heredity of Mental Qualities in Man."

Mr. Edward K. Rand: "The Script of Tours."

The meeting was dissolved at 10.35 P. M.

One thousand one hundred and ninety-third meeting.

DECEMBER 9, 1931.—STATED MEETING.

The Academy met at its House at 8.30 P. M.

The PRESIDENT in the Chair.

There were present thirty-seven Fellows and Associates and eleven guests.

The records of the meeting of November 11 were read and approved.

The Corresponding Secretary reported that a letter had been received from the Permanent Secretary of the American Council of Learned Societies announcing that he had been authorized to include in the tentative budget for 1932 an item of $1,500 as a second contribution from the American Council to the amount expended for the publication of three papers by Edward Sapir on native American languages as Volume 65 of the Proceedings of the Academy.

The President announced the death of two Fellows:—Clarence Lemuel Elisha Moore (Class I, Section 1) and Robert DeCourcy Ward (Class II, Section 1); and of one Foreign Honorary Member, John Briquet (Class II, Section 2).

The following communications were presented:

Dr. Harvey Cushing: "Experimental Gigantism."

Messrs. John W. M. Bunker, Nicholas A. Milas, and Robert S.

Harris: "The Use of Synthetic Organic Peroxides for the Cure of Experimental Rickets."

One paper was presented by title: "Postulates for Separation of Point-Pairs (Reversible Order on a Closed Line)," by E. V. Huntington and K. E. Rosinger.

The meeting was dissolved at 9.57 P. M.

One thousand one hundred and ninety-fourth meeting.

JANUARY 13, 1932.—STATED MEETING.

The Academy met at its House at 8.25 P. M.

The PRESIDENT in the Chair.

There were present thirty-eight Fellows and Associates.

The records of the meeting of December 9 were read and approved.

The Corresponding Secretary reported the receipt of a letter from Dr. Henry Jackson, resigning Fellowship, to take effect at the end of the current academic year; of three copies of a printed paper on the "Caffeine Effect of the Crest Uniformity of Muscular Fatigue Curves," by Professor Ralph H. Cheney, of Long Island University, embodying the results of a research aided by a grant from the Permanent Science Fund; and of a number of copies of a "Summary of Activities in 1931" of the American Council of Learned Societies.

The President announced the death of a Fellow, Richard Henry Dana (Class III, Section 4) and of an Associate, Frederick Pickering Cabot.

The following communication was presented:

Mr. H. Addington Bruce: "Sources of American Discontent."

The meeting was dissolved at 10.04 P. M.

One thousand one hundred and ninety-fifth meeting.

FEBRUARY 10, 1932.—STATED MEETING.

The Academy met at its House at 8.15 P. M.

The PRESIDENT in the Chair.

There were present forty-eight Fellows and Associates.

The records of the meeting of January 13 were read and approved.

The Corresponding Secretary reported the receipt of biographical memoirs of Ira Nelson Hollis, by L. S. Marks, and of Clarence L. E. Moore, by Philip Franklin; also the resignations from Fellowship of William H. Dunbar and Ralph Barton Perry.

The President announced the death of Arthur Anthony Macdonell, Foreign Honorary Member in Class IV, Section 3.

On recommendation of the Council, it was

Voted, To make a supplementary appropriation of $300 from the income of the General Fund, for the expenses of the House.

The following communications were presented:

Mr. Kirk Bryan: "The Physical Features of Arid Lands as Illustrated in Arizona and New Mexico."

Mr. Arthur M. Schlesinger: "George Washington."

The meeting was dissolved at 9.40 P. M.

One thousand one hundred and ninety-sixth meeting.

MARCH 9, 1932.—STATED MEETING.

The Academy met at its House at 8.20 P. M.

The PRESIDENT in the Chair.

There were present thirty-five Fellows and Associates and two guests.

The records of the meeting of February 10 were read and approved.

The Corresponding Secretary announced the appointment of Messrs. Harry W. Tyler, George D. Birkhoff, and Marston Morse as delegates to the International Congress of Mathematicians at Zurich, September 4–12, 1932; and of Professor Arthur E. Kennelly to the National Academy's celebration of the One Hundredth Anniversary of the electrical discoveries of Joseph Henry, at Washington, on April 25, 1932.

He reported the receipt of a letter from Admiral William S. Sims, resigning Fellowship.

The President announced the death of one Fellow, William Wallace Fenn, Class IV, Section 1.

He also announced the appointment of the Nominating Committee:

Charles A. Kraus, of Class I
George R. Minot, of Class II
William C. Wait, of Class III
George H. Chase, of Class IV

On recommendation of the Council the following appropriations were made for the ensuing year:

From the income of the General Fund, $7,950, to be used as follows:

for General and Meeting expenses	$ 900
for Library expenses	2,000
for Books, Periodicals, and Binding	1,400
for House expenses	2,700
for Treasurer's expenses	950
	$7,950

From the income of the Publication Funds, $3,282.25, to be used for publication.

From the income of the Rumford Fund, $3,696.07, to be used as follows:

for Research	$2,250.00
for Books, Periodicals, and Binding	350.00
for use at the discretion of the Committee	1,096.07
	$3,696.07

From the income of the C. M. Warren Fund, $1,159, to be used at the discretion of the Committee.

The following communication was presented:

Mr. George H. Parker: "The Flow of Trophic Influences over Nerves."

The meeting was dissolved at 9.45 P. M.

One thousand one hundred and ninety-seventh meeting.

APRIL 13, 1932.—STATED MEETING.

The Academy met at its house at 8.25 P. M.

The PRESIDENT in the Chair.

There were present thirty-nine Fellows and Associates and four guests.

The records of the meeting of March 9 were read and approved.

The Corresponding Secretary announced the appointment of Mr. John D. Black as delegate to the Centenary Celebration of the University of Richmond, May 8–10, and of Monsignor James H. Ryan as delegate to the dedication of the Folger Shakespeare Library at Washington, April 23.

He also reported that the Council had reappointed Professor F. N. Robinson as delegate of the Academy to the American Council of Learned Societies for a term of four years.

The President announced the death of three Fellows:—Bernard Arthur Behrend (Class I, Section 4), Harvey Monroe Hall (Class II, Section 2), and Frederick Jackson Turner (Class IV, Section 2).

The following communication was presented:

Mr. Tenney L. Davis: "Chinese Alchemy."

The meeting was dissolved at 9.35 P. M.

One thousand one hundred and ninety-eighth meeting.

MAY 11, 1932.—ANNUAL MEETING.

The Academy met at its House at 8.20 P. M.

The PRESIDENT in the Chair.

There were present fifty-eight Fellows and three guests.

The records of the meeting of April 13 were read and approved.

The Corresponding Secretary announced that the Council had made the following grants from the Permanent Science Fund:

1. To Ralph E. Cleland, Goucher College, Baltimore, Md., $300, to aid in a study of wild races of Oenothera.

2. To Joseph A. Cushman, Sharon, Mass., $250, for a study of the foraminiferal faunas of Europe and America.

3. To Frederick L. Hisaw, University of Wisconsin, Madison, Wis., $500, to aid in a study of the physiology of the endometrium of *Macacus rhesus* monkeys.

4. To W. J. Luyten, University of Minnesota, Minneapolis, $300, to aid in a research concerning the motions of the stars.

5. To Reginald H. Painter, Kansas State College, Manhattan, $125, for a taxonomic investigation of American Bombyliidae (Diptera) and publication of monographs in this field.

6. To James T. Penney, University of South Carolina, Columbia, $300, for a study of the cellular behavior of sponges during the formation of gemmules and the formation of sponges from gemmules.

7. To Guilford B. Reed, Queen's University, Kingston, Ont., $300, for the extension of an analysis of the antigenic content of variant or dissociated strains of tubercle bacilli.

8. To Jay R. Traver, Cornell University, Ithaca, N. Y., $350, to aid in the completion of a study of North Carolina May flies.

9. To W. D. Urry, Johns Hopkins University, Baltimore, Md., $500, for the continuation of a study of the geologic age of the Keweenawan rocks.

10. To Howard H. Beard, Louisiana State University, New Orleans, $400, for an investigation on the nutritional anemia of the rat.

11. To J. L. Hough, University of Chicago, $75, to aid in a study of the bottom deposits of Lake Michigan.

The President announced the death of five Fellows:—Louis Agricola Bauer (Class I, Section 2), Charles Leavitt Edgar (Class I, Section 4), Benjamin Kendall Emerson (Class II, Section 1), William Barclay Parsons (Class I, Section 4), Roland Thaxter (Class II, Section 2); and of two Foreign Honorary Members:—Wilhelm Ostwald (Class I, Section 3) and Johan Herman Lie Vogt (Class II, Section 1).

The following report of the Council was presented:

REPORT OF THE COUNCIL.

Since the last report of the Council there have been reported the deaths of twenty-one Fellows:—Francis Greenleaf Allinson, Solon Irving Bailey, Louis Agricola Bauer, Bernard Arthur Behrend, Roland William Boyden, Richard Henry Dana, Charles Leavitt Edgar, Benjamin Kendall Emerson, William Wallace Fenn, Daniel Chester French, Harvey Monroe Hall, Edward Skinner King, Clarence Lemuel Elisha Moore, Clifford Herschel Moore, George Foot Moore, William Barclay Parsons, Samuel Wesley Stratton, George Fillmore Swain, Roland Thaxter, Frederick Jackson Turner, Robert DeCourcy Ward; six Foreign Honorary Members:— John Briquet, Louis Dollo, Arthur Anthony Macdonell, Wilhelm Ostwald, Johan Herman Lie Vogt, Richard Wettstein; and two Associates:—Charles Francis Dorr Belden, Frederick Pickering Cabot.

Fifty-five Fellows and eleven Foreign Honorary Members were elected by the Council and announced to the Academy in May 1931.

The roll now includes 613 Fellows, 97 Foreign Honorary Members, and 25 Associates (not including those elected in May 1932).

The annual report of the Treasurer, Ingersoll Bowditch, was read, of which the following is an abstract:

General Fund.

Receipts.

Income on hand April 1, 1931	$ 1,386.80	
From Investments	2,921.91	
From Assessments	5,490.00	
From Admissions	330.00	
From Interest on Deposits	189.95	
From Rumford Fund Income	200.00	$10,518.66

Expenditures.

Expenses of Library	$2,088.96		
Treasurer's Expenses	864.04		
Books and Binding	1,167.10		
General Expenses	1,009.87		
House Expenses	3,019.08		
President's Expenses	24.66		
Interest on Bonds Bought	136.67	$ 8,310.38	
Income transferred to Principal		369.62	$ 8,680.00

Rumford Fund.

Receipts.

Income on hand April 1, 1931	$ 4,778.65	
From Investments	3,957.10	$ 8,735.75

Expenditures.

Transferred to Publication Account	$2,430.42		
Purchase and Binding of Books	214.39		
Research	1,975.00		
United States Mint-medals	315.29		
Cases and engraving medals	33.00		
Mass. Hospital Life Ins. Co.	41.20		
Transferred to General Fund Income for care of books	200.00		
Books and Periodicals—bal. 1930 transferred to Capital	22.61		
Discretion of Committee—bal. 1930 transferred to Capital	2,039.21	$ 7,271.12	
Interest on bonds bought	42.17		
Income transferred to Principal	201.18	243.35	$ 7,514.47

Publication Account.

Receipts.

Income on hand April 1, 1931		$12,807.03	
From Income Appleton Fund	$ 1,443.76		
From Income Centennial Fund	2,666.93		
From Authors' Reprints	452.59		
From Sale of Publications	659.94		
From Amer. Council of Learned Societies	1,500.00		
From Farlow Herbarium for Memoirs	2,340.00		
Balance from Rumford Fund Publications	2,430.42	11,493.64	$24,300.67

Expenditures.

Publications	$12,731.94		
Vault Rent-part	10.00	$12,741.94	
Interest on Bonds bought	8.19		
Income transferred to Principal	207.20	215.39	$12,957.33

C. M. WARREN FUND.

Receipts.

Income on hand April 1, 1931		$ 344.48	
From Investments		1,256.51	$ 1,600.99

Expenditures.

Research	$ 550.00		
Vault Rent-part	3.00	$ 553.00	
Interest on bonds bought	$ 24.44		
Income transferred to Principal	62.75	87.19	$ 640.19

FRANCIS AMORY FUND.

Receipts.

From Investments		$ 2,493.00

Expenditures.

Publishing Statement	$ 55.75	
Income transferred to Principal	2,276.14	
Interest on bonds bought	161.11	$ 2,493.00

PERMANENT SCIENCE FUND.

Receipts.

Received for above fund	$ 5,850.00

Expenditures.

Grants from above fund	$ 5,850.00

The following reports were also presented:

REPORT OF THE LIBRARY COMMITTEE.

During the year 184 volumes and 25 unbound numbers of periodicals have been borrowed by 21 different Fellows and 16 different libraries, and many more have been consulted at the Academy. All books taken out have been returned or satisfactorily accounted for.

The number of bound volumes on the shelves at the time of the last report was 42,578. During the year 451 volumes were added, making the number now 43,029. This includes 46 purchased from the General Fund, 24 from the Rumford Fund, and 381 received by gift or exchange. The number of pamphlets added was 85.

No changes have been made in the periodical subscription list this year, but several institutions have been added to the exchange list. We now subscribe to 43 scientific journals and exchange publications with 450 institutions. By exchange, subscription, or gift, we receive regularly 827 separate serial publications. It may be well in this connection to remind the Fellows that it has been the policy for a number of years to use all available funds for keeping up the collection of serials, of which the Academy has many long and valuable sets, making no attempt to add to the book collection.

The following appropriations were placed at the disposal of the Librarian during the past year:

Balance from General Fund	$ 246.15
Appropriation from General Fund	3,400.00
Appropriation from Rumford Fund	350.00
Total	$3,996.15

The expenses charged to the library during the financial year ending March 31, 1932, were:

Salaries	$2,000.00
Binding:	
General Fund	$ 713.49
Rumford Fund	90.06
	$ 803.55
Purchase of Books and Periodicals:	
General Fund	$ 453.61
Rumford Fund	124.33
	$ 577.94
Miscellaneous	88.96
Total	$3,470.45

There remained an unexpended balance of $525.70, as follows:

General Fund	$ 390.09
Rumford Fund	135.61
Total	$ 525.70

This unusually large balance is due to the fact that the 1932 bill for German periodicals was received too late to be paid before the close of the fiscal year. As usual, however, all balances will be added to the appropriations for the ensuing year.

Respectfully submitted,
ALFRED C. LANE, *Librarian.*

May 11, 1932.

REPORT OF THE RUMFORD COMMITTEE.

The Rumford Committee held five meetings during the academic year 1931–1932; on May 13, Oct. 14, Nov. 11, Dec. 9, and March 9.

The Committee made the following grants from the Rumford Fund, during the year, in aid of researches on light and heat.

1931		
Oct. 14.	No. 293. Prof. P. W. Bridgman, Harvard University, for Equipment in Researches on Thermal and Optical Properties of Matter	$ 500
Nov. 11.	No. 294. Prof. Harlan T. Stetson, Ohio Wesleyan University, Delaware, Ohio, for Equipment for Measuring Solar Radiation	300
Dec. 9.	No. 295. Profs. Ross and Van Biesbroeck, Yerkes Observatory, Williams Bay, Wisconsin, for Thermoelectric Pyrometer Equipment	400
1932		
Mar. 9.	No. 296. Mr. H. M. O'Bryan, Mass. Institute of Technology, Cambridge, Mass., for Equipment for Spectroscopic Researches	500
Mar. 9.	No. 297. Prof. J. C. Stearns, Denver University, Denver, Col., for Equipment in Researches on Cosmic Rays	300
		$2,000

Reports of progress in research aided by grants from the Rumford Fund have been received from Messrs. G. H. Dieke, J. W. McBain, D. H. Crawford, P. W. Bridgman, W. J. Luyten, H. H. Plaskett, H. Shapley and Harlan T. Stetson.

The following recent publications concern researches that have been aided by grants from the Rumford Fund.

Nos.

281. By Prof. P. W. Bridgman, Harvard University, "P. V. T. Relations of NH_4Cl and NH_4Br, and in Particular the Effect of Pressure on the Volume Anomalies," *Phys. Review*, Vol. 38, No. 1, pp. 182–191, July 1, 1931.

281. By Prof. P. W. Bridgman, "Volume-Temperature-Pressure Relations for Several Non-volatile Liquids," *Proc. Am. Ac. Arts & Sc.*, Vol. 67, No. 1, Jan. 1932.

281. By Prof. P. W. Bridgman, "Physical Properties of Single Crystal Magnesium," *Proc. Am. Ac. Arts & Sc.*, Vol. 67, No. 2, Jan. 1932.

290. By George Harrison, Mass. Institute of Technology, and Philip Leighton, Stanford University, "Spectral Fluorescence Efficiencies of Certain Substances with Applications to Heterochromatic Photographic Photometry," *Phys. Review*, Vol. 38, No. 5, pp. 899–908, Sept. 1, 1931.

290. By George Harrison, "A Twenty-one Foot Vacuum Spectrograph for the Extreme Ultraviolet," *Rev. of Scientific Instruments*, Vol. 2, No. 10, Oct. 1931.

263. By Prof. Paul Kirkpatrick, University of Hawaii, "A Generating Voltmeter for the Measurement of High Potentials," *Rev. of Sci. Instruments*, Vol. 3, No. 1, Jan. 1932.

263. By Prof. Kirkpatrick, "Lateral Space Distribution of X-ray Photo-electrons," *Phys. Review*, Vol. 38, No. 11, pp. 1938–1942, Dec. 1, 1931.

286. By Prof. Harlow Shapley, Ernst J. Öpik, and Samuel L. Boothroyd, "The Arizona Expedition for the Study of Meteors," *Proc. Nat. Ac. Sciences*, Vol. 18, pp. 16–23, Jan. 1932.

Respectfully submitted,
A. E. KENNELLY, *Chairman.*

May 11, 1932.

Report of the C. M. Warren Committee.

The Committee had at its disposal at the beginning of the fiscal year 1931–1932, $1,523.11, of which $550 has been appropriated during the past year.

Since the last annual report grants have been made as follows:

June 10, 1931: to Professor E. H. Huntress, M. I. T., $250, to be used for the purchase of materials necessary in the preparation of substances to be investigated in connection with the development of a scheme for the qualitative analysis of organic compounds.

June 10, 1931: to Professor Henry Gilman, Iowa State College, $300, to be used in the purchase of expensive chemicals needed in his study of the Grignard reaction.

The Committee considered with care four applications for assistance but found it inadvisable to make grants in these cases.

At the next meeting of the Committee applications for seven grants, amounting to $2,410, will be considered.

Reports of progress, during the current year, have been made by Professors Allen, Huntress, Richtmyer, Schmidt.

The papers listed below, which have been published since the last report of the Committee, describe the results of investigations aided by the Warren Fund. In each case there is an acknowledgment by the author of the assistance received.

C. F. H. Allen and W. E. Barker:

"Addition Reactions of Vinyl Phenyl Ketone. II. Desoxybenzoin." J. Am. Chem. Soc. *54*, 736 (1932).

Sadaichi Miyamoto and Carl L. A. Schmidt:

"On the Equilibria between the Zwitterion and the Classical Amino Acids." University of California Publications in Physiology, Vol. 8, No. 1, pp. 1–7, March 4, 1932.

Respectfully submitted,

James F. Norris, *Chairman.*

May 11, 1932.

Report of the Committee of Publication.

The Committee of Publication has held one meeting during the year, the rest of its business being carried on as usual by correspondence. At the meeting of March 9, 1932, the Committee authorized the publication of a new Memoir by Professor M. L. Fernald, and left on the table for further discussion the question of publication of a statistical report submitted by Professor E. B. Wilson. On account of what seemed extremely high cost of printing, the Editor was given discretionary power to change the printer of Academy publications. The Intelligencer Press has, however, agreed to a discount of ten percent on all its bills, and as yet no change of printer has been made.

Since the last report of the Committee, Volume 65 of the Proceedings has been published, Volume 66—including the records number—has been completed, and four numbers of Volume 67 have been issued. In addition to this, the fifth of the Thaxter Memoirs has appeared. Several manuscripts submitted for publication by non-members of the Academy have been declined.

According to the Treasurer, the Committee has spent during the fiscal year—April 1, 1931 to April 1, 1932—the sum of $12,733.87. On May 1, 1932, the balance available for publications during the current year is $14,200.08. This includes an unexpended balance of $1,004.87 appropriated by the Rumford Committee for publications in the field of heat and light. The cost of the Sapir volume was $6,168.27, towards which the American Council of Learned Societies contributed $3,000.00. The expense of the Thaxter Memoir was defrayed by Professor Thaxter.

The enlargement of Classes III and IV and the prospect that demand for publication in the fields of the humanities would be increased has led President Ford to appoint a special committee consisting of Professor G. H. Chase, Professor E. F. Gay and the Editor, to consider the interests of Classes III and IV in reference to publication. This committee has held one meeting in conference with President Ford and Dr. Waldo G. Leland, Secretary of the American Council of Learned Societies. At this meeting Dr. Leland assured the Committee of his interest in publications in the field of the humanities and suggested that the Academy might perform useful service by the publication of original documents. The Council, he said; would be glad to consider applications for the financial support of such publications. Encouraged by such an attitude, Professor Kirsopp Lake has already requested the Committee to accept for publication a collection of Greek manuscripts.

Respectfully submitted,

H. V. NEAL, *Chairman.*

May 11, 1932.

REPORT OF THE HOUSE COMMITTEE.

The House Committee has had funds at its disposal amounting to $3,659.43, made up as follows:

Balance from previous year	$ 319.43
Appropriations for 1931–32	3,100.00
Received for use of rooms	240.00
	$3,659.43

Of this amount the sum of $2,586.68 has been spent for the routine expenses, janitor, light, power, heat, telephone, etc., and $672.40 has been spent for upkeep and equipment, making a total of $3,259.08, and leaving an unexpended balance of $400.35. The chief items of upkeep this year have been the painting of the reading room, the upper hall, and the janitor's suite. A new vacuum cleaner has also been purchased.

Meetings have been held as follows:

The Academy	8
American Antiquarian Society	1
American Chemical Society, Northeastern Section	8
Archæological Institute of America, Boston Society	3
Copley Society	1
Harvard-Technology Chemical Club	7
Japan Society of Boston	2
Mediæval Academy of America	1
New England Botanical Club	9
	40

The Council Chamber has been used for Academy Council and Committee meetings, and also by the Trustees of the Children's Museum, the New England Farm and Garden Association, the Thursday Evening Club, etc.

A detailed list of expenditures follows:

Janitor		$ 970.00
Electricity	Light	240.60
	Power	112.68
Coal		890.86
Elevator		107.36
Gas		56.40
Telephone		107.22
Water		55.20
Ash tickets		25.10
Upkeep		526.10
Furnishing and equipment		146.30
Janitor's supplies and sundries		21.26
Total expenditure		$3,259.08

Respectfully submitted,

S. Burt Wolbach, *Chairman.*

May 11, 1932.

On the recommendation of the Treasurer, it was *Voted*, That the annual assessment for 1932–1933 be $15.

The annual election resulted in the choice of the following officers and committees:

Jeremiah D. M. Ford, *President*
Harry M. Goodwin, *Vice-President for Class I*
Walter B. Cannon, *Vice-President for Class II*
Edwin F. Gay, *Vice-President for Class III*
Charles B. Gulick, *Vice-President for Class IV*
Tenney L. Davis, *Corresponding Secretary*
Walter E. Clark, *Recording Secretary*
Ingersoll Bowditch, *Treasurer*
Alfred C. Lane, *Librarian*
Herbert V. Neal, *Editor*

Councillors for Four Years.

Elmer P. Kohler, of Class I
Roger I. Lee, of Class II
Thomas Nelson Perkins, of Class III
William S. Ferguson, of Class IV

Councillor for Two Years.

Roland G. D. Richardson, of Class I

Finance Committee.

Thomas Barbour
Paul J. Sachs
Alfred L. Ripley

Rumford Committee.

Arthur E. Kennelly
Elihu Thomson
Percy W. Bridgman
Harry M. Goodwin
Charles L. Norton
Harlow Shapley
Norton A. Kent

C. M. Warren Committee.

James F. Norris
Gregory P. Baxter
Walter L. Jennings
Arthur D. Little
Frederick G. Keyes
Reid Hunt
Charles A. Kraus

Committee of Publication.

Herbert V. Neal, *Chairman*
Edwin C. Kemble, of Class I
Frederic T. Lewis, of Class II
Joseph H. Beale, of Class III
Robert P. Blake, of Class IV

Committee on the Library.

Alfred C. Lane, *Chairman*
Raymond C. Archibald, of Class I
Thomas Barbour, of Class II
William Y. Elliott, of Class III
Henry B. Washburn, of Class IV

Auditing Committee.

George R. Agassiz
John E. Thayer

House Committee.

S. Burt Wolbach, *Chairman*
William H. Lawrence
Robert P. Bigelow
Davis R. Dewey

Committee on Meetings.

THE PRESIDENT
THE RECORDING SECRETARY
DANIEL F. COMSTOCK GEORGE H. PARKER CHARLES J. BULLOCK
JOSHUA WHATMOUGH

The Corresponding Secretary announced that the following gentlemen had been elected members of the Academy:

FELLOWS.

CLASS I.

Section 1.	Clarence Raymond Adams, Providence, R. I.
	Albert Arnold Bennett, Providence, R. I.
	Einar Hille, Princeton, N. J.
	David Vernon Widder, Cambridge
Section 3.	Kenneth Lamartine Mark, Boston
	Walter Cecil Schumb, East Milton
Section 4.	Glennon Gilboy, Cambridge
	Albert Haertlein, Watertown
	Ralph Restieaux Lawrence, Brookline

CLASS II.

Section 1.	Frederick Kuhne Morris, Cambridge
Section 2.	Ralph Hartley Wetmore, Belmont
Section 3.	David Rapport, Cambridge
Section 4.	Joseph Charles Aub, Belmont
	Franklin Greene Balch, Boston
	Elliott Carr Cutler, Brookline

CLASS III.

Section 1.	Francis Noyes Balch, Jamaica Plain
	Felix Frankfurter, Cambridge
	Walter Perley Hall, Fitchburg
	Eldon Revare James, Cambridge
	James Arnold Lowell, Newton
	Sayre Macneil, Cambridge

Calvert Magruder, Cambridge
John MacArthur Maguire, Cambridge
Odin Roberts, Boston
Francis Bowes Sayre, Cambridge
Warren Abner Seavey, Arlington
Edward Sampson Thurston, Cambridge
Edmund Allen Whitman, Cambridge
Section 2. Charles Austin Beard, New Milford, Conn.
William Richards Castle, Jr., Washington, D. C.
Joseph Perkins Chamberlain, New York, N. Y.
Joseph Clark Grew, Tokyo, Japan
Philip Caryl Jessup, New York, N. Y.
Charles Edward Merriam, Chicago, Ill.
Ogden Livingston Mills, Washington, D. C.
William Phillips, Boston
William Franklin Willoughby, Washington, D. C.
Section 3. John Candler Cobb, Milton
William James Cunningham, Cambridge
Arthur Stone Dewing, Cambridge
Wallace Brett Donham, Boston
Norman Scott Brien Gras, Cambridge
Henry Wyman Holmes, Cambridge
Walter Wallace McLaren, Williamstown
Leon Carroll Marshall, Baltimore, Md.
Frederick Cecil Mills, New York, N. Y.
Harold Glenn Moulton, Washington, D. C.
William Fielding Ogburn, Chicago, Ill.
George Byron Roorbach, Cambridge
Leo S. Rowe, Washington, D. C.
Carl Snyder, New York, N. Y.
John Henry Williams, Cambridge
Section 4. Charles Francis Adams, Concord
Charles Foster Batchelder, Cambridge
Howard Coonley, Milton
John Albert Cousens, Tufts College
Henry Sturgis Dennison, Framingham
William Lusk Webster Field, Milton

Francis Russell Hart, Boston
Edward Jackson Holmes, Boston
Nathaniel Thayer Kidder, Milton
Thomas William Lamont, New York, N. Y.
Everett Morss, Boston
Robert Lincoln O'Brien, Dedham
Andrew James Peters, Jamaica Plain
Alfred Lawrence Ripley, Andover
Payson Smith, Brookline
Charles Henry Taylor, Boston
Eliot Wadsworth, Boston
Edwin Sibley Webster, Brookline
Benjamin Loring Young, Weston

Class IV.

Section 1. Michael Joseph Ahern, Weston
James Rowland Angell, New Haven, Conn.
Leonard Carmichael, Providence, R. I.
George Croft Cell, Reading
William Healy, Boston
Karl Spencer Lashley, Chicago
Arthur Darby Nock, Cambridge
William Cardinal O'Connell, Boston
Section 2. William Brooks Cabot, Boston
Waldo Gifford Leland, Washington, D. C.
Charles Holt Taylor, Cambridge
Section 3. Ronald Salmon Crane, Chicago, Ill.
Morris William Croll, Princeton, N. J.
Ralph Hayward Keniston, Chicago, Ill.
Ernest Felix Langley, Cambridge
Fred Bates Lund, Newton
George Rapall Noyes, Berkeley, Cal.
Milman Parry, Belmont
Arthur Stanley Pease, Cambridge
Horatio Elwin Smith, Providence, R. I.
William Thomson, Cambridge

Section 4. William Welles Bosworth, New York, N. Y.
Chalmers Dancy Clifton, New York, N. Y.
Kenneth John Conant, Cambridge
George Harold Edgell, Cambridge
William James, Cambridge
Anthony John Philpott, Arlington
Henry Richardson Shepley, Brookline

FOREIGN HONORARY MEMBERS.

CLASS II.

Section 2. Frederick Orpen Bower, Ripon, England
Kingo Miyabe, Sapporo, Japan
Albert Charles Seward, Cambridge, England

CLASS III.

Section 2. Mineichiro Adatci, The Hague, Holland
Paul Claudel, Washington, D. C.
Hu Shih, Peiping, China
Section 3. Sir William Henry Beveridge, London, England
Arthur Lyon Bowley, London, England
Ralph George Hawtrey, London, England
Charles Rist, Versailles, France

CLASS IV.

Section 1. Pierre Janet, Paris, France
Section 3. Paul Mazon, Paris, France
Frederick William Thomas, Oxford, England

The following communication was presented:

Mr. John R. Freeman: "Earthquake Damage."

Two papers were presented by title: "Bacterial Detoxification," by Robert S. Harris and John W. M. Bunker, presented by J. W. M. Bunker; and "Further Studies on the Fauna of North American Hot Springs," by C. T. Brues.

The meeting was dissolved at 10.15 P. M.

BIOGRAPHICAL NOTICES.

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IRA NELSON HOLLIS (1856–1930).

Fellow in Class I, Section 4, 1897.

Ira Nelson Hollis was born at Mooresville, Floyd County, Indiana, on March 7, 1856, the son of Ephriam Joseph Hollis (1825–1910) and Mary Kearns Hollis. His father became Captain in the 59th Indiana Regiment, serving at Vicksburg, Corinth, Shiloh, Chickamauga, Atlanta, and on the march to the sea, returning in command of the regiment, and later became owner and operator of a quarry at Louisville, Kentucky. His mother was the daughter of a farmer in Steubenville, Ohio.

Hollis' youth was spent at Louisville under straitened circumstances. He attended the local High School and then went as an apprentice in a machine shop. He later secured a clerical position with a railroad, and then with a cotton commission house in Memphis. At the age of eighteen he took the examination for admission to the Naval Academy at Annapolis and came out at the head of the list, a position which he retained throughout the course. After graduating as Cadet-Engineer in 1878 he spent three years on the Cruiser Quinnebaug in the Mediterranean and North Sea, and on the coast of Africa. He had been made Assistant Engineer in 1880, and on the conclusion of the cruise he was detailed as Professor of Marine Engineering at Union College, Schenectady. In 1884 he served with the Advisory Board for the construction of the ships of the White Squadron. Going to the Pacific coast, he spent three years at the Union Iron Works, supervising the construction of the Charleston. He then served three years on board that vessel in charge of her machinery, with the rank of Passed Assistant Engineer, mainly on the China Station, but, later taking part in the chase of the Itata.

In 1892 Hollis was designated to lecture on naval engineering at the Naval War College at Newport; his lectures being subsequently published as a text book for the Navy. He then became Assistant to the Chief of the Bureau of Steam Engineering. From this position he was called in 1893 to take charge of the development of instruction in engineering at Harvard University.

During his twenty years as Professor of Mechanical Engineering at Harvard University Hollis built up a reputation as educator and administrator. His breadth of experience, energy and sanity of judgement were also brought into play in numerous nonacademic activities. As Chairman of the Athletic Committee he converted the marsh land (now known as Soldiers Field) into a well equipped playing field and constructed on it the colossal Stadium, the first structure of its character in America. His courage in building that structure of reinforced concrete in the face of the grave doubts then existing as to its durability in the New England climate was characteristic of the man. He was active also in improving intercollegiate athletic relations, in establishing the Harvard Union (a students' club), in founding the Engineers' Club of Boston, of which he was the first president, and in numerous other enterprises demanding organizing power and leadership. His election later to the Board of Overseers of Harvard University—a unique honor for a non-graduate of Harvard College—gave evidence of the confidence and respect with which he was regarded by the great body of Harvard alumni. In 1913 Hollis became President of the Worcester Polytechnic Institute, a position which gave wide scope to his administrative powers. He soon became a leading citizen of the community and during the war was on the Committee of Public Safety and in the New England Fuel Administration. In this period also he was elected President of the American Society of Mechanical Engineers and in that position did valuable work for national preparedness. He resigned in 1925 on account of ill health, returned to Cambridge, Massachusetts, and devoted himself to writing until his death on August 14, 1930.

Hollis was the recipient of honorary degrees of A.M. (Harvard), L.H.D. (Union), Sc.D. (University of Pittsburgh), and Eng.D. (Worcester Polytechnic Institute). He was a Fellow of the American Academy of Arts and Sciences, and a member of the American Society of Mechanical Engineers, the Society of U. S. Naval Engineers, the American Society of Naval Architects and Marine Engineers, the Society for the Promotion of Engineering Education, and the Boston Society of Civil Engineers.

His publications include "The Frigate Constitution," and various scientific papers. He drafted the Personnel Act of 1898, reorganizing the line and staff of the Navy and establishing the present system.

The large influence of Hollis in all his associations was the result not only of his energy, character and good judgement but also of his geniality and capacity for comradeship and sympathetic helpfulness. He is survived by four children, Janette Ralton, Oliver Nelson, Eleanor Vernon and Carolyn. His wife, Caroline Lorman Hollis, the daughter of Charles Lorman of Detroit, died in 1925.

LIONEL S. MARKS.

CLARENCE LEMUEL ELISHA MOORE[1] (1876–1931).

Fellow in Class I, Section 1, 1914.

Clarence Lemuel Elisha Moore, the son of George Taylor and Lydia Ann (Bradshaw) Moore, was born in Bainbridge, Ohio, on May 12, 1876. His father was a grain dealer and was descended from the early settlers. After some years of school teaching, he entered the Ohio State University, from which he was graduated in 1901. His college course was followed by graduate study in mathematics at Cornell University which granted him a master's degree in 1902, and the doctorate in 1904. From this time until his death, on December 5, 1931, he was connected with the Massachusetts Institute of Technology, where he was an instructor for five years, assistant professor for seven years, associate professor for four years and professor for eleven years. Shortly after coming to the Institute, he spent a year in study abroad, principally in Turin under Segre, and the influence of the Italian geometers profoundly affected his later mathematical career. He married Belle Pease Fuller of Springfield, Massachusetts, on June 11, 1913. She and their daughter, Hazel Ann, survive him.

Professor Moore was a member of the American Mathematical Society and of the Circolo Mathematico di Palermo, and a charter member of the Mathematical Association of America. He was a Fellow of this Academy and at the time of his death one of its Councillors. Many of his early papers were published in the Proceedings of this Academy. All of his later ones were published in the Journal of Mathematics and Physics which was founded at the Massachusetts

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[1] Compare the biography by D. J. Struik, in the Bulletin of the American Mathematical Society, vol. 38 (1932), p. 155, and the longer account by the same author in the Journal of Mathematics and Physics, vol. XI (1932), p. 1. The latter includes a detailed list of publications, and a critical survey of his mathematical contributions.

Institute of Technology in 1921, largely through his efforts, and of which he was an editor for ten years.

Professor Moore distinguished himself in all three of the fields of activity of an American educator: teaching, administration and research. But perhaps his most important service to the community in which he labored was in the rôle of friend and, in the case of his younger colleagues and students, father confessor and adviser. For his juniors, "to talk it over with Moore" was the natural treatment of any difficulty whether it concerned practical life, academic matters or abstract mathematics.

In recent years he conducted classes in mathematics as applied to aeronautical engineering. For eleven years he was in charge of the course in general science and engineering which included an undergraduate option in mathematics. This course, unlike most of those at Technology, left considerable freedom to the student in his choice of subjects, and Professor Moore gave countless hours to counselling undergraduates. He opposed, with equal firmness, diffuse superficiality and narrow specialization. His advice was nearly always followed by the students, not under compulsion but through the effectiveness of kindly argument. While Professor Moore realized the value of discipline and the importance of enforcing rules in general, no one was quicker than he to cut red tape in the interest of justice or common sense.

In creative mathematical work, C. L. E. Moore had a vigorous interest, unusual in mathematicians past their first youth. This accounts for the amount and importance of his contributions, in spite of the physical handicap of impaired vision, which made prolonged periods of reading an impossibility. His papers[1] for the most part deal with geometry. He began with problems concerning algebraic geometry in euclidean space of three and higher dimensions, and later went on to the differential geometry of Riemannian manifolds. He also (partly in collaboration with H. B. Phillips) did some work on the applications of vector analysis to geometry. His work, taken as a whole, possesses unusual unity, and gives evidence of a real feeling for vital problems and a keen geometric insight into them. In mathematics; as in other aspects of life, C. L. E. Moore was little influenced by current fashions. This independence led him, for example (partly in collaboration with E. B. Wilson) to apply the methods of Ricci to the geometry of hyperspace several years before the body of American geometers became interested in the absolute calculus through its application to relativity by Einstein.

Besides the original contributions made directly by himself, or in collaboration with his colleagues, Professor Moore had a very stimulating influence on the general activity of the mathematical department at Technology. He continually encouraged his colleagues to do creative work, even in the face of difficulties, and to publish their results. When most of the mathematical journals in this country were inactive owing to a printers' strike, it was he who urged the foundation of the Journal of Mathematics and Physics to give the work of the staff an outlet. In questions of mathematical appointments, he was inclined to favor the candidate whom it might be "impossible to keep" rather than the one whom it might prove "impossible to lose." On all questions of departmental policy he always had a decided opinion, and even those who disagreed with him found their ideas clarified by his comments. In a controversy he stood by his guns without being unreasonably stubborn and however emphatic his point of view might be, he avoided all note of personal hostility.

C. L. E. Moore leaves, at his untimely death, many scientific and academic mementos and in the minds of his associates so vivid a memory of his vigorous, friendly personality that it is hard for them to realize that he is no longer with them.

Philip Franklin.

American Academy of Arts and Sciences.

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(Corrected to February 1, 1933.)

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Section I.—*Mathematics and Astronomy.*—52.

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CLASS I., SECTION II.—*Physics.*—57.

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George Russell Harrison	Cambridge
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George Shannon Forbes	Cambridge
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Martin André Rosanoff	Valencia, Pa.
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Class I., Section IV.—*Technology and Engineering*.—41.

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Arthur Edwin Kennelly	Cambridge
Ralph Restieaux Lawrence	Brookline
William Henry Lawrence	Jamaica Plain
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Lionel Simeon Marks	Cambridge
Edward Furber Miller	Newton
Frederick Law Olmsted	Brookline
Charles Francis Park	Taunton
Langdon Pearse	Chicago, Ill.
Harold Pender	Philadelphia, Pa.
Greenleaf Whittier Pickard	Newton Center
George Edmond Russell	Arlington
Albert Sauveur	Cambridge
Henry Lloyd Smyth	Watertown
Charles Milton Spofford	Brookline
John F. Stevens	Baltimore, Md.
Edward Pearson Warner	New York, N. Y.
Arthur Edward Wells	New York, N. Y.
Robert Spurr Weston	Boston
Joseph Ruggles Worcester	Boston

Class II.—*Natural and Physiological Sciences.*—202.

Section I.—*Geology, Mineralogy, and Physics of the Globe.*—50.

Wallace Walter Atwood	Worcester
Norman Levi Bowen	Washington, D. C.
Isaiah Bowman	New York, N. Y.
Kirk Bryan	Cambridge
Henry Helm Clayton	Canton
Herdman Fitzgerald Cleland	Williamstown
Reginald Aldworth Daly	Cambridge
Edward Salisbury Dana	New Haven, Conn.
William Morris Davis	Palo Alto, Cal.
William Ebenezer Ford	New Haven, Conn.
James Walter Goldthwait	Hanover, N. H.
Louis Caryl Graton	Cambridge
Herbert Ernest Gregory	Honolulu, T. H.
William Jackson Humphreys	Washington, D. C.
Ellsworth Huntington	New Haven, Conn.
Robert Tracy Jackson	Peterborough, N. H.
Thomas Augustus Jaggar	Honolulu, T. H.
Douglas Wilson Johnson	New York, N. Y.
Arthur Keith	Washington, D. C.
Alfred Church Lane	Cambridge
Esper Signius Larsen, Jr.	Cambridge
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Frederic Brewster Loomis	Amherst
Alexander George McAdie	Hampton, Va.
George Francis McEwen	La Jolla, Cal.
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Kirtley Fletcher Mather	Cambridge
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Frederick Kuhne Morris	Cambridge
Charles Palache	Cambridge
Percy Edward Raymond	Lexington

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Charles Schuchert	New Haven, Conn.
William Berryman Scott	Princeton, N. J.
Hervey Woodburn Shimer	Hingham
Frank Bursley Taylor	Fort Wayne, Ind.
Thomas Wayland Vaughan	La Jolla, Cal.
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David White	Washington, D. C.
Herbert Percy Whitlock	New York, N. Y.
Bailey Willis	Palo Alto, Cal.
Arthur Winslow	Boston
John Eliot Wolff	Pasadena, Cal.
Frederick Eugene Wright	Washington, D. C.

Class II., Section II.—*Botany.*—38.

LeRoy Abrams	Stanford University, Cal.
Oakes Ames	North Easton
Joseph Charles Arthur	Lafayette, Ind.
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Liberty Hyde Bailey	Ithaca, N. Y.
Edward Wilber Berry	Baltimore, Md.
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Edward Murray East	Jamaica Plain
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Joseph Horace Faull	Cambridge
Merritt Lyndon Fernald	Cambridge
Richard Thornton Fisher	Petersham
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John George Jack	East Walpole
Edward Charles Jeffrey	Cambridge

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Ivan Murray Johnston	Jamaica Plain
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Karl Sax	Jamaica Plain
William Albert Setchell	Berkeley, Cal.
Arthur Bliss Seymour	Cambridge
Elvin Charles Stakman	St. Paul, Minn.
William Codman Sturgis	Annisquam
William Trelease	Urbana, Ill.
Charles Alfred Weatherby	Cambridge
William Henry Weston, Jr.	Cambridge
Ralph Hartley Wetmore	Belmont

CLASS II., SECTION III.—*Zoology and Physiology.*—64.

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Francis Gano Benedict	Boston
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William Ernest Castle	Belmont
Charles Value Chapin	Providence, R. I.
Benjamin Preston Clark	Boston
Lemuel Roscoe Cleveland	Jamaica Plain
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Manton Copeland	Brunswick, Me.
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Samuel Randall Detwiler	New York, N. Y.
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Henry Clinton Fall	Tyngsboro
Alexander Forbes	Milton
William King Gregory	New York, N. Y.
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Leigh Hoadley	Cambridge
Samuel Jackson Holmes	Berkeley, Cal.
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George Howard Parker	Cambridge
Raymond Pearl	Baltimore, Md.
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Henry Augustus Pilsbry	Philadelphia, Pa.
Frederick Haven Pratt	Wellesley Hills
Herbert Wilbur Rand	Cambridge

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Alexander Grant Ruthven	Ann Arbor, Mich.
Percy Goldthwait Stiles	Newtonville
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Simon Flexner	New York, N. Y.
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Robert Battey Greenough	Boston
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Elliott Proctor Joslin	Boston
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James Howard Means	Boston
George Richards Minot	Boston
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Robert Bayley Osgood	Boston
Joseph Hershey Pratt	Boston
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Joseph Treloar Wearn	Cleveland, Ohio
William Henry Welch	Baltimore, Md.
Benjamin White	Boston
Francis Henry Williams	Boston
Simeon Burt Wolbach	Sudbury
Hans Zinsser	Boston

CLASS III.—*The Social Arts.*—107.

SECTION I.—*Jurisprudence.*—30.

George Weston Anderson	Boston
Francis Noyes Balch	Jamaica Plain
Joseph Henry Beale	Cambridge
Fred Tarbell Field	Newton
Felix Frankfurter	Cambridge
Thomas Hovey Gage	Worcester
Walter Perley Hall	Fitchburg
Charles Evans Hughes	Washington, D. C.
Nathan Isaacs	Cambridge
Eldon Revare James	Cambridge

Frederick Lawton	Boston
James Arnold Lowell	Newton
Sayre Macneil	Cambridge
Calvert Magruder	Cambridge
William DeWitt Mitchell	Washington, D. C.
Edmund Morris Morgan	Arlington
Herbert Parker	South Lancaster
George Wharton Pepper	Philadelphia, Pa.
Roscoe Pound	Watertown
Thomas Reed Powell	Cambridge
Odin Roberts	Boston
Elihu Root	New York, N. Y.
Arthur Prentice Rugg	Worcester
Francis Bowes Sayre	Cambridge
Austin Wakeman Scott	Cambridge
Edward Sampson Thurston	Cambridge
William Cushing Wait	Medford
Eugene Wambaugh	Cambridge
Edmund Allen Whitman	Cambridge
George Woodward Wickersham	New York, N. Y.

Class III., Section II.—*Government, International Law, and Diplomacy*.—20.

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William Yandell Elliott	Belmont
Sidney Bradshaw Fay	Cambridge
William Cameron Forbes	Milton
Joseph Clark Grew	Tokyo, Japan
Albert Bushnell Hart	Cambridge
Arthur Norman Holcombe	Cambridge
Manley Ottmer Hudson	Cambridge
Philip Carryl Jessup	New York, N. Y.
Abbott Lawrence Lowell	Cambridge
William MacDonald	New York, N. Y.
Charles Edward Merriam	Chicago, Ill.
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John Bassett Moore	New York, N. Y.

William Bennett Munro	Pasadena, Cal.
Westel Woodbury Willoughby	Baltimore, Md.
William Franklin Willoughby	Washington, D. C.
George Grafton Wilson	Cambridge
Quincy Wright	Chicago, Ill.

Class III., Section III.—*Economics and Sociology.*—34.

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Charles Jesse Bullock	Cambridge
Richard Clarke Cabot	Cambridge
Thomas Nixon Carver	Cambridge
John Bates Clark	New York, N. Y.
John Candler Cobb	Milton
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William Leonard Crum	Cambridge
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Wesley Clair Mitchell	New York, N. Y.
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William Fielding Ogburn	Chicago, Ill.
Warren Milton Persons	New York, N. Y.
George Byron Roorbach	Cambridge
Leo S. Rowe	Washington, D. C.
Carl Snyder	New York, N. Y.
Pitirim Alexandrovich Sorokin	Winchester

Oliver Mitchell Wentworth Sprague	Cambridge
Frank William Taussig	Cambridge
Donald Skeele Tucker	Cambridge
John Henry Williams	Cambridge

CLASS III., SECTION IV.—*Administration and Affairs.*—23.

Charles Francis Adams	Concord
Charles Foster Batchelder	Cambridge
Ingersoll Bowditch	Jamaica Plain
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Henry Sturgis Dennison	Framingham
William Lusk Webster Field	Milton
Frank Johnson Goodnow	Baltimore, Md.
Francis Russell Hart	Boston
Edward Jackson Holmes	Boston
Nathaniel Thayer Kidder	Cambridge
Thomas William Lamont	New York, N. Y.
Everett Morss	Boston
Robert Lincoln O'Brien	Dedham
Thomas Nelson Perkins	Westwood
Andrew James Peters	Jamaica Plain
Herbert Putnam	Washington, D. C.
Alfred Lawrence Ripley	Andover
Jeremiah Smith, Jr.	Cambridge
Payson Smith	Brookline
Charles Henry Taylor	Boston
Eliot Wadsworth	Boston
Edwin Sibley Webster	Brookline
Benjamin Loring Young	Weston

CLASS IV.—*The Humanities.*—176.

SECTION I.—*Theology, Philosophy, and Psychology.*—39.

Michael Joseph Ahern	Weston
James Rowland Angell	New Haven, Conn.
Edwin Garrigues Boring	Cambridge
Edgar Sheffield Brightman	Newton
Henry Addington Bruce	Cambridge

Leonard Carmichael	Providence, R. I.
George Croft Cell	Reading
Walter Fenno Dearborn	Cambridge
Edmund Burke Delabarre	Providence, R. I.
Raymond Dodge	New Haven, Conn.
Edward Staples Drown	Cambridge
William Henry Paine Hatch	Cambridge
William Healy	Boston
William Arthur Heidel	Middletown, Conn.
William Ernest Hocking	Cambridge
Frederick John Foakes Jackson	Englewood, N. J.
Truman Lee Kelley	Cambridge
Albert Cornelius Knudson	Cambridge
Karl Spencer Lashley	Chicago, Ill.
William Lawrence	Boston
Clarence Irving Lewis	Cambridge
William McDougall	Durham, N. C.
Edward Caldwell Moore	Cambridge
Paul Elmer More	Princeton, N. J.
Arthur Darby Nock	Cambridge
William Cardinal O'Connell	Boston
Johnson O'Connor	Boston
George Herbert Palmer	Cambridge
Charles Edwards Park	Boston
Leighton Parks	London, England
Francis Greenwood Peabody	Cambridge
James Hugh Ryan	Washington, D. C.
Henry Knox Sherrill	Boston
Willard Learoyd Sperry	Cambridge
Russell Henry Stafford	Brookline
Henry Bradford Washburn	Cambridge
John Broadus Watson	New York, N. Y.
James Haughton Woods	Cambridge
Robert Mearns Yerkes	New Haven, Conn.

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Charles McLean Andrews	New Haven, Conn.
James Phinney Baxter, 3d	Cambridge

Carl Lotus Becker	Ithaca, N. Y.
Robert Pierpont Blake	Cambridge
Franz Boas	New York, N. Y.
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George Henry Chase	Cambridge
Roland Burrage Dixon	Cambridge
Wilberforce Eames	New York, N. Y.
Ephraim Emerton	Cambridge
Max Farrand	San Marino, Cal.
William Scott Ferguson	Cambridge
Worthington Chauncey Ford	Cambridge
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Charles Homer Haskins	Cambridge
Charles Downer Hazen	New York, N. Y.
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Earnest Albert Hooton	Cambridge
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Kirsopp Lake	Cambridge
George LaPiana	Cambridge
Waldo Gifford Leland	Washington, D. C.
Charles Howard McIlwain	Cambridge
Roger Bigelow Merriman	Cambridge
Samuel Eliot Morison	Boston
George Andrew Reisner	Boston
Michael Ivanovich Rostovtzeff	New Haven, Conn.
George Sarton	Cambridge
Arthur Meier Schlesinger	Cambridge
Herbert Joseph Spinden	Brooklyn, N. Y.
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Charles Cutler Torrey	New Haven, Conn.
Alfred Marston Tozzer	Cambridge
Clark Wissler	New York, N. Y.

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Eugene Watson Burlingame	Albany, N. Y.
Edward Capps	Princeton, N. J.
Walter Eugene Clark	Cambridge
Ronald Salmon Crane	Chicago, Ill.
Morris William Croll	Princeton, N. J.
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Frank Edgar Farley	Middletown, Conn.
Jeremiah Denis Mathias Ford	Cambridge
James Geddes, Jr.	Brookline
Charles Hall Grandgent	Cambridge
Louis Herbert Gray	New York, N. Y.
William Chase Greene	Cambridge
Charles Burton Gulick	Cambridge
Roy Kenneth Hack	Cincinnati, Ohio
Raymond Dexter Havens	Baltimore, Md.
George Lincoln Hendrickson	New Haven, Conn.
William Guild Howard	Cambridge
Eugene Xavier Louis Henry Hyvernat	Washington, D. C.
Carl Newell Jackson	Cambridge
James Richard Jewett	Cambridge
(Ralph) Hayward Keniston	Chicago, Ill.
George Lyman Kittredge	Cambridge
Henry Roseman Lang	New Haven, Conn.
Ernest Felix Langley	Cambridge
Charles Rockwell Lanman	Cambridge
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William Thomson	Cambridge
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Chester Noyes Greenough	Cambridge
Edward Burlingame Hill	Cambridge
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Charles Hopkinson	Manchester
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Henry James	New York, N. Y.
William James	Cambridge
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Curtis Hidden Page	Gilmanton, N. H.
William Lyon Phelps	New Haven, Conn.
Anthony John Philpott	Arlington
Arthur Kingsley Porter	Cambridge
Chandler Rathfon Post	Cambridge
Edwin Arlington Robinson	New York, N. Y.
Denman Waldo Ross	Cambridge
Paul Joseph Sachs	Cambridge
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Henry Dwight Sedgwick	Boston
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Edmund C. Tarbell	Boston
Charles Howard Walker	Boston
George Parker Winship	Charles River
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Charles Henry Conrad Wright	Cambridge

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Jacques Salomon Hadamard	Paris
Godfrey Harold Hardy	Cambridge, England
Ejnar Hertzsprung	Leyden
Tullio Levi-Civita	Rome
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Charles Jean de la Vallée Poussin	Louvain
Hermann Weyl	Zürich

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James Franck	Berlin
Abram F. Joffé	Leningrad
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Max Planck	Berlin
Sir Ernest Rutherford	Cambridge, England
Sir Joseph John Thomson	Cambridge, England

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August Krogh	Copenhagen
Louis Édouard Lapicque	Paris

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D'Arcy Wentworth Thompson	St. Andrews

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Rufus Daniel Isaacs, Marquess of Reading	London

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Edmund Husserl	Freiburg i. B.
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Hermann Georg Jacobi	Bonn
Sylvain Lévi	Paris
Paul Mazon	Paris
Antoine Meillet	Paris
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Section IV.—*The Fine Arts and Belles Lettres.*—8.

Alfredo Casella	Rome
Henry Guy	Grenoble
Paul Hazard	Paris
Rudyard Kipling	Burwash, Sussex
Victor Laloux	Paris
Gilbert Murray	Oxford
Edgar Allison Peers	Liverpool
Henri Rabaud	Paris

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The letters F and FHM refer to the lists of Fellows and Foreign Honorary Members, respectively. The class and section are indicated by the numerals following.

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Huntington, A. M. F, IV: 4
Huntington, E. F, II: 1
Huntington, E. V. F, I: 1
Husserl, E. FHM, IV: 1
Hutchins, C. C. F, I: 2
Hyvernat, E. X. L. H. F, IV: 3
Isaacs, N. F, III: 1

Ives, F. E. F, I: 2
Ives, J. E. F, I: 2
Jack, J. G. F, II: 2
Jack, J. R. F, I: 4
Jackson, C. L. F, I: 3
Jackson, C. N. F, IV: 3
Jackson, D. F, I: 1
Jackson, D. C. F, I: 4
Jackson, F. J. F. F, IV: 1
Jackson, R. T. F, II: 1
Jacobi, H. G. FHM, IV: 3
Jaggar, T. A. F, II: 1
James, E. R. F, III: 1
James, H. F, IV: 4
James, W. F, IV: 4
Janet, P. FHM, IV: 1
Jeffrey, E. C. F, II: 2
Jennings, H. S. F, II: 3
Jennings, W. L. F, I: 3
Jepson, W. L. F, II: 2
Jessup, P. C. F, III: 2
Jewett, F. B. F, I: 4
Jewett, J. R. F, IV: 3
Joffé, A. F. FHM, I: 2
Johnson, D. W. F, II: 1
Johnson, L. J. F, I: 4
Johnston, I. M. F, II: 2
Jones, G. F, I: 3
Joslin, E. P. F, II: 4
Keith, A. F, II: 1
Kelley, T. L. F, IV: 1
Kemble, E. C. F, I: 2
Keniston, H. F, IV: 3
Kennelly, A. E. F, I: 4
Kent, N. A. F, I: 2
Keyes, F. G. F, I: 3
Kidder, A. V. F, IV: 2
Kidder, N. T. F, III: 4
Kipling, R. FHM, IV: 4
Kittredge, G. L. F, IV: 3
Knudson, A. C. F, IV: 1
Köhler, W. FHM, IV: 1
Kofoid, C. A. F, II: 3
Kohler, E. P. F, I: 3
Kraus, C. A. F, I: 3
Kroeber, A. L. F, IV: 2
Krogh, A. FHM, II: 3
Lake, K. F, III: 2
Laloux, V. FHM, IV: 4
Lamb, A. B. F, I: 3
Lamont, T. W. F, III: 4
Lampland, C. O. F, I: 1
Lane, A. C. F, II: 1
Lang, H. R. F, IV: 3
Langley, E. F. F, IV: 3
Langmuir, I. F, I: 3
Lanman, C. R. F, IV: 3
La Piana, G. F, IV: 2
Lapicque, L. E. FHM, II: 3
Larmor, Sir J. FHM, I: 2
Larsen, E. S. F, II: 1
Lashley, K. S. F, IV: 1
Lawrence, R. R. F, I: 4
Lawrence, W. F, IV: 1
Lawrence, W. H. F, I: 4
Laws, F. A. F, I: 2
Lawson, A. C. F, II: 1
Lawton, F. F, III: 1
Le Chatelier, H. L. FHM, I: 3
Lee, R. I. F, II: 4
Lefavour, H. F, I: 2
Leith, C. K. F, II: 1
Leland, W. G. F, IV: 2
Lévi, S. FHM, IV: 3
Levi-Civita, T. FHM, I: 1
Lewis, C. I. F, IV: 1
Lewis, F. T. F, II: 3
Lewis, G. N. F, I: 3
Lewis, W. K. F, I: 3
Lillie, R. S. F, II: 3
Lindgren, W. F, II: 1
Lindsay, R. B. F, I: 2
Lipman, J. G. F, II: 2
Little, A. D. F, I: 3
Livingston, B. E. F, II: 2
Locke, E. A. F, II: 4
Lodge, J. E. F, IV: 4
Loeffler, C. M. T. F, IV: 4
Longcope, W. T. F, II: 4
Loomis, F. B. F, II: 1

Lowell, A. L. F, III: 2
Lowell, J. A. F, III: 1
Lowes, J. L. F, IV: 4
Lull, R. S. F, II: 3
Lund, F. B. F, II: 4
Luyten, W. J. F, I: 1
Lyman, T. F, I: 2
Lyon, D. G. F, IV: 3
McAdie, A. G. F, II: 1
Macallum, A. B. FHM, I: 3
MacDonald, W. F, III: 2
McDougall, W. F, IV: 1
McEwen, G. F. F, II: 1
McIlwain, C. H. F, IV: 2
MacInnes, D. A. F, I: 3
McLaren, W. W. F, III: 3
McLaughlin, D. H. F, II: 1
Macneil, S. F, III: 1
Maginnis, C. D. F, IV: 4
Magoun, F. P., Jr. F, IV: 3
Magruder, C. F, III: 1
Main, C. T. F, I: 4
Mallory, F. B. F, II: 4
Manship, P. F, IV: 4
Margerie, E. de. FHM, II: 1
Mark, E. L. F, II: 3
Mark, K. L. F, I: 3
Marks, L. S. F, I: 4
Marshall, L. C. F, III: 3
Martin, E. G. F, II: 3
Mason, D. G. F, IV: 4
Mather, F. J. F, IV: 4
Mather, K. F. F, II: 1
Mathews, E. B. F, II: 1
Matthews, A. F, IV: 3
Mayo, W. J. F, II: 4
Mazon, P. FHM, IV: 3
Mead, A. D. F, II: 3
Means, J. H. F, II: 4
Meillet, A. FHM, IV: 3
Melander, A. L. F, II: 3
Mendel, L. B. F, I: 3
Merriam, C. E. F, III: 2
Merriam, J. C. F, II: 1
Merrill, E. D. F, II: 2
Merriman, R. B. F, IV: 2
Merritt, E. G. F, I: 2
Miller, D. C. F, I: 2
Miller, E. F. F, I: 4
Miller, G. A. F, I: 1
Miller, G. S. F, II: 3
Miller, J. A. F, I: 1
Miller, W. J. F, II: 1
Millikan, R. A. F, I: 2
Mills, F. C. F, III: 3
Mills, O. L. F, III, 2
Minot, G. R. F, II: 4
Mitchell, S. A. F, I: 1
Mitchell, W. C. F, III: 3
Mitchell, W. DeW. F, III: 1
Miyabe, K. FHM, II: 2
Miyajima, M. FHM, II: 4
Molengraaff, G. A. F. FHM, II: 1
Moore, E. C. F, IV: 1
Moore, J. B. F, III: 2
More, P. E. F, IV: 1
Morgan, E. M. F, III: 1
Morgan, T. H. F, II: 3
Morison, S. E. F, IV: 2
Morley, F. F, I: 1
Morris, F. K. F, II, 1
Morse, H. W. F, I: 2
Morse, M. F, I: 1
Morss, E. F, III: 4
Moss, W. L. F, II: 4
Moulton, F. R. F, I: 1
Moulton, H. G. F, III: 3
Mueller, E. F, I: 3
Müller, F. von. FHM, II: 4
Mueller, J. H. F, II: 4
Mulliken, S. P. F, I: 3
Munro, W. B. F, III: 2
Munroe, C. E. F, I: 3
Murdock, H. F, IV: 4
Murdock, K. B. F, IV: 4
Murray, G. FHM, IV: 4
Neal, H. V. F, II: 3
Neilson, W. A. F, IV: 4
Newell, L. C. F, I: 3
Nichols, E. L. F, I: 2

Nitze, W. A. F, IV: 3
Nock, A. D. F, IV: 1
Norris, J. F. F, I: 3
Norton, C. L. F, I: 2
Noyes, A. A. F, I: 3
Noyes, G. R. F, IV: 3
Noyes, W. A. F, I: 3
Noyes, W. A., Jr. F, I: 3
Nuttall, G. H. F. FHM, II: 3
O'Brien, R. L. F, III: 4
d'Ocagne, M. FHM, I: 4
O'Connell, W. H. F, IV: 1
O'Connor, J. F, IV: 1
Oertel, H. FHM, IV: 3
Ogburn, W. F. F, III: 3
Oldenberg, O. F, I: 2
Olmsted, F. L. F, I: 4
Osborn, H. F. F, II: 3
Osgood, R. B. F, II: 4
Osterhout, W. J. V. F, II: 2
Page, C. H. F, IV: 4
Palache, C. F, II: 1
Palmer, G. H. F, IV: 1
Park, C. E. F, IV: 1
Park, C. F. F, I: 4
Parker, G. H. F, II: 3
Parker, H. F, III: 1
Parks, L. F, IV: 1
Parry, M. F, IV: 3
Paschen, F. FHM, I: 2
Peabody, F. G. F, IV: 1
Pearl, R. F, II: 3
Pearse, L. F, I: 4
Pearson, K. FHM, IV: 2
Pease, A. S. F, IV: 3
Peers, E. A. FHM, IV: 4
Peirce, G. J. F, II: 2
Pender, H. F, I: 4
Pepper, G. W. F, III: 1
Perkins, T. N. F, III: 4
Persons, W. M. F, III: 3
Peters, A. J. F, III: 4
Phelps, W. L. F, IV: 4
Phillips, H. B. F, I: 1
Phillips, J. C. F, II: 3
Philpott, A. J. F, IV: 4
Picard, C. E. FHM, I: 1
Pickard, G. W. F, I: 4
Pickering, W. H. F, I: 1
Pidal, R. M. FHM, IV: 3
Pierce, G. W. F, I: 2
Pigou, A. C. FHM, III: 3
Pilsbry, H. A. F, II: 3
Pirenne, H. FHM, IV: 2
Planck, M. FHM, I: 2
Poincaré, R. FHM, III: 4
Pollock, Sir F. FHM, III: 1
Poor, C. L. F, I: 1
Porter, A. K. F, IV: 4
Post, C. R. F, IV: 4
Pound, R. F, III: 1
Powell, T. R. F, III: 1
Prandtl, L. FHM, I: 4
Pratt, F. H. F, II: 3
Pratt, J. H. F, II: 4
Prescott, S. C. F, I: 3
Probst, E. FHM, I: 4
Pupin, M. I. F, I: 2
Putnam, H. F, III: 4
Rabaud, H. FHM, IV: 4
Rand, E. K. F, IV: 3
Rand, H. W. F, II: 3
Rapport, D. F, II: 3
Raymond, P. E. F, II: 1
Reading, Lord. FHM, III: 4
Redfield, A. C. F, II: 3
Redlich, J. FHM, III: 1
Regan, C. T. FHM, II: 3
Rehder, A. F, II: 2
Reisner, G. A. F, IV: 2
Rendle, A. B. FHM, II: 2
Renner, O. FHM, II: 2
Richards, R. H. F, I: 3
Richardson, R. G. D. F, I: 1
Ripley, A. L. F, III: 4
Rist, C. FHM, III: 3
Ritter, W. E. F, II: 3
Roberts, O. F, III: 1
Robinson, B. L. F, II: 2
Robinson, E. A. F, IV: 4

Robinson, F. N. F, IV: 3
Rogers, A. F. F, II: 1
Roorbach, G. B. F, III: 3
Root, E. F, III: 1
Root, R. K. F, IV: 3
Rosanoff, M. A. F, I: 3
Rosenau, M. J. F, II: 4
Ross, D. W. F, IV: 4
Rostovtzeff, M. I. F, IV: 2
Rowe, A. W. F, I: 3
Rowe, L. S. F, III: 3
Rugg, A. P. F, III: 1
Russell, G. E. F, I: 4
Russell, H. N. F, I: 1
Rutherford, Sir E. FHM, I: 2
Ruthven, A. G. F, II: 3
Rutledge, G. F, I: 1
Ryan, J. H. F, IV: 1
Sachs, P. J. F, IV: 4
Sarton, G. F, IV: 2
Saunders, F. A. F, I: 2
Sauveur, A. F, I: 4
Sax, K. F, II: 2
Sayles, R. W. F, II: 1
Sayre, F. B. F, III: 1
Scatchard, G. F, I: 3
Schaller, W. T. F, II: 1
Schevill, R. F, IV: 3
Schlesinger, A. M. F, IV: 2
Schlesinger, F. F, I: 1
Schuchert, C. F, II: 1
Schumb, W. C. F, I: 3
Scott, A. W. F, III: 1
Scott, W. B. F, II: 1
Sedgwick, E. F, IV: 4
Sedgwick, H. D. F, IV: 4
Sellards, A. W. F, II: 4
Setchell, W. A. F, II: 2
Seward, A. C. FHM, II: 2
Seymour, A. B. F, II: 2
Shapley, H. F, I: 1
Shaw, Sir W. N. FHM, II: 1
Shepley, H. R. F, IV: 4
Sherrill, H. K. F, IV: 1
Sherrill, M. S. F, I: 3
Sherrington, Sir C. S. FHM, II: 4
Shimer, H. W. F, II: 1
Sitter, W. de. FHM, I: 1
Slater, J. C. F, I: 2
Slipher, V. M. F, I: 1
Slocum, F. F, I: 1
Smith, H. E. F, IV: 3
Smith, H. M. F, I: 3
Smith, J., Jr. F, III: 4
Smith, P. F, III: 4
Smith, T. F, II: 4
Smyth, H. L. F, I: 4
Smyth, H. W. F, IV: 3
Snyder, C. F, III: 3
Snyder, V. F, I: 1
Sörensen, S. P. L. FHM, I: 3
Sollmann, T. H. F, II: 4
Sorokin, P. A. F, III: 3
Sperry, W. L. F, IV: 1
Spinden, H. J. F, IV: 2
Spofford, C. M. F, I: 4
Sprague, O. M. W. F, III: 3
Stafford, R. H. F, IV: 1
Stakman, E. C. F, II: 2
Stebbins, J. F, I: 1
Stein, Sir A. FHM, IV: 2
Stephenson, F. B. F, IV: 3
Stetson, H. T. F, I: 1
Stevens, J. F. F, I: 4
Stieglitz, J. O. F, I: 3
Stiles, C. W. F, II: 4
Stiles, P. G. F, II: 3
Stodola, A. FHM, I: 4
Stone, J. S. F, I: 2
Strong, R. P. F, II: 4
Struik, D. J. F, I: 1
Sturgis, W. C. F, II: 2
Sumner, J. O. F, IV: 2
Taber, H. F, I: 1
Talbot, F. B. F, II: 4
Tamarkin, J. D. F, I: 1
Tarbell, E. C. F, IV: 4
Taussig, F. W. F, III: 3
Taylor, C. H. F, III: 4
Taylor, C. H. F, IV: 2

Taylor, F. B. F, II: 1
Terry, C. S. FHM, IV: 2
Thayer, J. E. F, II: 3
Theiler, Sir A. FHM, II: 4
Thomas, F. W. FHM, IV: 3
Thompson, D'A. W. FHM, II: 3
Thompson, M. deK. F, I: 2
Thomson, E. F, I: 2
Thomson, Sir J. J. FHM, I: 2
Thomson, W. F, IV: 3
Thurston, E. S. F, III: 1
Timonoff, V. E. FHM, I: 4
Tolman, R. C. F, I: 3
Torrey, C. C. F, IV: 2
Tozzer, A. M. F, IV: 2
Trelease, W. F, II: 2
Trevelyan, G. M. FHM, IV: 2
Tucker, D. S. F, III: 3
Tyler, H. W. F, I: 1
Tyzzer, E. E. F, II: 4
Unwin, W. C. FHM, I: 4
Vallarta, M. S. F, I: 2
Vallée Poussin, C. J. de la. FHM, I: 1
Vaughan, T. W. F, II: 1
Veblen, O. F, I: 1
Verhoeff, F. H. F, II: 4
Vries, H. de. FHM, II: 2
Wadsworth, E. F, III: 4
Wagner, K. W. FHM, I: 4
Wait, W. C. F, III: 1
Walker, C. H. F, IV: 4
Walker, W. H. F, I: 3
Walsh, J. L. F, I: 1
Wambaugh, E. F, III: 1
Warner, E. P. F, I: 4
Warren, C. H. F, II: 1
Washburn, H. B. F, IV: 1
Watson, J. B. F, IV: 1
Wearn, J. T. F, II: 4
Weatherby, C. A. F, II: 2
Webster, D. L. F, I: 2
Webster, E. S. F, III: 4
Welch, W. H. F, II: 4
Wells, A. E. F, I: 4
Weston, R. S. F, I: 4
Weston, W. H. F, II: 2
Wetmore, R. H. F, II: 2
Weyl, H. FHM, I: 1
Weysse, A. W. F, II: 3
Whatmough, J. F, IV: 3
Wheeler, W. M. F, II: 3
White, B. F, II: 4
White, D. F, II: 1
Whitehead, A. N. F, I: 1
Whitlock, H. P. F, II: 1
Whitman, E. A. F, III: 1
Whitney, W. R. F, I: 3
Wickersham, G. W. F, III: 1
Widder, D. V. F, I: 1
Wieland, H. FHM, I: 3
Wilkins, E. H. F, IV: 3
Williams, F. H. F, II: 4
Williams, J. H. F, III: 3
Williams, R. S. F, I: 3
Willis, B. F, II: 1
Willoughby, W. F. F, III: 2
Willoughby, W. W. F, III: 2
Wilson, E. B. F, II: 3
Wilson, E. B. F, I: 2
Wilson, G. G. F, III: 2
Winship, G. P. F, IV: 4
Winslow, A. F, II: 1
Wissler, C. F, IV: 2
Wister, O. F, IV: 4
Wolbach, S. B. F, II: 4
Wolff, J. E. F, II: 1
Wood, R. W. F, I: 2
Woodman, A. G. F, I: 3
Woods, F. A. F, II: 3
Woods, F. S. F, I: 1
Woods, J. H. F, IV: 1
Worcester, J. R. F, I: 4
Wright, C. H. C. F, IV: 4
Wright, F. E. F, II: 1
Wright, Q. F, III: 2
Yerkes, R. M. F, IV: 1
Young, B. L. F, III: 4
Zeleny, J. F, I: 2
Zinsser, H. F, II: 4

STATUTES AND STANDING VOTES.

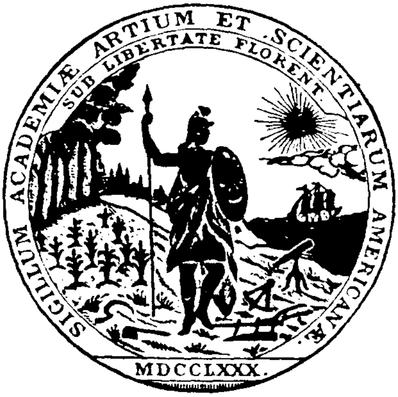
STATUTES.

Adopted November 8, 1911: *amended May* 8, 1912, *January* 8, *and May* 14, 1913, *April* 14, 1915, *April* 12, 1916, *April* 10, 1918, *May* 14, 1919, *February* 8, *April* 12, *and December* 13, 1922, *February* 14, *March* 14, *and October* 10, 1923, *March* 10, 1926, *May* 9, 1928, *April* 8, 1931, *and November* 11, 1931.

CHAPTER I.

THE CORPORATE SEAL.

ARTICLE 1. The Corporate Seal of the Academy shall be as here depicted:


SIGILLUM ACADEMIÆ ARTIUM ET SCIENTIARUM AMERICANÆ
SUB LIBERTATE FLORENT
MDCCLXXX.

ARTICLE 2. The Recording Secretary shall have the custody of the Corporate Seal.

See Chap. v, art. 3; chap. vi, art. 2.

CHAPTER II.

Fellows and Foreign Honorary Members and Dues.

Article 1. The Academy consists of Fellows, who are either citizens or residents of the United States of America, and Foreign Honorary Members. They are arranged in four Classes, according to the Arts and Sciences in which they are severally proficient, and each Class is divided into four Sections, namely:

Class I. *The Mathematical and Physical Sciences*

Section 1. Mathematics and Astronomy
Section 2. Physics
Section 3. Chemistry
Section 4. Technology and Engineering

Class II. *The Natural and Physiological Sciences*

Section 1. Geology, Mineralogy, and Physics of the Globe
Section 2. Botany
Section 3. Zoölogy and Physiology
Section 4. Medicine and Surgery

Class III. *The Social Arts*

Section 1. Jurisprudence
Section 2. Government, International Law, and Diplomacy
Section 3. Economics and Sociology
Section 4. Administration and Affairs

Class IV. *The Humanities*

Section 1. Theology, Philosophy, and Psychology
Section 2. History, Archæology, and Anthropology
Section 3. Philology
Section 4. The Fine Arts and Belles Lettres

Article 2. The number of Fellows shall not exceed Eight hundred, of whom not more than Six hundred shall be residents of Massachusetts, nor shall there be more than Two hundred and ten in any one Class.

Article 3. The number of Foreign Honorary Members shall not exceed One hundred and thirty. They shall be chosen from among citizens of foreign countries most eminent for their discoveries and attainments in any of the Classes above enumerated. There shall not be more than Thirty-five in any one Class.

ARTICLE 4. If any person, after being notified of his election as Fellow, shall neglect for six months to accept in writing, or, if a Fellow resident within fifty miles of Boston shall neglect to pay his Admission Fee, his election shall be void; and if any Fellow resident within fifty miles of Boston shall neglect to pay his Annual Dues for six months after they are due, provided his attention shall have been called to this Article of the Statutes in the meantime, he shall cease to be a Fellow; but the Council may suspend the provisions of this Article for a reasonable time.

With the previous consent of the Council, the Treasurer may dispense (*sub silentio*) with the payment of the Admission Fee or of the Annual Dues or both whenever he shall deem it advisable. In the case of officers of the Army or Navy who are out of the Commonwealth on duty, payment of the Annual Dues may be waived during such absence if continued during the whole financial year and if notification of such expected absence be sent to the Treasurer. Upon similar notification to the Treasurer, similar exemption may be accorded to Fellows subject to Annual Dues, who may temporarily remove their residence for at least two years to a place more than fifty miles from Boston.

If any person elected a Foreign Honorary Member shall neglect for six months after being notified of his election to accept in writing, his election shall be void.

See Chap. vii, art. 2.

ARTICLE 5. Every Fellow resident within fifty miles of Boston hereafter elected shall pay an Admission Fee of Ten dollars, unless previously as an Associate he has paid an Admission Fee of like amount.

Every Fellow resident within fifty miles of Boston shall, and others may, pay such Annual Dues, not exceeding Fifteen dollars, as shall be voted by the Academy at each Annual Meeting, when they shall become due; but any Fellow shall be exempt from the annual payment if, at any time after his admission, he shall pay into the treasury Two hundred dollars in addition to his previous payments. Any

Fellow shall also be exempt from Annual Dues who has paid such dues for forty years, or, having attained the age of seventy-five has paid dues for twenty-five years.

All Commutations of the Annual Dues shall be and remain permanently founded, the interest only to be used for current expenses.

Any fellow not previously subject to Annual Dues who takes up his residence within fifty miles of Boston, shall pay to the Treasurer within three months thereafter Annual Dues for the current year, failing which his Fellowship shall cease; but the Council may suspend the provisions of this Article for a reasonable time.

Only Fellows who pay Annual Dues or have commuted them may hold office in the Academy or serve on the Standing Committees or vote at meetings.

ARTICLE 6. Fellows who pay or have commuted the Annual Dues and Foreign Honorary Members shall be entitled to receive gratis one copy of all Publications of the Academy issued after their election.

See Chap. xi, art. 2.

ARTICLE 7. Diplomas signed by the President and the Vice-President of the Class to which the member belongs, and countersigned by the Secretaries, shall be given to Foreign Honorary Members and to Fellows on request.

ARTICLE 8. If, in the opinion of a majority of the entire Council, any Fellow or Foreign Honorary Member shall have rendered himself unworthy of a place in the Academy, the Council shall recommend to the Academy the termination of his membership; and if three-fourths of the Fellows present, out of a total attendance of not less than fifty at a Stated Meeting, or at a Special Meeting called for the purpose, shall adopt this recommendation, his name shall be stricken from the Roll.

See Chap. iii; chap.vi, art. 1; chap. x, art. 1, 7; chap. xi, art. 2.

CHAPTER III.

ELECTION OF FELLOWS AND FOREIGN HONORARY MEMBERS.

The procedure in the election of Fellows and Foreign Honorary Members shall be as follows:

Nominations to Fellowship or Foreign Honorary Membership in any Section must be signed by Two Fellows of that Section, or by three Fellows of any Sections, and sent to the Corresponding Secretary accompanied by a statement of the qualifications of the nominee and brief biographical data.

Notice shall be sent to every Fellow not later than the fifteenth of January in each year, reminding him that all nominations must be in the hands of the Corresponding Secretary before the fifteenth of February following.

A list of the nominees, giving a brief account of each, with the names of the nominators, shall be sent to every Fellow with a request that he return the list with such confidential comments and indications of preference as he may choose to make.

All the nominations, with any comments thereon and with expressions of preference on the part of the Fellows, shall be referred to the appropriate Class Committees, which shall canvass them, and report their recommendations in writing to the Council before the Stated Meeting of the Academy in April.

Elections of Fellows and Foreign Honorary Members shall be made by the Council before the Annual Meeting in May, and announced at that meeting.

Persons nominated in any year, but not elected, may be carried over to the list of nominees for the next year at the discretion of the Council, but shall not be further continued unless renominated.

See Chap. ii; chap. vi, art. 1; chap. x, art. 1.

CHAPTER IV.

OFFICERS.

ARTICLE 1. The Officers of the Academy shall be a President (who shall be Chairman of the Council), four Vice-Presidents (one from each Class), a Corresponding Secretary (who shall be Secretary of the Council), a Recording Secretary, a Treasurer, a Librarian, and an Editor, all of whom shall be elected by ballot at the Annual Meeting, and shall hold their respective offices for one year, and until others are duly chosen and installed.

There shall be also sixteen Councillors, one from each Section of each Class. At each Annual Meeting four Councillors, one from each Class, shall be elected by ballot to serve for the full term of four years and until others are duly chosen and installed. The same Fellow shall not be eligible for two successive terms.

The Councillors, with the other officers previously named, and the Chairman of the House Committee, *ex officio*, shall constitute the Council.

See Chap. xi, art. 1.

Article 2. If any officer be unable, through death, absence, or disability, to fulfill the duties of his office, or if he shall resign, his place may be filled by the Council in its discretion for any part or the whole of the unexpired term.

Article 3. At the Stated Meeting in March, the President shall appoint a Nominating Committee of Four Fellows having the right to vote, one from each Class. This Committee shall prepare a list of nominees for the several offices to be filled, and for the Standing Committees, and file it with the Recording Secretary not later than four weeks before the Annual Meeting.

See Chap. vi, art. 2.

Article 4. Independent nominations for any office, if signed by at least twenty Fellows having the right to vote, and received by the Recording Secretary not less than ten days before the Annual Meeting, shall be inserted in the call therefor, and shall be mailed to all the Fellows having the right to vote.

See Chap. vi, art. 2.

Article 5. The Recording Secretary shall prepare for use in voting at the Annual Meeting a ballot containing the names of all persons duly nominated for office.

CHAPTER V.

The President.

Article 1. The President, or in his absence the senior Vice-President present (seniority to be determined by length of continuous fellowship in the Academy), shall preside at all meetings of the Academy. In the absence of all these officers, a Chairman of the meeting shall be chosen by ballot.

Article 2. Unless otherwise ordered, all Committees which are not elected by ballot shall be appointed by the presiding officer.

Article 3. Any deed or writing to which the Corporate Seal is to be affixed, except leases of real estate, shall be executed in the name of the Academy by the President or, in the event of his death, absence, or inability, by one of the Vice-Presidents, when thereto duly authorized.

See Chap. ii, art. 7; chap. iv, art. 1, 3; chap. vi, art. 2; chap. vii, art. 1; chap. x, art. 6; chap. xi, art. 1, 2; chap. xii, art. 1.

CHAPTER VI.

The Secretaries.

Article 1. The Corresponding Secretary shall conduct the correspondence of the Academy and of the Council, recording or making an entry of all letters written in its name, and preserving for the files all official papers which may be received. At each meeting of the Council he shall present the communications addressed to the Academy which have been received since the previous meeting, and at the next meeting of the Academy he shall present such as the Council may determine.

He shall notify all persons who may be elected Fellows or Foreign Honorary Members, send to each a copy of the Statutes, and on their acceptance issue the proper Diploma. He shall also notify all meetings of the Council; and in case of the death, absence, or inability of the Recording Secretary he shall notify all meetings of the Academy.

Under the direction of the Council, he shall keep a List of the Fellows and Foreign Honorary Members, arranged in their several Classes and Sections. It shall be printed anually and issued as of the first day of July.

See Chap. ii, art. 7; chap. iii; chap. iv, art. 1; chap. x, art. 6; chap. xi, art. 1; chap. xii, art. 1.

Article 2. The Recording Secretary shall have the custody of the Charter, Corporate Seal, Archives, Statute-Book, Journals, and all literary papers belonging to the Academy.

Fellows borrowing such papers or documents shall receipt for them to their custodian.

The Recording Secretary shall attend the meetings of the Academy and keep a faithful record of the proceedings with the names of the Fellows present; and after each meeting is duly opened, he shall read the record of the preceding meeting.

He shall notify the meetings of the Academy to each Fellow and by mail at least seven days beforehand, and in his discretion may also cause the meetings to be advertised; he shall apprise Officers and Committees of their election or appointment, and inform the Treasurer of appropriations of money voted by the Academy.

After all elections, he shall insert in the Records the names of the Fellows by whom the successful nominees were proposed.

He shall send the Report of the Nominating Committee in print to every Fellow having the right to vote at least three weeks before the Annual Meeting.

See Chap. iv, art. 3.

In the absence of the President and of the Vice-Presidents he shall, if present, call the meeting to order, and preside until a Chairman is chosen.

See Chap. 1; chap. ii, art. 7; chap. iv, art. 3, 4, 5; chap. x, art. 6; chap. xi, art. 1, 2; chap. xii, art. 1, 3.

ARTICLE 3. The Secretaries, with the Editor, shall have authority to publish such of the records of the meetings of the Academy as may seem to them likely to promote its interests.

CHAPTER VII.

THE TREASURER AND THE TREASURY.

ARTICLE 1. The Treasurer shall collect all money due or payable to the Academy, and all gifts and bequests made to it. He shall pay all bills due by the Academy, when approved by the proper officers, except those of the Treasurer's office, which may be paid without such approval; in the name of the Academy he shall sign all leases of real estate; and, with the written consent of a member of the Committee on Finance, he shall make all transfers of stocks, bonds, and other securities belonging to the Academy, all of which shall be in his official custody.

He shall keep a faithful account of all receipts and expenditures, submit his accounts annually to the Auditing Committee, and render them at the expiration of his term of office, or whenever required to do so by the Academy or the Council.

He shall keep separate accounts of the income of the Rumford Fund, and of all other special Funds, and of the Appropriation thereof, and render them annually.

His accounts shall always be open to the inspection of the Council.

ARTICLE 2. He shall report annually to the Council at its March meeting on the expected income of the various Funds and from all other sources during the ensuing financial year. He shall also report the names of all Fellows who may be then delinquent in the payment of their Annual Dues.

ARTICLE 3. He shall give such security for the trust reposed in him as the Academy may require.

ARTICLE 4. With the approval of a majority of the Committee on Finance, he may appoint an Assistant Treasurer to perform his duties, for whose acts, as such assistant, he shall be responsible; or, with like approval and responsibility, he may employ any Trust Company doing business in Boston as his agent for the same purpose, the compensation of such Assistant Treasurer or agent to be fixed by the Committee on Finance and paid from the Funds of the Academy.

ARTICLE 5. At the Annual Meeting he shall report in print all his official doings for the preceding year, stating the amount and condition of all the property of the Academy entrusted to him, and the character of the investments.

ARTICLE 6. The Financial Year of the Academy shall begin with the first day of April.

ARTICLE 7. No person or committee shall incur any debt or liability in the name of the Academy, unless in accordance with a previous vote and appropriation therefor by the Academy or the Council, or sell or otherwise dispose of any property of the Academy, except cash or invested funds, without previous consent and approval of the Council.

See Chap. ii, art. 4, 5; chap. vi, art. 2; chap. x, art. 6; chap. xi, art. 1, 2, 3; chap. xii, art. 1.

CHAPTER VIII.

The Librarian and the Library.

Article 1. The Librarian shall have charge of the printed books, keep a correct catalogue thereof, and provide for their delivery from the Library.

At the Annual Meeting, as Chairman of the Committee on the Library, he shall make a Report on its condition.

Article 2. In conjunction with the Committee on the Library he shall have authority to expend such sums as may be appropriated by the Academy for the purchase of books, periodicals, etc., and for defraying other necessary expenses connected with the Library.

Article 3. All books procured from the income of the Rumford Fund or of other special Funds shall contain a book-plate expressing the fact.

Article 4. Books taken from the Library shall be receipted for to the Librarian or his assistant.

Article 5. Books shall be returned in good order, regard being had to necessary wear with good usage. If any book shall be lost or injured, the Fellow to whom it stands charged shall replace it by a new volume or by a new set, if it belongs to a set, or pay the current price thereof to the Librarian, whereupon the remainder of the set, if any, shall be delivered to the Fellow so paying, unless such remainder be valuable by reason of association.

Article 6. All books shall be returned to the Library for examination at least one week before the Annual Meeting.

Article 7. The Librarian shall have the custody of the Publications of the Academy. With the advice and consent of the President, he may effect exchanges with other associations.

See Chap. ii, art. 6; chap. xi, art. 1, 2.

CHAPTER IX.

The Editor and the Publications.

Article 1. The Editor shall have charge of the conduct through the press of the Proceedings and the Memoirs, and all correspondence relative thereto, and shall have power to fix the price at which individual numbers of the Proceedings and Memoirs are sold.

ARTICLE 2. In conjunction with the Committee of Publication, he shall have authority to expend such sums as may be appropriated by the Academy for printing the publications and for defraying other expenses therewith connected.

ARTICLE 3. All publications which are financed in whole or in part from the income of the Rumford Fund or from the income of other special funds, and all publications of work done with the aid of the Rumford Fund or other special funds, shall contain a conspicuous statement of this fact.

ARTICLE 4. Two hundred extra copies of each paper printed in the Proceedings or Memoirs shall be placed at the disposal of the author without charge.

If, on account of the number of communications offered for publication, it shall be necessary to decline for publication communications otherwise acceptable, members of the Academy shall be given preference in each of the several Classes over non-members; but whenever it shall be necessary to exercise this preference, the Editor shall inform the Council of the fact.

See Chap. iv, art. 1; chap. vi, art. 3; chap. x, art. 6; chap. xi, art. 4.

CHAPTER X.

THE COUNCIL.

ARTICLE 1. The Council shall exercise a discreet supervision over all nominations and elections to membership, and in general supervise all the affairs of the Academy not explicitly reserved to the Academy as a whole or entrusted by it or by the Statutes to standing or special committees.

It shall consider all nominations duly sent to it by any Class Committee, and act upon them in accordance with the provisions of Chapter III.

With the consent of the Fellow interested, it shall have power to make transfers between the several Sections, reporting its action to the Academy.

See Chap. iii, art. 2, 3; chap. xi, art. 1.

ARTICLE 2. Nine members shall constitute a quorum.

ARTICLE 3. It shall establish rules and regulations for the transaction of its business, and provide all printed and engraved blanks and books of record.

ARTICLE 4. It shall act upon all resignations of officers, and all resignations and forfeitures of Fellowship; and cause the Statutes to be faithfully executed.

It shall appoint all agents and subordinates not otherwise provided for by the Statutes, prescribe their duties, and fix their compensation. They shall hold their respective positions during the pleasure of the Council.

ARTICLE 5. It may appoint, for terms not exceeding one year, and prescribe the functions of, such committees of its number, or of the Fellows of the Academy, as it may deem expedient, to facilitate the administration of the affairs of the Academy or to promote its interests.

ARTICLE 6. At its March meeting it shall receive reports from the President, the Secretaries, the Treasurer, and the Standing Committees, on the appropriations severally needed for the ensuing financial year. At the same meeting the Treasurer shall report on the expected income of the various Funds and from all other sources during the same year.

A report from the Council shall be submitted to the Academy, for action, at the March meeting, recommending the appropriation which in the opinion of the Council should be made.

On the recommendation of the Council, special appropriations may be made at any Stated Meeting of the Academy, or at a Special Meeting called for the purpose.

See Chap. xi, art. 3.

ARTICLE 7. After the death of a Fellow or Foreign Honorary Member, it shall appoint a member of the Academy to prepare a biographical notice for publication in the Proceedings.

ARTICLE 8. It shall report at every meeting of the Academy such business as it may deem advisable to present.

See Chap. ii, art. 4, 5, 8; chap. iv, art. 1, 2; chap. vi, art. 1; chap. vii, art. 1; chap. xii, art. 1, 4.

CHAPTER XI.

STANDING COMMITTEES.

ARTICLE 1. The Class Committee of each Class shall consist of the Vice-President, who shall be chairman, and the four Councillors of the Class, together with such other officer or officers annually elected as may belong to the Class. It shall consider nominations to Fellowship in its own Class, and report in writing to the Council such as may receive at a Class Committee Meeting a majority of the votes cast, provided at least three shall have been in the affirmative.

See Chap. iii, art. 2.

ARTICLE 2. At the Annual Meeting the following Standing Committees shall be elected by ballot to serve for the ensuing year:

(i) *The Committee on Finance*, to consist of four Fellows, who, through the Treasurer, shall have full control and management of the funds and trusts of the Academy, with the power of investing the funds and changing the investments thereof in their discretion.

See Chap. iv, art. 3; chap. vii, art. 1, 4; chap. x, art. 6.

(ii) *The Rumford Committee*, to consist of seven Fellows, who shall report to the Academy on all applications and claims for the Rumford Premium. It alone shall authorize the purchase of books, publications and apparatus at the charge of the income from the Rumford Fund, and generally shall see to the proper execution of the trust.

See Chap. iv, art. 3; chap. x, art. 6.

(iii) *The Cyrus Moors Warren Committee*, to consist of seven Fellows, who shall consider all applications for appropriations from the income of the Cyrus Moors Warren Fund, and generally shall see to the proper execution of the trust.

See Chap. iv, art. 3; chap. x, art. 6.

(iv) *The Committee of Publication*, to consist of the Editor, *ex officio*, as Chairman, and four other Fellows, one from each Class, to whom all communications submitted to the Academy for publication shall be referred, and to whom the printing of the Proceedings and the Memoirs shall be entrusted.

It shall fix the price at which volumes of the publications shall be sold; but Fellows may be supplied at half price with volumes which may be needed to complete their sets, but which they are not entitled to receive gratis.

It shall determine when the pressure of material offered for publication makes it necessary to give preference to members of the Academy as compared with non-members, or to give priority to certain members as compared with others, and to what extent this preference or priority shall be applied in each of the four Classes, to the end that a proper balance of the facilities of publication with respect to subject matter and authors may be maintained.

See Chap. iv, art. 3; chap. vi, art. 1, 3; chap. ix, chap. x, art. 6.

(v) *The Committee on the Library,* to consist of the Librarian, *ex officio,* as Chairman, and four other Fellows, one from each Class, who shall examine the Library and make an annual report on its condition and management.

See Chap. iv, art. 3; chap. viii, art. 1, 2; chap. x, art. 6.

(vi) *The House Committee,* to consist of four Fellows, who shall have charge of all expenses connected with the House, including the general expenses of the Academy not specifically assigned to the care of other Committees or Officers.

See Chap. iv, art. 1, 3; chap. x, art. 6.

(vii) *The Committee on Meetings,* to consist of the President, the Recording Secretary, and four other Fellows, who shall have charge of plans for meetings of the Academy.

See Chap. iv, art. 3; chap. x, art. 6.

(viii) *The Auditing Committee,* to consist of two Fellows, who shall audit the accounts of the Treasurer, with power to employ an expert and to approve his bill.

See Chap. iv, art. 3; chap. vii, art. 1; chap. x, art. 6.

ARTICLE 3. The Standing Committees shall report annually to the Council in March on the appropriations severally needed for the ensuing financial year; and all bills incurred on account of these Committees, within the limits of the several appropriations made by the Academy, shall be approved by their respective Chairmen.

In the absence of the Chairman of any Committee, bills may be approved by any member of the Committee whom he shall designate for the purpose.

See Chap. vii, art. 1, 7; chap. x, art. 6.

CHAPTER XII.

MEETINGS, COMMUNICATIONS, AND AMENDMENTS.

ARTICLE 1. There shall be annually eight Stated Meetings of the Academy, namely, on the second Wednesday of October, November, December, January, February, March, April, and May. Only at these meetings, or at adjournments thereof regularly notified, or at Special Meetings called for the purpose, shall appropriations of money be made or amendments of the Statutes or Standing Votes be effected.

The Stated Meeting in May shall be the Annual Meeting of the Corporation.

Special Meetings shall be called by either of the Secretaries at the request of the President, of a Vice-President, of the Council, or of ten Fellows having the right to vote; and notifications thereof shall state the purpose for which the meeting is called.

A meeting for receiving and discussing literary or scientific communications may be held on the fourth Wednesday of each month, excepting July, August, and September; but no business shall be transacted at said meetings.

ARTICLE 2. Twenty-five Fellows having the right to vote shall constitute a quorum for the transaction of business at Stated or Special Meetings. Eighteen Fellows shall be sufficient to constitute a meeting for literary or scientific communications and discussions.

ARTICLE 3. Upon the request of the presiding officer or the Recording Secretary, any motion or resolution offered at any meeting shall be submitted in writing.

ARTICLE 4. No report of any paper presented at a meeting of the Academy shall be published by any Fellow without the consent of the author; and no report shall in any case be published by any Fellow in a newspaper as an account of the proceedings of the Academy without the previous consent and approval of the Council. The Council, in its discretion, by a duly recorded vote, may delegate its authority in this regard to one or more of its members.

ARTICLE 5. No Fellow shall introduce a guest at any meeting of the Academy until after the business has been transacted, and especially until after the result of the balloting upon nominations has been declared.

Article 6. The Academy shall not express its judgment on literary or scientific memoirs or performances submitted to it, or included in its Publications.

Article 7. All proposed Amendments of the Statutes shall be referred to a committee, and on its report, at a subsequent Stated Meeting or at a Special Meeting called for the purpose, two-thirds of the ballot cast, and not less than twenty-five, must be affirmative to effect enactment.

Article 8. Standing Votes may be passed, amended, or rescinded at a Stated Meeting, or at a Special Meeting called for the purpose, by a vote of two-thirds of the members present. They may be suspended by a unanimous vote.

See Chap. ii, art. 5, 8; chap. iii; chap. iv, art. 3, 4, 5; chap. v, art. 1; chap. vi, art. 1, 2; chap. x, art. 8.

STANDING VOTES.

1. Communications of which notice has been given to either of the Secretaries shall take precedence of those not so notified.

2. Fellows may take from the Library six volumes at any one time, and may retain them for three months, and no longer. Upon special application, and for adequate reasons assigned, the Librarian may permit a larger number of volumes, not exceeding twelve, to be drawn from the Library for a limited period.

3. Works published in numbers, when unbound, shall not be taken from the Hall of the Academy without the leave of the Librarian.

4. The Council, under such rules respecting nominations as it may prescribe, may elect as Associates of the Academy a limited number of men of mark in affairs or of distinguished service in the community.

Associates shall be entitled to the same privileges as Fellows, but shall not have the right to vote.

The admission fee and annual dues of Associates shall be the same as those of Fellows residing within fifty miles of Boston.

5. Communications offered for publication in the Proceedings or Memoirs of the Academy shall not be accepted for publication before the author shall have informed the Committee on Meetings of his readiness, either himself or through some agent, to use such time as the Committee may assign him at such meeting as may be convenient both to him and to the Committee, for the purpose of presenting to the Academy a general statement of the nature and significance of the results contained in his communication.

RUMFORD PREMIUM.

In conformity with the terms of the gift of Sir Benjamin Thompson, Count Rumford, of a certain Fund to the American Academy of Arts and Sciences, and with a decree of the Supreme Judicial Court of Massachusetts for carrying into effect the general charitable intent and purpose of Count Rumford, as expressed in his letter of gift, the Academy is empowered to make from the income of the Rumford Fund, as it now exists, at any Annual Meeting, an award of a gold and a silver medal, being together of the intrinsic value of three hundred dollars, as a Premium to the author of any important discovery or useful improvement in light or heat, which shall have been made and published by printing, or in any way made known to the public, in any part of the continent of America, or any of the American islands; preference always being given to such discoveries as, in the opinion of the Academy, shall tend most to promote the good of mankind; and, if the Academy sees fit, to add to such medals, as a further Premium for such discovery and improvement, a sum of money not exceeding three hundred dollars.

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  K. S. Lashley, 600.
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  W. G. Leland, 600.
  J. A. Lowell. 598.
  F. B. Lund, 600.
  W. W. McLaren, 599.
  Sayre Macneil, 598.
  Calvert Magruder, 599.
  J. M. Maguire, 599.
  K. L. Mark, 598.
  L. C. Marshall, 599.
  C. E. Merriam, 599.
  F. C. Mills, 599.
  O. L. Mills, 599.
  F. K. Morris, 598.
  Everett Morss, 600.
  H. G. Moulton, 599.
  A. D. Nock, 600.
  G. R. Noyes, 600.
  R. L. O'Brien, 600.
  W. H. O'Connell, 600.
  W. F. Ogburn, 599.
  Milman Parry, 600.
  A. S. Pease, 600.
  A. J. Peters, 600.
  William Phillips, 599.
  A. J. Philpott, 601.
  David Rapport, 598.
  A. L. Ripley, 600.
  Odin Roberts, 599.
  G. B. Roorbach, 599.
  L. S. Rowe, 599.
  F. B. Sayre, 599.
  W. C. Schumb, 598.
  W. A. Seavey, 599.
  H. R. Shepley, 601.
  H. E. Smith, 600.
  Payson Smith, 600.
  Carl Snyder, 599.
  C. Henry Taylor, 600.
  C. Holt Taylor, 600.
  William Thomson, 600.
  E. S. Thurston, 599.
  Eliot Wadsworth, 600.
  E. S. Webster, 600.
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  E. A. Whitman, 599.
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  Richard Wettstein, 579.
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  F. O. Bower, 601.
  A. L. Bowley, 601.
  Paul Claudel, 601.
  R. G. Hawtrey, 601.
  Hu Shih, 601.
  Pierre Janet, 601.
  Paul Mazon, 601.
  Kingo Miyabe, 601.
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